

## A HYBRID MULTI-CRITERIA DECISION METHOD FOR PERFORMANCE IMPROVEMENT OF BUILDING MANAGEMENT SYSTEMS

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*This paper aims to apply a new hybrid method of supplier selection to the integration of building management system for a residential assembly by solving multi-criteria decision problems to find optimal provider.*

*As it is based on Analytic Hierarchy Process (matrix calculation) and Data Envelopment Analytic Hierarchy Process (linear programming), the final results will be more accurate and useful in practice.*

*The research concludes that the first method is outperformed by the second one which uses optimal values through the entire algorithm and Analytic Hierarchy Process shall be used for lower value components (where a certain error can be accepted)*

*The novelty of this research lies in the application of the original hybrid approach to a real case, since a similar approach in Romania doesn't exist to modern buildings.*

**Keywords:** data envelopment, analytic hierarchy process, supplier selection, decision making, building management system, linear programming

### 1. Introduction

One of the most important goals of the modern world is the continuous improvement of commercial or residential assemblies in two aspects: service quality and operational optimization.

They are directly both responsible for reducing the level of pollution (buildings are one of the main causes) and cost (initial, operation and decommissioning) [1].

A supply chain is formed by the relations between suppliers, manufacturers, distributors, and customers. Each member of this connection has potential competitors in order to secure suitable supplies and deliveries [2], so various factors have been used as sub-criteria supplier selection: degree and level of automation, integration, compatibility, connection to internet, reliability, response time, modularity, future modernization, maintenance, remote control,

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lifetime and various costs. The application of these depends on the two main criteria: operational correctness and minimizing costs.

Till its end, this extended research is focused on developing a hybrid method to assist supplier selection in building industry. Once identified the decision problem then was carried its formulation by establishing selection criteria, sub-criteria and the identification of the five alternatives. The following steps are composed of application of integrated methods, interpretation of results and final conclusions. Input data were derived from the decision matrices corresponding weights of decision criteria and alternatives for each criterion. Output data consist of the final weights of alternatives that offer and their classification according to the percentage obtained by applying algorithms.

## 2. Analytic Hierarchy Process

Developed by Thomas Saaty [3] in the middle 80's, the **Analytic Hierarchy Process (AHP)** algorithm is widespread due to the ease of application, speed and the existence of a large number of software programs.

This method is based on organizing the problem - that is intended to be solved - in a hierarchical structure. Through this reduction, complex situations are reduced to understandable comparisons and classifications, leading to find the best solutions.

Hierarchical structure layout divides the problem in several different levels, each with a finite number of elements. The first level is the goal followed by the levels of criteria, sub-criteria and alternatives as shown in fig.1.

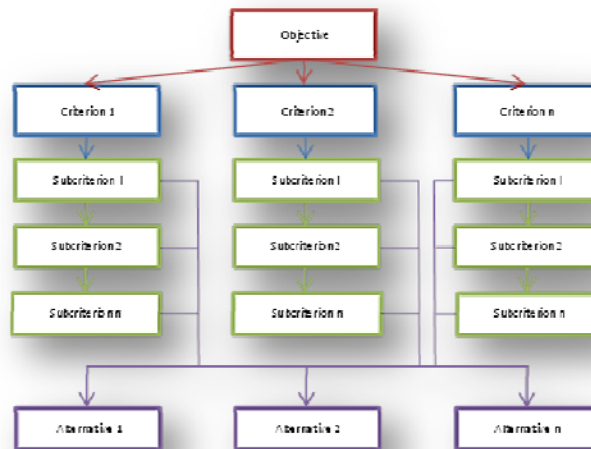


Fig. 1 The general form of a hierarchical structure for decision problems

To make a decision in an organized way to generate priorities we need to decompose the decision into the following steps:

Define the problem and determine the kind of knowledge needed.

Structure the decision hierarchy from the top with the goal of the decision, then the objectives, through the intermediate levels to the lowest level (which usually is a set of the alternatives).

Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.

The obtained priorities from the comparisons are used to weight the priorities in the level immediately below (for every element). Then for each element in the level below add its weighed values and obtain its overall or global priority.

The process of weighing and adding continues until the final priorities of the alternatives in the bottom most level are obtained.

In order to compute the weights for the different criteria, the AHP starts creating a pairwise comparison matrix  $\mathbf{A}$  which is an  $m \times m$  real matrix, where  $m$  is the number of evaluation criteria considered. Each entry  $a_{ij}$  of the matrix represents the importance of the  $i^{th}$  criterion relative to the  $j^{th}$  criterion. If  $a_{ij} > 1$ , then the  $i^{th}$  criterion is more important than the  $j^{th}$  criterion, while if  $a_{ij} < 1$ , then the  $i^{th}$  criterion is less important than the  $j^{th}$  criterion. If two criteria have the same importance, then  $a_{ij}$  is 1.

Step 1: computation of  $a_{ij}$  will be based on a conventional numeric scale which contains values from 1 to 9 as described on table 1:

Table 1

**Numeric scale used for  $a_{ij}$  computation**

$a_{ij}$	<i>Linguistic Approximation</i>
1	i element has the same importance as j
3	i element is somehow more important than j
5	i element is more important than j
7	i element is much more important than j
9	i element is extremely important than j

The phrases in the “Linguistic Approximation” column are only suggestive, and may be used to translate the decision maker’s qualitative evaluations of the relative importance between two criteria into numbers.

Step 2: contains the computation of normalized pairwise matrix  $\bar{A}$  which  $\bar{a}_{ij}$  elements are determined by the ration between  $a_{ij}$  element and the sum of the elements of the column that contains it as follows:

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_{i=1}^n a_{ij}} ; j \in (1 \dots n) \quad (1)$$

The criteria weight vector  $w_r$  (that is an  $m$ -dimensional column vector) is built by averaging the entries for each row of  $\bar{A}$ :

$$w_r = \frac{1}{m} \sum_{i=1}^n \bar{a}_{ij} ; r, m, j \in (1 \dots n) \quad (2)$$

Steps 1 and 2 will be repeated for each alternative, each criterion and for assessments between criteria and the result will be noted as  $w_{Ai}$ ,  $w_{Ci}$ .

Step 3 after the alternatives are compared with each other in terms of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the columns of the decision matrix (blue line in table 2) (not to be confused with the judgment matrices with the pairwise comparisons). The weights of importance of the criteria are also determined by using pairwise comparisons (red line on table 2). Therefore, if a problem has  $M$  alternatives and  $N$  criteria, then the decision maker is required to construct  $N$  judgment matrices (one for each criterion) of order  $M \times M$  and one judgment matrix of order  $N \times N$  (for the  $N$  criteria). Finally, given a decision matrix the final priorities (green column on table 2), denoted by  $W_i$ , of the alternatives in terms of all the criteria combined are determined according to the following formula (3).

Table 2

The general form of the power matrix and global weights column

The relative weights of the criteria					
<div><div></div><div><div><div><div><div><math>w_{C1}</math></div><div><math>w_{C2}</math></div><div><math>\cdots</math></div><div><math>w_{Ci}</math></div></div></div><div><div><div><math>w_{A1}</math></div><div><math>w_{A1}</math></div><div><math>\cdots</math></div><div><math>w_{A1}</math></div><div><math>W_1</math></div></div><div><div><div><math>w_{A2}</math></div><div><math>w_{A2}</math></div><div><math>\cdots</math></div><div><math>w_{A2}</math></div><div><math>W_2</math></div></div><div><div><div><math>\vdots</math></div><div><math>\vdots</math></div><div><math>\vdots</math></div><div><math>\vdots</math></div><div><math>\vdots</math></div></div><div><div><div><math>w_{Ai}</math></div><div><math>w_{Ai}</math></div><div><math>\cdots</math></div><div><math>w_{Ai}</math></div><div><math>W_n</math></div></div></div></div></div></div></div></div></div>					
The relative weights of alternatives for each criterion					Global weights column

Final overall weights which give the alternative scoring are determined by the formula [3]:

$$W_n = \sum_{i=1}^M w_{Ci} w_{Ai}, n \in (1 \dots N) \quad (3)$$

### 3. Data Envelopment Analysis

Developed by Charnes Cooper and Rhodes in 1978, **Data Envelopment Analysis (DEA)** is widely used to determine the efficiency and comparative analysis of **Decision Making Unit (DMU)** in order to maximize the outputs [4].

Let  $X_i$  be the vector of inputs into  $i^{\text{th}}$  DMU. Also let  $Y_i$  be the corresponding vector of outputs. Let  $X_0$  be inputs for a DMU that will be analyzed to determine its efficiency and  $Y_0$  be the outputs. Knowing that  $X$  and  $Y$  are determined data values, the following linear programming will measure the efficiency  $\max \theta$  with constrains system [5]:

$$\begin{aligned} \sum_{i=1}^n \lambda_i X_i &\leq \theta X_0 \\ \sum_{i=1}^n \lambda_i Y_i &\geq Y_0 \\ \lambda &\geq 0 \end{aligned} \quad (4)$$

$\lambda_i \rightarrow$  the weight given to DMU  $i$  to dominate DMU 0;

$\theta \rightarrow$  the efficiency of DMU 0.

The described algorithm is applied "n" times identifying the efficiency score for the DMU considered. Each of these will select the input and output that maximizes the final weight.

Starting with the usual definition of efficiency stated in (5) can be easily deducted that the maximum is 1 (the number of the resources used equals to the goods provided). In all other cases, DMU are considered inefficient.

$$efficiency = \frac{output\ value}{input\ value} \quad (5)$$

### 4. Data Envelopment Analytic Hierarchy Process

**Data Envelopment Analytic Hierarchy Process (DEAHP)** is a powerful tool for the analysis of efficiency proposed by Ramathan [5] who found similarities with AHP (table 3). Following them was discovered that both DEAHP and AHP can share initial hypotheses if the inputs and outputs will be considered as criteria for evaluating the effectiveness by minimizing the first and maximizing the last.

Considering that the primary point of departure the square matrix  $A$  defined by AHP's step 1, its components will be converted in DMU. In the last column of Table 3 DEAHP matrix will have "dummy" inputs equal to 1.

Converting the general form of (4) equation to DEAHP particular form will be obtained [10]:

$$\text{Max } Z = \sum_{i=1}^m b_{ih} y_{ih}, h \in (1, k)$$

Subject to:

$$\left\{ \begin{array}{l} \sum_{i=1}^n a_{ih} x_{ih} = 1, h \in (1, k) \\ \sum_{h=1}^k \left( \sum_{i=1}^m b_{ih} y_{ih} - \sum_{i=1}^n a_{ih} x_{ih} \right) \leq 0 \end{array} \right. \quad (6)$$

$Z$  – efficiency score function

$x_{ih}$  – observed value of input  $i$  for the DMU  $h$

$y_{ih}$  – observed value of output  $i$  for the DMU  $h$

$a_{ih}, b_{ih}$  – weight attached to inputs and outputs of DMU  $h$

$m$  – number of output variables

$n$  – number of input variables

$k$  – number of DMU

Table 3

Comparison of input data for AHP and DEAHP

	AHP			DEAHP				
	Crit 1	Crit 2	Crit n		Output 1	Output 2	Output n	Dummy Input
Alt 1	1	$a_{12}$	$a_{1n}$	DMU <sub>1</sub>	1	$a_{12}$	$a_{1n}$	1
Alt 2	$1/a_{12}$	1	$a_{2n}$	DMU <sub>2</sub>	$1/a_{12}$	1	$a_{2n}$	1
...	...	...	...	...	...	...	..	1
Alt m	$1/a_{m1}$	$1/a_{m2}$	1	DMU <sub>m</sub>	$1/a_{m1}$	$1/a_{m2}$	1	1

The function  $Z$  is considered in turn in each member left inequality constraints system components. The resulting values can be used in two ways: either as a partial share values or additional constraints necessary to calculate total weights. The final results of the new constraints will be placed in the system (6) and the results are what will decide ranking the most effective alternative.

## 5. Application of AHP and DEAHP in determination of suitable suppliers for smart buildings

The objective of this paper is to find a suitable supplier selection model for a construction company which aims to implement a building management system to a further complete integration of intelligent equipment.

Supplier selection is a multiple criteria decision-making (MCDM) problem which is affected by several conflicting factors [7].

Consequently, a purchasing manager must analyze the trade-off between the several criteria. MCDM techniques support the DMUs in evaluating a set of alternatives. Supplier selection problem has become one of the most important issues for establishing an effective supply chain system. The supplier selection problem in a supply chain system is a group decision according to multiple criteria from which a number of criteria have been considered for supplier selection in previous and present decision models.

Supplier selection methods are the models or approaches used to conduct the selection process. The methods chosen are extremely important to the overall selection process and can have a significant influence on the selection results. It is important to understand why a firm chooses one method (or a combination of different methods) over another. Several well-known selection methods have been developed and classified by numerous scholars over the years. Certain methods have been popular selection choices for years, while other methods have only emerged recently. Usually when a company sets out to develop or choose a supplier selection method, the result is a combination of several different methods with different strengths suited to meet the company's specific selection needs.

The main elements [8] considered are **operational correctness** and **cost reduction**, meaning "the most efficient and cheaper."

**Operational correctness**, defining the technical performance is determined by a number of features: *the degree and level of automation involved, BMS integration possibility, compatibility with existing standards, connection to INTERNET, reliability, response time, modularity, the possibility of further modernization, ease of maintenance and troubleshooting, remote control and monitoring, lifetime.*

**Minimizing costs** is the economic component of implementation and is characterized by: the initial cost of the investment, operating cost, decommissioning costs (where applicable).

The main steps in the model design are: *define* the criteria and sub-criteria for selection to achieve hierarchical structure, *compute* the criteria weights providers previously determined; *determine* the final scores for each supplier; *compare* the solutions obtained by AHP, respectively DEAHP [9].

**Defining the selection criteria to create hierarchical structure** has already been achieved by exposure characteristics of two main elements, and the resulting hierarchical structure is described in Figure 2.

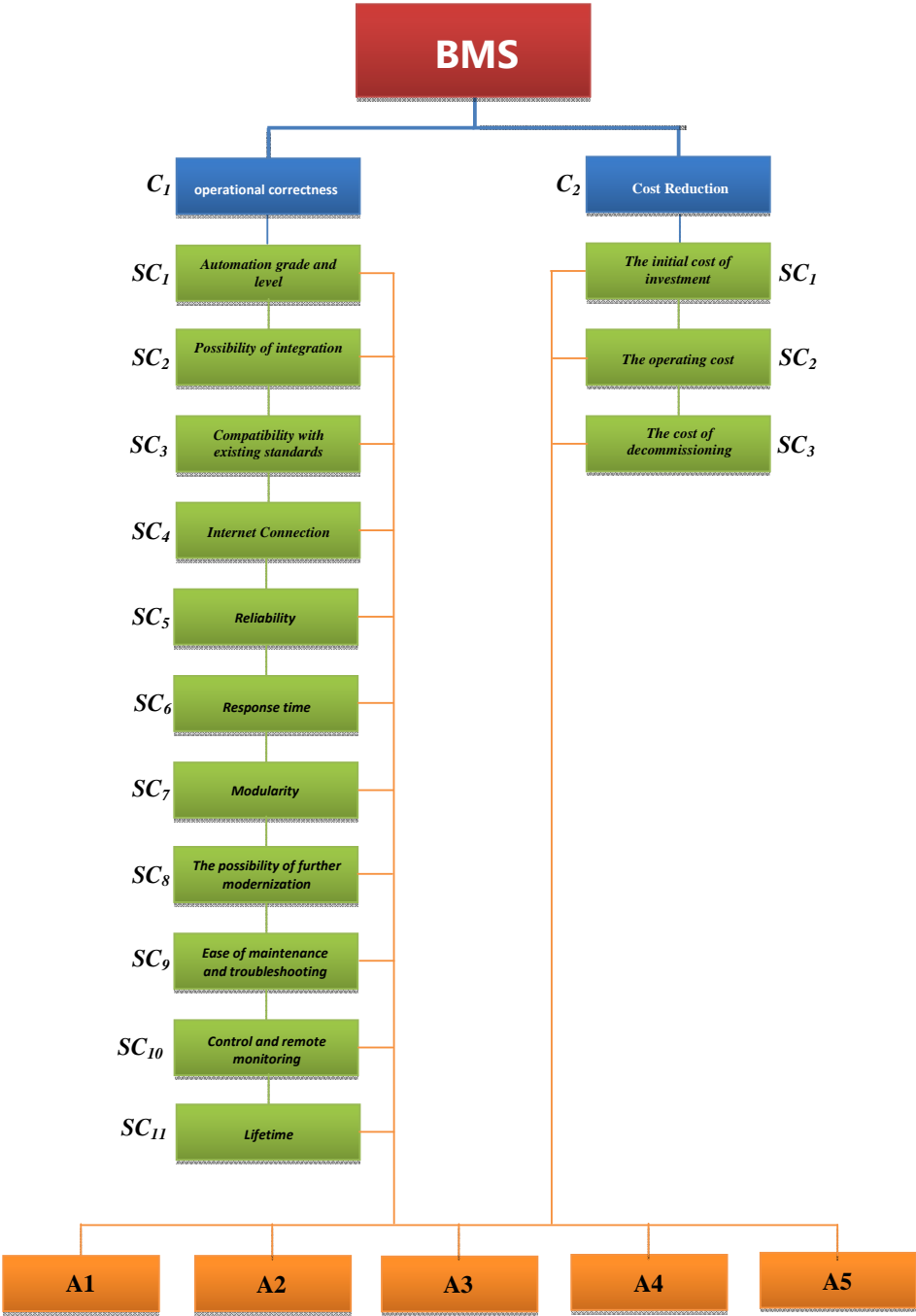


Fig. 2 Hierarchical selection provider for the Building Management System



### 5.1 Application of Analytic Hierarchy Process

Considering figure 2, the data needed to apply AHP algorithm are: the number of main criteria (2 main criteria), the number of sub criteria (11 for the first main criteria and 3 for the second) and the number of alternatives which is set arbitrary to 5 (for complex enough computations and quite various providers scenario).

A number of 19 matrices will be determined (14 for each sub criteria, 1 for establishing the weights between sub-criteria, 1 for the main criteria, 2 for each main criteria and one for the objective).

Because of the small space available for this paper, will be represented just the matrix of sub-criteria for operational correctness and its normalized values (Tables 4 and 5) together with the values of local weights for alternatives for each criteria (Table 6), final results for operational correctness (Table 7), cost reduction and goal (Table 8).

Table 4

Matrix of sub-criteria decision making for operational correctness

	$SC_1$	$SC_2$	$SC_3$	$SC_4$	$SC_5$	$SC_6$	$SC_7$	$SC_8$	$SC_9$	$SC_{10}$	$SC_{11}$
$SC_1$	1	1/2	7	3	8	3	4	9	7	4	2
$SC_2$	2	1	8	3	4	2	8	6	8	6	5
$SC_3$	1/7	1/8	1	1/4	2	1/3	1/2	3	2	1/2	1/2
$SC_4$	1/3	1/3	4	1	5	2	3	4	3	5	2
$SC_5$	1/8	1/4	1/2	1/5	1	1/3	1/4	5	3	1/3	1/2
$SC_6$	1/3	1/2	3	1/2	3	1	4	5	4	5	1/4
$SC_7$	1/4	1/8	2	1/3	4	1/4	1	2	2	1/2	1/6
$SC_8$	1/9	1/6	1/3	1/4	1/5	1/5	1/2	1	3	1/3	1/2
$SC_9$	1/7	1/8	1/2	1/3	1/3	1/4	1/2	1/3	1	1/2	1/6
$SC_{10}$	1/4	1/6	2	1/5	3	1/5	2	3	2	1	1/4
$SC_{11}$	1/2	1/5	2	1/2	1/5	2	4	6	2	4	1

Table 5

Normalized decision matrix and column of local weights for the sub-criteria

0,1927	0,1432	0,2308	0,3136	0,2603	0,2594	0,1441	0,2030	0,1892	0,1472	0,1622	20,42%
0,3855	0,2864	0,2637	0,3136	0,1302	0,1729	0,2883	0,1353	0,2162	0,2209	0,4054	25,62%
0,0275	0,0358	0,0330	0,0261	0,0651	0,0288	0,0180	0,0677	0,0541	0,0184	0,0405	3,77%
0,0642	0,0955	0,1319	0,1045	0,1627	0,1729	0,1081	0,0902	0,0811	0,1840	0,1622	12,34%
0,0241	0,0716	0,0165	0,0209	0,0325	0,0288	0,0090	0,1128	0,0811	0,0123	0,0405	4,09%
0,0642	0,1432	0,0989	0,0523	0,0976	0,0865	0,1441	0,1128	0,1081	0,1840	0,0203	10,11%
0,0482	0,0358	0,0659	0,0348	0,1302	0,0216	0,0360	0,0451	0,0541	0,0184	0,0135	4,58%
0,0214	0,0477	0,0110	0,0261	0,0065	0,0173	0,0180	0,0226	0,0811	0,0123	0,0405	2,77%
0,0275	0,0358	0,0165	0,0348	0,0108	0,0216	0,0180	0,0075	0,0270	0,0184	0,0135	2,11%
0,0482	0,0477	0,0659	0,0209	0,0976	0,0173	0,0721	0,0677	0,0541	0,0368	0,0203	4,99%
0,0964	0,0573	0,0659	0,0523	0,0065	0,1729	0,1441	0,1353	0,0541	0,1472	0,0811	9,21%

These two steps are repeated for comparisons between the five alternatives for each sub-criterion separately, resulting in local weights in Table 6:

Table 6

**The weights of the 5 local alternatives to 11 sub-function of correctness in operation**

	$SC_1$	$SC_2$	$SC_3$	$SC_4$	$SC_5$	$SC_6$	$SC_7$	$SC_8$	$SC_9$	$SC_{10}$	$SC_{11}$
A1	0,5206	0,4419	0,1436	0,2554	0,3020	0,3173	0,2018	0,2357	0,2287	0,3361	0,2857
A2	0,1376	0,1541	0,4866	0,4781	0,0525	0,0764	0,0445	0,0461	0,1576	0,1612	0,0847
A3	0,2085	0,2928	0,0967	0,0974	0,3976	0,3904	0,0779	0,1003	0,0779	0,0641	0,1318
A4	0,0734	0,0634	0,0608	0,0645	0,2041	0,1736	0,4899	0,4596	0,0413	0,0362	0,0616
A5	0,0599	0,0479	0,2122	0,1046	0,0437	0,0422	0,1859	0,1583	0,4945	0,4024	0,4361

Table 7

**Global weights of sub-criteria alternatives depending on operational accuracy**

$SC_1$	$SC_2$	$SC_3$	$SC_4$	$SC_5$	$SC_6$	$SC_7$	$SC_8$	$SC_9$	$SC_{10}$	$SC_{11}$	Final results
0,2042	0,2562	0,0377	0,1234	0,0409	0,1011	0,0458	0,0277	0,0211	0,0499	0,0921	

A1	0,5206	0,4419	0,1436	0,2554	0,3020	0,3173	0,2018	0,2357	0,2287	0,3361	0,2857	36,45%
A2	0,1376	0,1541	0,4866	0,4781	0,0525	0,0764	0,0445	0,0461	0,1576	0,1612	0,0847	17,73%
A3	0,2085	0,2928	0,0967	0,0974	0,3976	0,3904	0,0779	0,1003	0,0779	0,0641	0,1318	21,23%
A4	0,0734	0,0634	0,0608	0,0645	0,2041	0,1736	0,4899	0,4596	0,0413	0,0362	0,0616	11,09%
A5	0,0599	0,0479	0,2122	0,1046	0,0437	0,0422	0,1859	0,1583	0,4945	0,4024	0,4361	13,50%

Table 8

**Global weights of alternatives for cost reduction sub-criteria**

	$SC_1$	$SC_2$	$SC_3$		$C_1$	$C_2$	
	0,0698	0,7644	0,1659		0,875	0,125	
	The final weights of the alternatives based on cost reduction				The final weights of the supplier selection		Final results
A1	0,0685	0,1137	0,0818	10,53%	0,0551	0,0413	13,6%
A2	0,1432	0,0518	0,3665	11,04%	0,0628	0,1065	5,57%
A3	0,2934	0,4534	0,1201	38,69%	0,1344	0,1549	32,37%
A4	0,4603	0,3010	0,3859	32,62%	0,4179	0,4585	8,77%
A5	0,0346	0,0801	0,0457	7,12%	0,3298	0,2388	39,69%

AHP determines *supplier 5* as deemed most appropriate for the project. If equipment will be purchased from many sources, classification other providers: 3,1,4,2 is a landmark.

## 5.2 Applying Data Envelopment Analytic Hierarchy Process

Based on decision matrix outlined in Table 4, the objective function and constraints will be:

$$\begin{aligned}
 \text{Max } Z_1 &= x_1 + 0,5x_2 + 7x_3 + 3x_4 + 8x_5 + 3x_6 + 4x_7 + 9x_8 + 7x_9 + 4x_{10} + 2x_{11} \\
 \left\{ \begin{array}{l}
 x_1 + 0,5x_2 + 7x_3 + 3x_4 + 8x_5 + 3x_6 + 4x_7 + 9x_8 + 7x_9 + 4x_{10} + 2x_{11} < 1 \\
 2x_1 + x_2 + 8x_3 + 3x_4 + 4x_5 + 2x_6 + 8x_7 + 6x_8 + 8x_9 + 6x_{10} + 5x_{11} < 1 \\
 0,1429x_1 + 0,125x_2 + x_3 + 0,25x_4 + 2x_5 + 0,3333x_6 + 0,5x_7 + 3x_8 + 2x_9 + 0,5x_{10} + 0,5x_{11} < 1 \\
 0,3333x_1 + 0,3333x_1 + 4x_1 + 1x_1 + 5x_1 + 2x_1 + 3x_1 + 4x_1 + 3x_1 + 5x_1 + 2x_1 < 1 \\
 0,125x_1 + 0,25x_2 + 0,5x_3 + 0,2x_4 + x_5 + 0,3333x_6 + 0,25x_7 + 5x_8 + 3x_9 + 0,3333x_{10} + 0,5x_{11} < 1 \\
 0,3333x_1 + 0,5x_2 + 3x_3 + 0,5x_4 + 3x_5 + x_6 + 4x_7 + 5x_8 + 4x_9 + 5x_{10} + 0,25x_{11} < 1 \\
 0,25x_1 + 0,125x_2 + 2x_3 + 0,3333x_4 + 4x_5 + 0,25x_6 + x_7 + 2x_8 + 2x_9 + 0,5x_{10} + 0,1667x_{11} < 1 \\
 0,1111x_1 + 0,1667x_2 + 0,3333x_3 + 0,25x_4 + 0,2x_5 + 0,2x_6 + 0,5x_7 + x_8 + 3x_9 + 0,3333x_{10} + 0,5x_{11} < 1 \\
 0,1429x_1 + 0,1250x_2 + 0,5x_3 + 0,3333x_4 + 0,3333x_5 + 0,25x_6 + 0,5x_7 + 0,3333x_8 + x_9 + 0,5x_{10} + 0,1667x_{11} < 1 \\
 0,25x_1 + 0,1667x_2 + 2x_3 + 0,2x_4 + 3x_5 + 0,2x_6 + 2x_7 + 3x_8 + 2x_{10} + 0,25x_{11} < 1 \\
 0,5x_1 + 0,2x_2 + 2x_3 + 0,5x_4 + 0,2x_5 + 2x_6 + 4x_7 + 6x_8 + 2x_9 + 4x_{10} + x_{11} < 1
 \end{array} \right. \quad (7)
 \end{aligned}$$

The solution is calculated using the software LIPS (Linear Program Solver) resulting optimum value of 1 for the first sub-criterion. Similarly calculate the 10 remaining values (table 9):

Table 9

Optimal values of objective functions

	The optimum value of the objective function
<b>SC<sub>1</sub></b>	<b>1</b>
<b>SC<sub>2</sub></b>	<b>1</b>
<b>SC<sub>3</sub></b>	0,3333
<b>SC<sub>4</sub></b>	0,9375
<b>SC<sub>5</sub></b>	0,5556
<b>SC<sub>6</sub></b>	0,8333
<b>SC<sub>7</sub></b>	0,5
<b>SC<sub>8</sub></b>	0,375
<b>SC<sub>9</sub></b>	0,125
<b>SC<sub>10</sub></b>	0,4167
<b>SC<sub>11</sub></b>	0,8

Unlike the AHP, these values are not local weights; they are used to provide additional constraints to compute the final result. If (like can be seen on line one and two of the table 9) the first and second objective function has value 1, one of these two will be taken as reference.

It is believed that the first reference function and additional constraints will be:

$$F_1 = F_2 = 3F_3 = 1,067F_4 = 1,8F_5 = 1,2F_6 = 2F_7 = 2,6667F_8 = 8F_9 = 2,4F_{10} = 1,25F_{11} \quad (8)$$

$$\left\{ \begin{array}{l} F_1 - F_2 = 0 \\ F_1 - 3F_3 = 0 \\ F_1 - 1,067F_4 = 0 \\ F_1 - 1,8F_5 = 0 \\ F_1 - 1,2F_6 = 0 \\ F_1 - 2F_7 = 0 \\ F_1 - 2,6667F_8 = 0 \\ F_1 - 8F_9 = 0 \\ F_1 - 2,4F_{10} = 0 \\ F_1 - 1,25F_{11} = 0 \end{array} \right. \quad (9)$$

The same applies in the case of the five alternatives for each sub-criterion to yield the optimal values in Table 10:

Table 10

Optimal values of the objective functions for each sub-criterion

	$SC_1$	$SC_2$	$SC_3$	$SC_4$	$SC_5$	$SC_6$	$SC_7$	$SC_8$	$SC_9$	$SC_{10}$	$SC_{11}$
A1	1	1	0,6	0,1667	0,875	0,6667	0,6667	0,6667	0,8	0,5	1
A2	0,8	0,556	1	1	0,2667	1	0,125	0,1667	0,625	0,875	0,6667
A3	0,6	0,7143	0,6	0,6	1	0,5	0,25	0,5	0,25	0,25	0,5
A4	0,4	0,5286	0,2	0,2	0,8334	0,25	1	1	0,125	0,125	0,1667
A5	0,2	0,1667	0,8	0,4	0,125	0,125	0,75	0,6667	1	1	0,6667

The final step is the calculation of the objective function with restrictions derived from the table above and the addition of extra constraints defined by equations (9).

Table 11 contains the final weights of alternatives depending on operational correctness (first column) and costs reduction (second column) criteria.

Table 11

The final weights of alternatives depending on operational correctness criterion

	Final ranking		
	1 <sup>st</sup> criterion	2 <sup>nd</sup> criterion	Final results
A1	1	1	0,565
A2	0,92524	0,746	0,257
A3	0,765629	0,966	0,9375
A4	0,575702	0,37	0,2222
A5	0,56623	0,347	1

DEAHP determines **alternative 5** as the best for the considered project. In case of buying equipment from several providers, the rank is: 3,1,2,4.

## 6. Conclusions

Although both methods are based on powerful algorithms used multi-criteria decision-making problem solving, they have several limitations which underlines the need of a new hybrid method which can avoid them:

**AHP method** uses comparative matrix normalization in the calculation of partial weights and the mathematical apparatus is relatively easy to understand and apply, especially by using personal computers. In the meantime, the weakest point of this approach is the error magnitude (given by calculating approximations and human subjectivity) and the important alteration of results while adding/removing criteria or/and alternatives [10].

While AHP uses matrix calculus, **DEAHP method** uses individual linear models for each objective ensuring more accurate results. The main disadvantage of this method remains its complexity when it's applied to simpler multi-criteria problems.

Final results shown in Table 8 and 11 indicate alternatives in terms of choice of more than one provider; for example can be chosen alternative 5 for luxury apartments and alternative 3 for the comfort one.

The novelty of this work consists, first providing a framework for all range of multi-criteria decision problems (and can also be used to validate initial data), and second by its application to modern buildings, with implications in reducing time, costs, and to resolve any problems arising in their field of decision.

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## REFERENCES

- [1]. *Ronnen Levinson, Hashem Akbari*, "Potential benefits of cool roofs on commercial buildings: conserving energy, saving money, and reducing emission of greenhouse gases and air pollutants", *Energy Efficiency*, Vol. 3, pp. 53-109, 2010.
- [2]. *Weng T., Agarwal, Y.*, "From Buildings to Smart Buildings — Sensing and Actuation to Improve Energy Efficiency", *Design & Test of Computers*, Vol. 29 Issue 4, pp.36-44, 2012.
- [3]. *Saaty, T.L.* The Analytical Hierarchy Process, Mc Graw-Hill, New York, NY, 1981.
- [4]. *M. R. Moazami Goudarzi, M. Reshadi, M. R. Mahdian and M. Pourghasem* "A DEA Ranking Method based on Crossing Evaluation of Efficient DMUs", *Applied Mathematical Sciences*, Vol. 6, Issue 29, pp. 1409 - 1417, 2012.
- [5]. *Ramakrishnan Ramanathan*, "Data envelopment analysis for weight derivation and aggregation", *Computers & Operations Research*, Vol.33, pp. 1289-1307, 2006.

- [6]. *Golany B.*, „ An interactive MOLP procedure for the extension of DEA effectiveness analysis”, *J. Oper., Res.Soc.*, 39, pp. 725-734. 1986.
- [7]. *Farzad Tahriri, Mohammad Rasid Osman, Aidy Ali and Rosnah Mohd Yusuff*, A review of supplier selection methods in manufacturing industries, *Suranaree J. Sci. Technol.* 15(3), pp 201, 208, 2008.
- [8]. *Johnny Wong, Heng Li*,” Development of a conceptual model for the selection of intelligent building systems”, *Building and Environment*, Vol. 41, pp. 1106–1123, 2006
- [9]. *Lin, R.C., Sir, M.Y., & Pasupathy, K.S.*, ”Multi-objective simulation optimization using data envelopment analysis and genetic algorithm: Specific application to determining optimal resource levels in surgical services.” *The International Journal Management Science-Omega*, 41(5), 881–892, 2013.
- [10]. *Sevklı, Mehmet, Koh, S C Lenny, Zaim Selim, Demirbag Mehmet, Tatoglu Ekrem*,“ An application of data envelopment analytic hierarchy process for supplier selection: a case study of BEKO in Turkey”, *International Journal of Production Research*, pp. 1-34, 2009.