

## THE GOLDEN SECTION AND PRODUCT DESIGN

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*The paper presents the results of an experiment with several phases carried out with the main aim of testing the hypothesis of golden section's supreme beauty. The experiment participants assessed the pleasantness of different rectangles and also of rectangular boxes. During another phase, other participants drew the perfect rectangle according to their opinion. Other participants measured the rectangles present in a room from their homes in search for the golden section. Finally, statistical surveys were carried out in order to discover the golden section in smartphones' bodies and displays and cars' body.*

**Keywords:** golden section, perfect proportion, product design

### 1. Introduction

Since the beginnings, the practitioners and the theorists of art and design were preoccupied with finding concepts and rules that eventually will lead to attainment of masterpieces. One of these concepts was the golden ratio. Different other names are used interchangeably, like golden section or perfect proportion.

Mathematically, two positive quantities  $x$  and  $y$  are in golden ratio, if the following equation is true ( $x > y$ ) [1]:

$$\frac{x+y}{x} = \frac{x}{y} \quad (1)$$

The golden ratio is represented by Greek letter  $\varphi$  and its value was calculated. The golden ratio is an irrational number, similar to  $\pi$  [1, 2]:

$$\varphi = \frac{x}{y} = \frac{x+y}{x} = 1.618 \quad (2)$$

Analysing Fibonacci series, the researchers discovered a relationship with the golden ratio. It is known that the Fibonacci series, called also the series of natural growth, has the following formula [1]:

$$x_i = x_{i-2} + x_{i-1} \quad (3)$$

Basically, the numbers in the Fibonacci series are [1]:

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1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 ...

The limit of ratios of Fibonacci series was discovered to be number  $\varphi$  [1]:

$$\lim_{i \rightarrow \infty} \frac{x_i}{x_{i-1}} = \varphi \quad (4)$$

The golden ratio  $\varphi$  has some noteworthy properties. These properties are presented in dedicated literature [1, 2]. For example:

$$\varphi = \frac{1}{\varphi - 1} \quad (5)$$

$$\sum_{i=0}^n x_i = x_{i-2} + 1 \quad (6)$$

The Romanian mathematician and philosopher Matyla Ghika discovered other properties, for example the following [3]:

$$\varphi = \lim_{i \rightarrow \infty} \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}} \quad (7)$$

$$\frac{1}{\varphi} = 1 + \lim_{i \rightarrow \infty} \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \quad (8)$$

The rectangle that have dimensions ratio equal to  $\varphi$  is the “perfect” rectangle (Figure 1). There is a graphical methodology to draw the perfect rectangle starting with a square and a semicircle [4]. Allegedly, this methodology was developed by the ancient Greeks. There are claims that this rectangle is the most beautiful to the human eye.



Fig. 1. The “perfect” rectangle

The golden ratio and the Fibonacci series can be found in nature – in plants, animals and even in human body. This is the reason why the Fibonacci series is called the law of natural growth [1].

For some plants, the array of leaves around the stem describes a spiral based on golden ratio. Also, the pineapple fruit and pine cone display such spiral.

Similar spiral can be found in the shell of nautilus. And several other examples can be indicated. So, it is undeniable that the golden ratio is a feature of living world [1, 5].

As was mentioned before, the rectangle with the golden ratio between its sides is considered by many experts to be the most beautiful, so it was called the golden section or the perfect rectangle. Some of the experts wrote textbooks, tracing evidence of its presence in the ancient pyramids, Greek temples, Renaissance paintings, etc. [6].

Most of the experts were concerned just to find evidence of the presence of perfect rectangle in different masterpieces of art or graphic design [7]. In many cases, there are no proofs that the authors of the masterpieces deliberately used the golden section. But on the other hand, there are creators that used it and advocated its use. One of them was the famous architect and designer Le Corbusier [6]. Some artists are so fond of the golden section that they used it to elaborate graphic constructions. For example, Janusz Kapusta made geometric experiments with circles, squares and triangles arranged in sequences based on golden ratio which finally lead to the creation of several works of art [8].

Also, some experts claimed that they discovered the golden section in product aesthetics. For example, Mike Baxter took into consideration the front view of cars and indicated that the width and height of some models are in the golden ratio [9].

But other experts expressed doubts, even fierce critiques, to the idea that the golden section is the most beautiful rectangle. They have several objections, one of them being the unscientific method of discovery of golden section in masterpieces. Basically, the fans of golden section are not measuring directly the masterpieces, but instead are measuring a photo of the masterpiece. More over, the criteria for selection of the elements that will be measured are subjective. In Figure 2, how was determined the position of the long vertical line near the middle? Why the small archway in Figure 3 is measured precisely in height, but not in width?

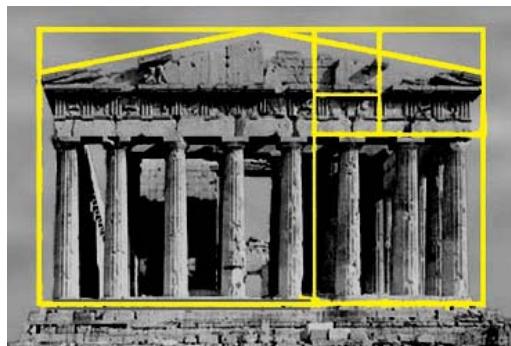


Fig. 2. Applying golden section to Athenian Parthenon [10]



Fig. 3. Applying golden section to Constantin's Arch in Rome [10]

The mathematician Underwood Dudley, specialised in crank mathematics, is against of the idea of the golden section being the most beautiful of rectangles. Reviewing a book on this subject, he provides strong arguments against many traditional claims in favour of the golden section. For example, the ancient Egyptians could not use the golden ratio in designing the pyramids as they did not know the irrational numbers, [11].

The Stanford academic Keith Devlin challenges the idea of golden ratio in an essay called significantly “The Myth That Will Not Go Away”. Among other arguments, he mentions that “a small (and not at all scientific) survey I once carried out myself revealed that all architects I asked knew about the golden ratio, and all believed that other architects used the golden ratio in their work, but none of them had ever used it themselves.” [12].

In an exhaustive article, Christopher Green analyses in depth the claims about the golden ratio. In the end, he did not reject the idea of the perfect proportion, but underlines that is a “fragile” idea [13].

The first scientific approach in researching the beauty of the golden section belongs to the German psychologist Gustav Theodor Fechner. He organised an experiment in which people were asked to select the most beautiful rectangle from a series. Only 33% of the 500 subjects had chosen the golden section as the most pleasing rectangle, which can not be considered a conclusive evidence [14].

In an experiment carried out later, the participants were required to slide two sheets of paper with rectangles drawn on them until they obtain the perfect rectangle according to their opinion. The results indicated that people whom acknowledged the idea of golden section and knew the value of  $\varphi$  were very precise in obtaining it [15].

Andrei Dumitrescu designed and carried out an experiment in order to rediscover the perfect proportion and to evaluate the parameters that might influence the perception of perfect proportion. There were used rectangles from cardboard of different ratios, different sizes, different colours and different connection radius. No perfect rectangle was used. The experiment's results indicated that [16]:

- the golden ratio was 1.618, and it resulted from interpolations;
- the golden ratio was perceived as just an acceptable proportion, because it obtained only a mean mark of 3.1 from the range of 1 to 5;
- parameters colour and size do not influence significantly the perception of perfect proportion;
- parameter ratio between connection radius and height influences the perception of perfect proportion in an inverse mode;
- the gender influence on perception was inconclusive.

Summarizing all the information presented above, there are three aspects to remember:

1. the golden ratio  $\varphi$  has remarkable and undeniable mathematical properties;
2. the golden ratio can be found in nature (plants, animals and humans);
3. the idea that the golden section is the most beautiful rectangle is challenged by several experts.

So, the researcher is tempted to search for answers to the following questions:

- Is the golden section the most beautiful?
- How is the golden section assessed in a three-dimensional space, considering that usually the evaluation is made in a two-dimensional space?
- In which degree is the golden section present in our environment and especially in our homes?



Fig. 4. Some of the rectangles used in experiment



Fig. 5. The rectangular boxes used in experiment

## 2. Experiment Design

After analysing the results and comments gathered during the previous experiment [16], it was noted that some participants complained that too many rectangles (225) were used in experiment, which was tiresome and exhausting, and they could not concentrate enough on the last rectangles. Also, it was thought that the mean as an indicator for the perfect proportion maybe was not the best choice.

So, a new experiment was designed. No data from the previous experiment was used. It was decided that the new experiment will have the following phases:

1. assessment of beauty of 27 rectangles;
2. assessment of beauty of 5 rectangular boxes;
3. drawing the perfect rectangle;
4. statistical survey of the presence of golden section in environment;
5. statistical survey of smartphones' dimensional ratios;
6. statistical survey of cars' dimensional ratios.

In the first phase of the experiment, 27 cardboard rectangles (varying in ratio, size, colour and connection radius – Figure 4) were given randomly to participants in series of one, so any comparison between rectangles was impossible. The variation of parameters was established using a fractional factorial design. The selected ratios of rectangle's sides were 1; 1.5; 2. Actually, no golden section was used in order to prevent participants in sensing that something is odd about this type of rectangle, biasing the assessment.

Each rectangle's beauty was assessed in isolation, according to the following scale:

- 1 – totally bad proportionate;
- 2 – bad proportionate;
- 3 – acceptable proportion;
- 4 – well proportionate;
- 5 – the perfect proportion.

In the second phase of the experiment, 5 rectangular boxes (varying only in sides' ratio – Figure 5) were assessed by participants from an aesthetic point of view. The sides ratio were: A (1:1:1); B (1:  $\varphi$ :  $\varphi$ ); C (1:3:4); D (1:  $\varphi$ :  $\varphi/2$ ); E (3:4:5). The same assessment scale as in first phase was used.

In the third phase of the experiment, the participants were asked to draw on a sheet of paper the perfect rectangle according to their opinion and taste. The participants were allowed to draw freehand or to use a ruler. Subsequent adjustments were permitted.

The fourth phase of the experiment consisted in counting and measuring of rectangles from a given environment. Actually, the participants selected a room

from their homes, identified all the rectangles present in the respective room, and measured them.

In the fifth phase of the experiment, series of smartphones from different manufacturers were selected and the dimensions of body and display were used in the quest for golden section.

In the sixth phase of the experiment, a series of cars from different manufacturers were selected and the overall dimensions of car body were used in search for golden section.

### 3. Experimental Results

The first phase of the experiment was carried out with 675 participants (381 female and 294 male participants). All participants were students (age: 20 – 23 years) at a large technical university. The experiment was supervised by the authors of the present paper.

The results were recorded in a spreadsheet and statistical calculations were carried out. There were calculated the means, the variances etc. The Cronbach Alpha coefficient was 0.77, so the results are relevant.

The percentage of mark “5”, amongst the total number of marks given to every single rectangle, was calculated. In Table 1, there are indicated the overall means, the percentages of mark “5” and difference between the means of marks given by female versus male participants.

It should be noted that the highest value for mean is 3.11 corresponding to the ratio of 1.5 (closest to the golden ratio), but the highest value for percentage of “5”s is 13.05% corresponding to the ratio of 1.

Table 1

Overall results of phase 1 (rectangles)

	Ratio	1	1.5	2
Mean	2.95	<b>3.11</b>	2.82	
Mark "5"	<b>13.05%</b>	8.79%	6.78%	
Difference between female and male assessment	0.13	0.09	0.09	

The value of 3.11 is relatively modest, considering that the assessment interval was 1 to 5. Taking into account the data displayed in Table 1, it appears that “perfect” (linked to mark “5”) is more likely to be associated with the square (ratio = 1) than to the rectangle closest to the golden section. Another important observation is that there is no significant difference between the way female and male participants assessed the rectangles.

In Table 2, the percentages of mark “5” are presented, as deployed on the three colours used in experiment: white, yellow and light blue. Again, the square (ratio = 1) scored better than the other rectangles. As the figures clearly indicated,

there are no significant differences between the colours, so colour is not a parameter to influence the perception of rectangles.

Table 2

**Results of phase 1 deployed on colours**

Ratio	Mark "5"		
	1	1.5	2
White	13.19%	8.89%	5.83%
Yellow	12.74%	8.44%	6.62%
Light blue	13.23%	9.04%	7.90%

In Table 3, there are presented the percentages of mark "5" deployed on the three connection radius used in the experiment. Because the rectangles were of different sizes, the ratio between connection radius and height was used as a parameter. As in the previous case, the square (ratio = 1) scored better than the other rectangles. It can be easily observed that as the relative radius increases as the scores decreases. So, the ratio between connection radius and height is an important parameter that influences the perception of rectangles.

Table 3

**Results of phase 1 deployed on ratio between connection radius and height**

Ratio	Mark "5"		
	1	1.5	2
0	19.60%	16.59%	12.54%
0.15	13.19%	5.98%	5.04%
0.3	6.37%	3.80%	2.77%

The second phase of the experiment was carried out with 723 participants (401 female and 322 male participants). All participants were students (age: 20 – 22 years) at a large technical university. The experiment was supervised by the authors of the present paper. The Cronbach Alpha coefficient was 0.83, so the results are relevant.

In Table 4, there are indicated the rectangular boxes' means, the percentages of mark "5", the total number of marks "5" and the difference between the mean of marks given by female versus male participants.

Table 4

**Results of phase 2 (rectangular boxes)**

	A (1:1:1)	B (1: $\varphi$ : $\varphi$ )	C (1:3:4)	D (1: $\varphi$ : $\varphi/2$ )	E (3:4:5)
Mean	3.98	3.29	2.48	3.05	3.29
Total marks "5"	357	57	32	97	64
Mark "5"	49.38%	7.88%	4.43%	13.42%	8.85%
Difference between female and male assessment	0.03	0.19	0.01	0.08	0.14

As the results displayed in Table 4 evidently indicates, the cube (1:1:1) was the absolute winner no matter the criteria used (mean or percentage of mark “5”). Almost half of the participants considered that the cube is the most well-proportionate geometrical body. Even the mean of cube was considerably higher and close to the maximum of 5 than the mean of the rectangle with 1.5 ratio (see Table 1). Another important remark is that there is no real difference between the way female and male participants assessed the rectangular boxes.

The third phase of the experiment was carried out with 63 participants (34 female and 29 male participants). All participants were students (age: 20 – 22 years) at a large technical university. The experiment was supervised by the authors of the present paper.

All the rectangles drawn on paper were measured with a ruler. If the sides were not parallel, the greater side was measured. Subsequently, the ratio of sides was calculated and compared with the golden ratio. It was considered an interval of acceptable variation of 95% – 105%. So, if the following mathematical relation was fulfilled (where  $r$  is the sides’ ratio), the rectangle was considered a golden section:

$$1.54 < r < 1.7 \quad (9)$$

*Table 5*

Results of phase 3 (drawn rectangles)	
	Participants that drew a golden section
Female	18.18%
Male	24.14%
Total	20.63%

In Table 5, there are indicated the results of phase 3. Only 1 in 5 participants drew a rectangle which was a golden section or nearby in an interval of 10%. The result is disappointing and against the hypothesis of golden section being the most beautiful rectangle. It should not be forgotten that the participants were allowed to alter their designs if they were not content with the first drawing. Interestingly, male participants were more inclined to draw the golden section than the female participants.

The fourth phase of the experiment was carried out with the help of 7 master students working for extra credits. They were living in university campus, in rented apartments or with parents. They chose a room in their home (bedroom, sitting room and even a kitchen). They produced photographs of the room (as proofs) and documents with tables containing the name of the product holder of rectangles and the dimensions of the rectangles.

The ratio of sides was calculated for each rectangle. If the ratio fulfilled relation (9), the rectangle was considered a golden section. The results of the fourth phase are displayed in Table 6.

Table 6

**Results of phase 4 (rectangles in homes)**

Room	Golden sections	Total rectangles	%
01	3	14	21.43
02	4	16	25.00
03	2	13	15.38
04	1	10	10.00
05	2	15	13.33
06	3	23	13.04
07	4	19	21.05
<b>Total</b>	<b>19</b>	<b>110</b>	<b>17.27</b>

The golden section was found on the following products:

- nightstand;
- cabinet;
- carpet;
- sink;
- cooking stove;
- electric radiator;
- radio;
- photographic camera;
- TV set;
- laptop.

According to the results, the golden section was rather scarce (17.27%) in student rooms and it can be assumed that the situation is similar for an average room. Even the highest result (25%) has a low value. The experimental results do not indicate however the origin of this state. It is not clear if people did not purchase products with golden section or if designers and manufacturers did not include the golden section in their products.

Even the students' rooms had a lot of furniture pieces and of different types, only two types displayed golden section: the nightstand and the cabinet. The carpet is not relevant, because it can be easily manufactured in any ratio. Most of the products displaying a golden section are technical products and especially electric or electronic. The last two types of products in the list (TV set and laptop) are interesting because they include an electronic display and this aspect was investigated in the following phase of the experiment.

The fifth phase of the experiment consisted in selecting 40 smartphones from 10 different manufacturers. There were selected famous manufacturers (with up to 11 models), but also relatively unknown producers (with up to 3 models).

The dimensions of body and display were recorded from online catalogues. (However, for some models, the dimensions of display were not available.) The length/width ratios were calculated for each model. If the ratio fulfilled the relation (9), the body or display was considered a golden section. The results are presented in Table 7.

Table 7

Results of phase 5 (smartphones)		
Models	Golden section - displays	Golden section – bodies
40	15.15%	0%

The most important observation is that no golden section was discovered for the body of the smartphone. This situation is explained by the ergonomic requirements the smartphone has to fulfil. The percentage of golden section in the display case was close to the value found in phase four. So, again the golden section scored rather poorly. It appears that the smartphone manufacturers are not interested in the golden section.

The sixth phase of the experiment was focused on the overall dimensions of cars. Some experts [9] claimed that the car's width and height are in golden ratio. There were selected 54 car models from seven manufacturers with headquarters located in five different countries. The types ranged from hatchbacks to convertibles and grand tourers. The dimensions were recorded from an online catalogue. The length/width, length/height and width/height ratios were calculated for each model. If the ratio fulfilled relation (9), the inscribing rectangle was considered a golden section. The results are presented in Table 8.

The data shows that the experts that had seen golden sections in cars were wrong. No golden sections were discovered in the rectangles described by car's length and width and car's length and height, and only 2 (3.7%) when width and height were considered. It was the case of 2 out of 13 grand tourers and out of 54 cars. When examining the last lines of Table 8, it can be observed that central tendency and span interval do not include the golden section.

Table 8

Results of phase 6 (cars)			
	Length / Width	Length / Height	Width / Height
Number of golden sections	0	0	2 (3.7%)
<i>Indicators calculated without the models with golden sections</i>			
Mean	2.46	3.00	1.22
Min	2.16	2.31	0.97
Max	2.67	3.63	1.53

#### 4. Conclusions

The idea that the golden section is the most pleasant rectangle has a rather weak support from experimental data. When considering the number of highest

marks obtained during aesthetic assessment, the square scored better than the golden section. When different rectangles (including square and golden section) were embodied in three-dimensional geometric bodies, the cube was perceived as being more pleasant by far in comparison to bodies with golden section on sides. There were found few golden sections among the “perfect” rectangles drew by participants. Even when drawn and not when selected, the golden section scored poorly. The statistical survey of rooms from home revealed a rather scarce presence of the golden section. The survey of two types of products (smartphones and cars) indicated also a weak presence of the famous rectangle.

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