

## OPTIMAL DISTRIBUTED GENERATORS PLACEMENT IN A RADIAL DISTRIBUTION NETWORK

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*The objective of this work is to improve the operation of radial distribution networks by adding distributed generators. An algorithm is proposed for finding the optimal locations and sizes of distributed generators (DG) that aims to reduce the power losses and maintain a voltage profile within acceptable limits, based on a sensitivity factor. The main objective function of the optimization problem is the minimization of the total investment cost. Case studies were performed on a 38 bus distribution network.*

**Keywords:** Distributed Generation, Optimization, Investment cost

### 1. Introduction

The distributed generation provides significant technical and economical benefits for the distribution network operators. The best economic method to inject the reactive power into the distribution network is to add capacitors. This paper assumes that about one capacitor bank exists at the network buses. Besides their purpose for active power generation, the distributed generators (DGs) are also used to reduce the losses and to maintain the voltage profile within acceptable limits [1]. This work presents an optimal DGs placement method which applies modified fast decoupled method for load flow calculation and sensitivity analysis based genetic algorithms for optimal placement of the DGs in various feeder buses, eventually at a given load pattern. A list of sensitive candidate nodes is achieved based on the sensitivity factors. The optimal solution between investment in DGs and the money savings achieved by reducing the energy losses, while maintaining an improved voltage profile in the network, is sought.

Genetic algorithms (GA) are satisfactory mechanisms for power system optimization. The design variables are encoded into a strings, as sets of genes that

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form the chromosomes. The concept of selective adaptation and survival of the best solution is employed to search the parameter space and determine the optimal string by randomized information exchange [2]. During each iterative procedure referred to as generation, a new set of strings is produced using rules of evolution with improved performance. GA has been successfully employed to the DG placement problem in which design variables are the DG sizes at the candidate locations during finite discretized load levels. The performance value of each string is composed of the total power losses and the capital cost of the installed DGs for a specific configuration of feeders [3, 4].

## 2. Power flow calculation

In this work, a modified fast-decoupled method is applied for load flow calculation. Let us consider a distribution network consisting of a radial main feeder comprising  $n$  branches, and one lateral branching out from the main feeder as shown in Fig. 1.

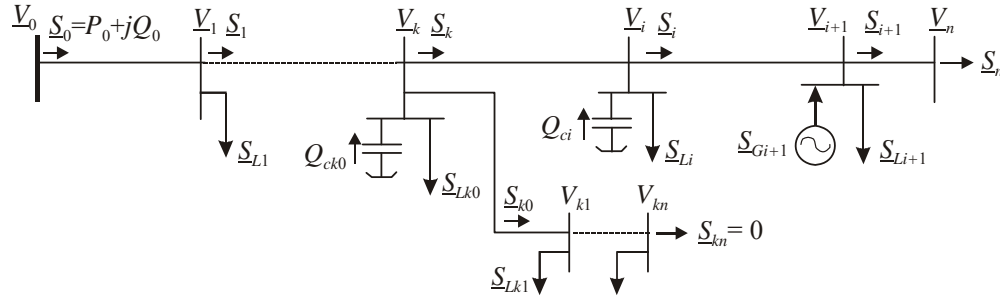


Fig. 1. One-line diagram for a radial distribution feeder

Lateral branching out from bus  $k$  will be referred to as the lateral  $k$ . The calculation is performed under the following assumptions [5]:

- the system is a balanced three-phase system;
- the distribution lines are modeled as series impedance,  $Z_i = R_i + jX_i$ ;
- the load at bus  $i$  is modeled as constant power,  $S_{Li} = P_{Li} + jQ_{Li}$ .

The shunt capacitors, that are considered as already installed in the network, are represented as reactive power injections. The distributed generators are modeled as  $PQ$  buses, but as negative load; their power factor was assumed to vary between 0.9 and 1. The voltage magnitude is limited to a desired value [6]. Generation at bus  $i$  is given by

$$\underline{S}_{Gi} = P_{Gi} + jQ_{Gi}$$

The complex power supplied from the main substation,  $\underline{S}_0 = P_0 + jQ_0$ , is first specified, then the active and reactive power of the first branch can be calculated as [7]:

$$\underline{S}_1 = \underline{S}_0 - \underline{Z}_1 \frac{P_0^2 + Q_0^2}{V^2} - \underline{S}_{L1} \quad (1)$$

and the receiving-end voltage is

$$V_1 \angle \theta_1 = V_0 - \underline{Z}_1 \frac{\underline{S}_0^*}{V_0} \quad (2)$$

The voltage at the substation bus is taken as reference, with  $\underline{V}_0 = V_0 \angle \theta_0 = 0$ .

Similarly the active and reactive power flowing at the receiving-end of branch  $i+1$ ,  $P_{i+1}$ , and  $Q_{i+1}$ , can be expressed as:

$$P_{i+1} = P_i - P_{loss,i+1} - P_{Li+1} + P_{Gi+1} \quad (3)$$

$$Q_{i+1} = Q_i - Q_{loss,i+1} - Q_{Li+1} + Q_{Gi+1} \quad (4)$$

A general term of the voltage magnitude at the sending-end resulted from equation (2) can be written as:

$$|\underline{V}_{i+1}|^2 = |\underline{V}_i|^2 - 2(R_{i+1}P_i + X_{i+1}Q_i) + (R_{i+1}^2 + X_{i+1}^2) \frac{P_i^2 + Q_i^2}{V_i^2} \quad (5)$$

At the end of the main feeder  $P_n = 0$  and  $Q_n = 0$ .

The active and reactive power losses on the  $i+1^{\text{th}}$  branch are:

$$P_{loss(i+1)} = R_{i+1} \frac{P_i^2 + Q_i^2}{V_i^2} \quad (6)$$

$$Q_{loss(i+1)} = X_{i+1} \frac{P_i^2 + Q_i^2}{V_i^2} \quad (7)$$

The total active power losses in the network are

$$P_{loss} = \sum_{i=1}^n R_{i+1} \frac{P_i^2 + Q_i^2}{V_i^2} \quad (8)$$

and the total reactive power losses are

$$Q_{loss} = \sum_{i=1}^n X_{i+1} \frac{P_i^2 + Q_i^2}{V_i^2} \quad (9)$$

where:  $P_i, Q_i$  are the active and reactive powers flowing into the sending-end of branch  $i+1$ .

- $n$  – the total number of branches in the system;
- $i$  – the index of a specific branch;
- $V_i$  – the voltage magnitude at bus  $i$ .

The total energy loss can now be defined as

$$E_{loss} = P_{loss} \times t$$

where  $t$  is the study period.

The corresponding cost of power losses are then achieved by

$$C_{Eloss} = E_{loss} \times k_E$$

where  $k_E$  is the energy price.

An iterative procedure was developed to obtain the unique solution for the bus voltages and hence for the branch active and reactive powers.

### 3. Sensitivity Factor Analysis

Power sensitivities are used to select the candidate buses for placing DGs in the distribution network. This helps reducing the search space during optimization procedures. The sensitivity analysis is a systematic procedure to determine the degree of influence between the reactive power placed at a bus  $i$  and change in the active power losses [7, 8]. The sensitivity factor associated with a bus is defined as:

$$S_{\Delta P} = \frac{\partial P_{loss}}{\partial kVA} \quad (10)$$

where  $S_{\Delta P}$  is the sensitivity factor which is estimated at each bus every time a configuration change occurs in the feeder such as the addition of a generator. The buses are ranked in the order of their sensitivity factors values [7, 8, 9]. Thereby the top-ranked bus in the priority list has the greatest sensitivity factor and is the one to be first considered for DG addition in the optimization process.

The bus which has the greatest sensitivity factor is tentatively added to it [7]. The total cost and bus voltages are then calculated. If both the reduction in the total cost and acceptable voltage profile are satisfied, the addition is accepted; otherwise, this addition is discarded, and a similar tentative addition to the bus

with the next greatest sensitivity factor is taken. Once the addition is accepted, the sensitivity factors of the nodes are re-evaluated. A new priority list is then established. Similar procedures for addition are repeated.

#### 4. Objective function

The objective function of the applied method is the total cost of adding  $n$  DGs in the distribution system, which can be formulated as:

$$TC = \sum_{k=1}^n C_{DG,k} (P_{DG,k}) \quad (11)$$

where, the cost of adding one DG at bus  $k$  in the network is given by:

$$C_{DG,k} (P_{DG,k}) = C_{C-DG,k} (P_{DG,k}) + C_{M-DG,k} (P_{DG,k}) + C_{F-DG,k} (P_{DG,k}) - C_{Eloss} \quad (12)$$

which, on the other hand, consists of four terms:

- The first term,  $C_{C-GD,k}$ , in \$, represents the capital cost of the distributed generator;
- The second term,  $C_{M-GD,k}$ , in \$, is the fixed operation and maintenance costs;
- The third term,  $C_{F-GD,k}$ , in \$/kWh, represents the fuel cost;
- The fourth term  $C_{Eloss}$  is the money savings by energy losses reduction; this term is obtained by multiplying the power losses of each load level by the duration of the load level.

The time related costs are evaluated on a 10 years study period. This is considered as a reasonable payback period of the investment costs [10, 11]. The annual cost is then obtained by dividing the total cost to 10. No bank interest or loans have been considered.

#### 5. Application of genetic algorithms for optimization

The solution procedure starts by applying a regular power flow calculation in order to determine the bus voltages and line losses. Then, determination of the optimal location and size for capacitors placement are performed by sensitivity analysis. The results obtained from the sensitivity analysis serve as the initial strings for the subsequent genetic algorithms (GA) application. GA employs its five operators to find the optimal solutions, which are not presented here because it is outside the purpose of this work. Since voltages at the network buses must be maintained within their upper and lower limits, any solution in which the voltages are not within limits eliminated. The chart flow of the algorithm applied in the optimization procedure is shown in Fig. 2.

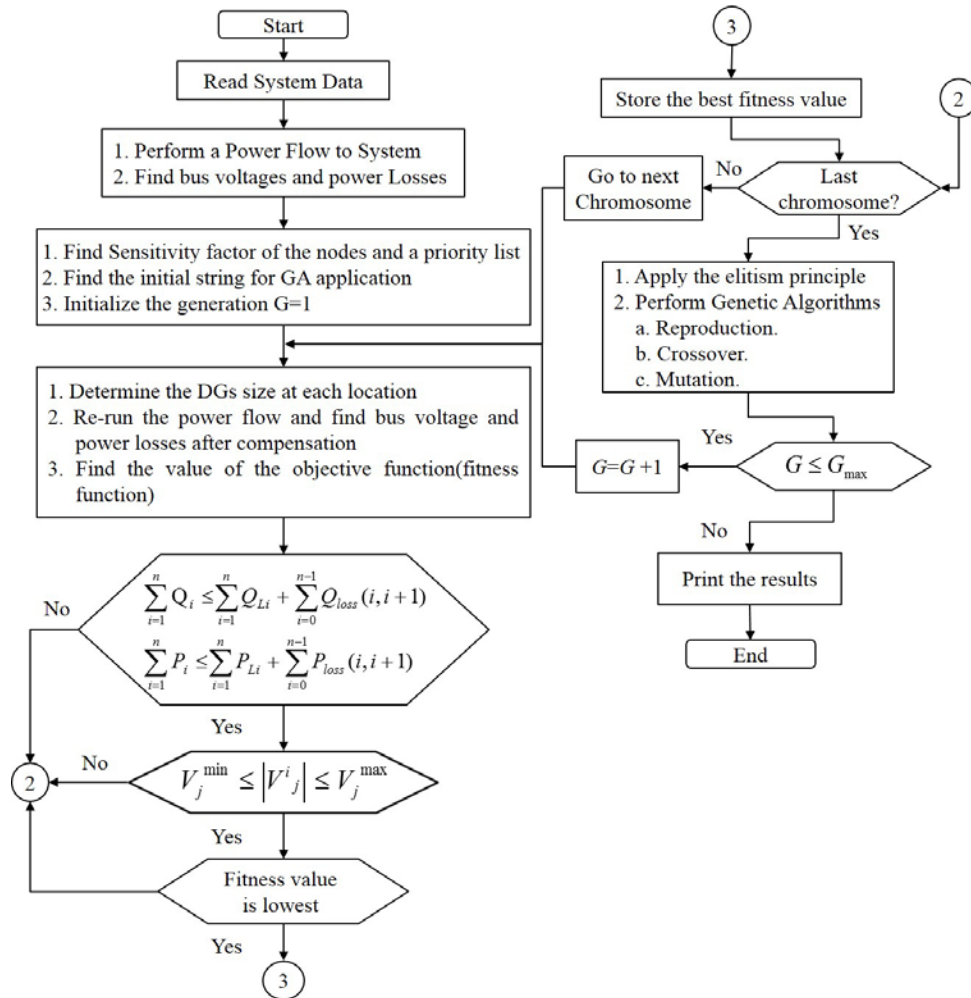


Fig. 2. Flow chart of the optimization algorithm

## 6. Case study

The case study is performed on a real electrical network, consisting of 38 buses of 11 kV nominal voltage [12]. The one line diagram of the studied network is shown in Fig. 3, and its data are given in tables 1 and 2.

The following values were considered for other quantities:

- Energy cost  $k_E = 0.06\$/\text{kWh}$ ;
- Generator cost:
  - capital price = 85  $\$/\text{kW}$ ;
  - annual cost of maintenance and operation of generator = 1000 \$;
  - fuel price = 4.5  $\$/\text{kW}$ ;

- limits of the bus voltages ( $V_{\min} = 0.9$  p.u. and  $V_{\max} = 1.1$  p.u.).

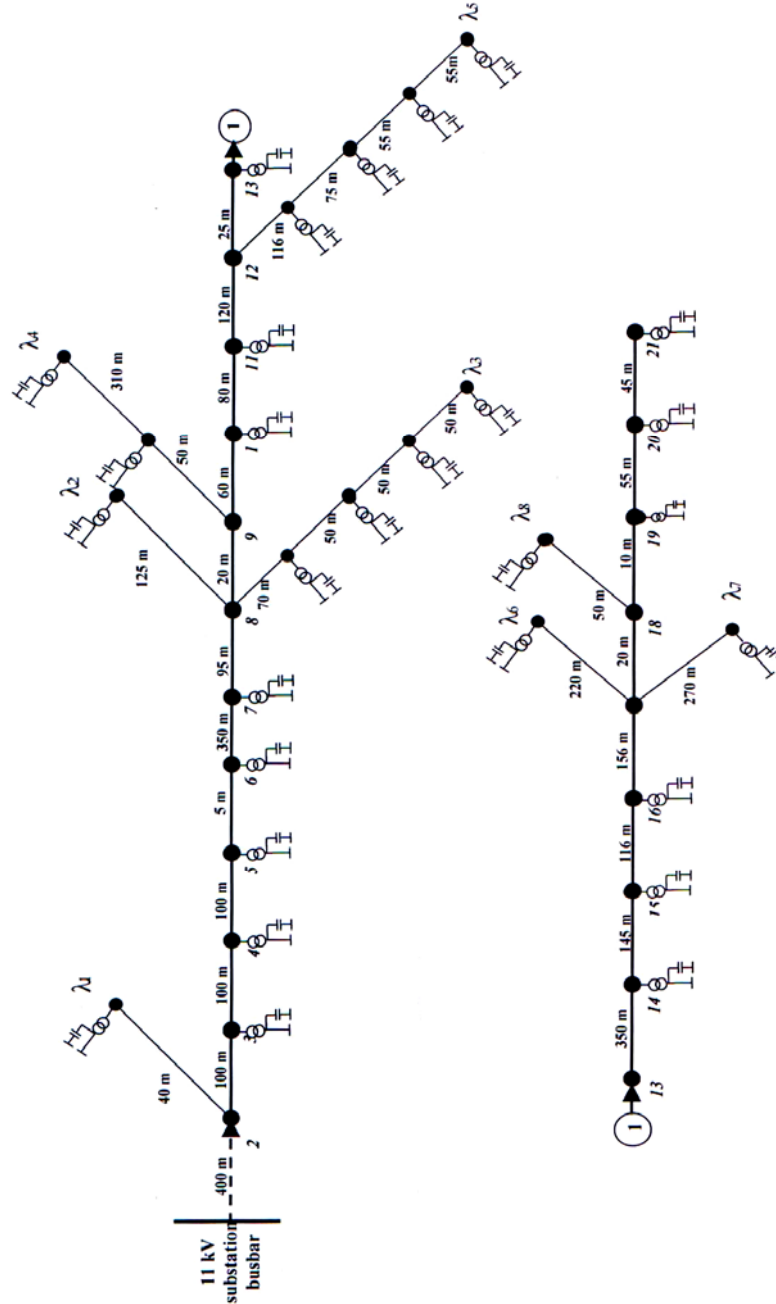


Fig. 3. The one line diagram of the 38 bus network

Table 1

Time (hrs)	Load data					
	Summer		Winter		Spring and autumn	
	I (A)	S (kVA)	I (A)	S (kVA)	I (A)	S (kVA)
1	232	4421	145	1	232	4421
2	213	4051	126	2	213	4051
3	213	4051	116	3	213	4051
4	213	4051	106	4	213	4051
5	213	4051	116	5	213	4051
6	213	4051	106	6	213	4051
7	213	4051	106	7	213	4051
8	203	3868	116	8	203	3868
9	203	3868	116	9	203	3868
10	222	4235	126	10	222	4235
11	222	4235	126	11	222	4235
12	271	5157	135	12	271	5157
13	271	5157	145	13	271	5157
14	271	5157	145	14	271	5157
15	280	5340	145	15	280	5340
16	280	5340	167	16	280	5340
17	271	5157	184	17	271	5157
18	271	5157	193	18	271	5157
19	271	5157	184	19	271	5157
20	271	5157	184	20	271	5157
21	251	4788	184	21	251	4788
22	251	4788	164	22	251	4788
23	232	4420	164	23	232	4420
24	232	4420	145	24	232	4420

Table 2

Load duration data for the test system						
System	Load level [p.u.]			Trimester [hrs]		
	S0	S1	S2	T0	T1	T2
	0.736	0.635	0.508	2920	2920	2920

The simulations were performed on a program implemented under Matlab. The program was divided into three parts: load flow solution, calculating the matrix of sensitivity factors, calculation of the optimal solution of adding distributed generators.

Table 3 shows the power flow results. The candidate buses for placing a distributed generator in terms of the sensitivity factor are shown in Table 4, for heavy load, medium load and light load as well.



Table 3

**Power flow results**

Branch <i>i-k</i>	Power flow		$V_i$ [kV]	Branch <i>i-k</i>	Power flow		$V_i$ [kV]
	$P$ [kW]	$Q$ [kVAr]			$P$ [kW]	$Q$ [kVAr]	
1-2	4123.48	3538.20	11.10	19-20	559.42	478.47	10.83
2-3	3971.74	3408.43	11.08	20-21	419.55	358.83	10.82
3-4	3824.53	3281.01	11.04	21-22	279.70	239.22	10.82
4-5	3677.82	3154.12	11.02	22-23	139.84	119.60	10.82
5-6	3531.60	3027.76	11.00	2-24	139.84	119.60	11.06
6-7	3391.47	2907.85	11.00	8-25	139.85	119.61	10.91
7-8	3232.58	2768.05	10.93	9-26	559.53	478.58	10.91
8-9	3087.98	2643.40	10.91	26-27	419.58	358.86	10.91
9-10	2528.45	2164.82	10.91	27-28	279.70	239.22	10.91
10-11	2248.11	1924.92	10.91	28-29	139.84	119.60	10.91
11-12	2106.81	1803.77	10.90	10-30	279.72	239.25	10.88
12-13	1965.26	1682.36	10.89	30-31	139.86	119.63	10.91
13-14	1403.41	1201.31	10.88	13-32	559.62	478.68	10.83
14-15	126.33	1081.46	10.87	32-33	419.60	358.89	10.87
15-16	1120.79	959.00	10.85	33-34	279.70	239.22	10.87
16-17	980.07	838.46	10.84	34-35	139.84	119.60	10.87
17-18	839.69	718.29	10.83	18-36	139.86	119.62	10.83
18-19	699.29	598.10	10.83				

Table 4

**The first 10 optimum sensitivity factor**

<i>heavy load</i>		<i>medium load</i>		<i>light load</i>	
Candidate bus no.	Best sensitivity factor	Candidate bus no.	Best sensitivity factor	Candidate bus no.	Best sensitivity factor
13	0.02596	13	0.01914	13	0.01211
14	0.02588	14	0.01909	14	0.01207
12	0.02542	12	0.01874	12	0.01187
11	0.02490	11	0.01835	11	0.01164
10	0.02440	10	0.01798	10	0.01141
32	0.02429	32	0.01793	32	0.01131
8	0.02416	15	0.01785	15	0.01130
9	0.02416	9	0.01780	9	0.01130
15	0.02415	8	0.01780	8	0.01121
30	0.02350	30	0.01732	30	0.01097

Let us first analyse the influence of reactive power that can be provided by distributed generators. Figs. 4, 5 and 6 provide a comparison of power losses when the reactive power only is considered as sensitivity.

If the total installed apparent power is considered in the analysis, the best solution for DG placement and size is:

- 1030 kVA generator installed at bus 18;
- 720 kVA generator installed at bus 23.

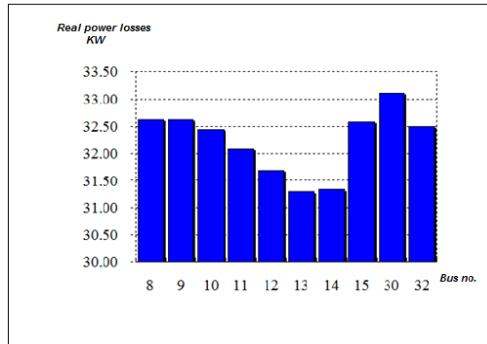


Fig. 4. The minimum total real power loss after injecting 2800 kVAr at heavy load

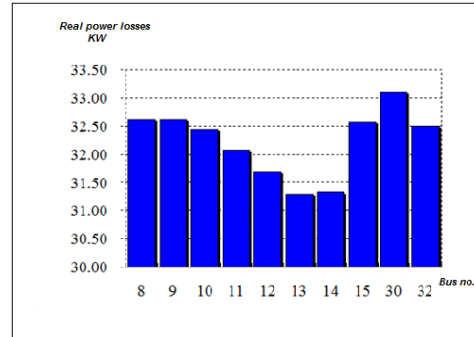


Fig. 5. The minimum total real power loss after injecting 2400 kVAr at medium load

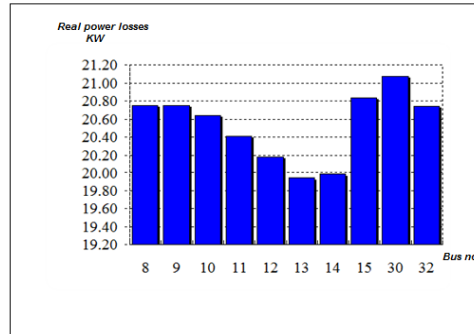


Fig. 6. The minimum total real power loss after injecting 1950 kVAr at low load

The optimal settings of these distribution generators for each load level (heavy, medium, and light load) are summarized in tables 5, 6 and 7, respectively. In each table, the power losses and the voltage profile of the system with and without distribution generator are also included.

The annual energy losses cost and annual system cost with/without addition DGs is outlined in table 8.

Table 5

Optimal distributed generator for heavy load		
System condition with/without distributed generator		
Heavy Load Case	Real Power Losses in (kW)	Voltage Profile
Without distributed generators	68.13	$V_{\min} = 0.975$ p.u. $V_{\max} = 0.993$ p.u.
With distributed generators at Bus 18=1030 kVA Bus 23=720 kVA	41.18	$V_{\min} = 0.982$ p.u. $V_{\max} = 1.00$ p.u.

Table 6

<b>Optimal distributed generator in medium load level</b>		
<b>System condition with/without distributed generator</b>		
<b>Medium Load Case</b>	<b>Real Power Losses in (kW)</b>	<b>Voltage Profile</b>
Without distributed generators	50.42	$V_{\min} = 0.978$ p.u. $V_{\max} = 0.996$ p.u.
With distributed generators at Bus 18=820 kVA Bus 23=580 kVA	28.03	$V_{\min} = 0.986$ p.u. $V_{\max} = 1.00$ p.u.

Table 7

<b>Optimal distributed generator in light load level</b>		
<b>System condition with/without distributed generator</b>		
<b>Light Load Case</b>	<b>Real Power Losses in (kW)</b>	<b>Voltage Profile</b>
Without distributed generators	32.04	$V_{\min} = 0.983$ p.u. $V_{\max} = 0.998$ p.u.
With distributed generators at Bus 18=710 kVA Bus 23=500 kVA	15.21	$V_{\min} = 0.990$ p.u. $V_{\max} = 1.00$ p.u.

Table 8

<b>The annual cost energy Losses and annual total system cost</b>		
<b>Annual System Cost: with/without distributed generator</b>		
<b>Annual System Cost =Annual Cost Energy Losses+Annual distributed generator cost</b>		
<b>Systems</b>	<b>Annual Cost Energy Losses (\$)</b>	<b>Annual System Cost (\$)</b>
Without distributed generators	26387.02	26387.02
With distributed generators	14794.51	23544.21

## 7. Conclusions

Distributed generation is one of the actual topics in the power system restructuring. They exist in various types, from renewable resource based to fossil fuel based. This paper assumed that the distributed generators are fossil fuel based, and show flexibility for changing the operation parameters.

The simulations have revealed that:

- The active power losses can be significantly reduced with the proper addition of the distributed generators;
- Voltage profile can be greatly improved when contribution of distributed generators is considered;
- The optimal settings of the distribution generator for various load levels is different;
- GAs can be successfully applied to reduce the total annual cost and the total power losses in the distribution network. The annual cost of energy losses can be reduced to almost half with respect to the value obtained when no generator exist in the network.

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