

## FLOW DYNAMICS REGIMES VIA NON-DIFFERENTIABILITY IN COMPLEX FLUIDS

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*A new topic in the analyses of complex fluid dynamics, considering that the movements of the complex fluid entities take place on continuum but non-differentiable curves is proposed. It results that in the dispersive approximation of motion, two distinguished flow regimes (non-quasi-autonomous and quasi-autonomous) by means of cnoidal modes of a velocity field can be established. The self-similarity of these modes specifies both the existence of some “cloning” mechanisms but also holographic behaviours. Some correlations with experimental data in plasma ablation are presented.*

**Keywords:** complex fluid, plasma ablation, flow regime, non-differentiability, fractals, holographic behaviours.

### 1. Introduction

The complex fluids dynamics is an interdisciplinary research topic that has been studied by means of a combination of basic theory, derived especially from physics and computer simulation. Such systems are composed of many interacting elemental units (called „agents”) and, among the most significant properties are the self-organization, the adaptability etc [1-3]. Examples of the most studied complex fluids are: colloidal fluids, polymers, foams, emulsions, gels,

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suspensions, micellar and liquid-crystal phases, molten materials, blood, etc. In this case the fluids do not obey to the hydrodynamic laws [4-6].

Plasma of electrical discharges shows complex dynamics as periodic transitions between multiple states [7-9], intermittency [10,11], chaos and hyperchaos[12]. The electrical response to different perturbations (periodic [13-16] or noisy[17]) can be explained using models involving transitions between solid-like and fluid-like behavior. Despite the large variety of the theoretical models that describe the complex dynamics [7, 9, 18-19], the physical origin of complexity is still a subject of interest. A possible solution can be the new approach based on the model of flow dynamics regimes via non-differentiability. In this way, both the complexity of interaction process that determines various temporal resolution scales and the patterns evolution that determines different degrees of freedom are taking into account.

In the framework of the multiscale approach of complex fluid dynamics, the main hypotheses are based on that, for temporal scales that are large with respect to the inverse of the highest Lyapunov exponent, the deterministic trajectories of the complex fluid particles are replaced by a collection of virtual trajectories and the concept of definite positions by that of probability density [20-22]. Since, in such context, the non-differentiability appears as a universal property of the complex fluids dynamics, it is necessary to construct a non-differentiable physics by considering that the complexity of the interactions processes is replaced by non-differentiability. This topic was developed using the Scale Relativity Theory (SRT) [23,24] and its extensions [25-26].

In the present paper we propose a new approach in the analyses of complex fluid dynamics, using SRT. Considering that the entities of the complex fluid are moving on continuous but non-differentiable curves, we show that in the dispersive approximation of SRT, two distinct flow regimes of the complex fluid appear. These regimes are controlled by means of a non-linear parameter associated to the modulus of the elliptic function  $cn$  (cnoidal oscillation modes of a velocity field).

## 2. Geodesics equations

We can simplify the dynamics of a complex fluid supposing that it displays chaotic behaviours (*i.e.* self-similarity and strong fluctuations at all possible scales [1-6,10-12,14,18,19,25,26]). This means that the complex fluid particles move on continuous but non-differentiable curves, *i.e.* fractal curves (for example, the Koch curve, the Peano curve or the Weierstrass curve [20,23,24]).

Once such hypothesis accepted, some consequences of non-differentiability through SRT become obvious [23,24]: i) physical quantities describing the complex fluid dynamics are fractal functions, *i.e.* functions that

simultaneously depend on spatial coordinates, time and scale resolution,  $\delta t/\tau$  (identified here with  $dt/\tau$  by substitution principle [23,24]). We mention that in classical physics, the physical quantities describing the dynamics of a complex fluid are continuous but differentiable functions merely depending on spatial coordinates and on time; ii) the complex fluid will behave as a special interaction-less „fluid” by means of geodesics in a fractal space (these geodesics are identified with the stream lines of complex fluid). In such conjecture, the dynamics of the complex fluid particles are given by the fractal operator  $\hat{d}/dt$

(the dispersive approximation of motion of SRT) [19,26]:

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \hat{\mathbf{V}} \cdot \nabla - i \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \Delta + \frac{\sqrt{2}}{3} \frac{\lambda^3}{\tau} \left( \frac{dt}{\tau} \right)^{(3/D_F)-1} \nabla^3 \quad (1)$$

where

$$\hat{\mathbf{V}} = \mathbf{V}_D - i \mathbf{V}_F \quad (2)$$

is the complex velocity,  $\mathbf{V}_D$  is the differentiable and resolution scale independent velocity,  $\mathbf{V}_F$  is the non-differentiable and resolution scale dependent velocity.  $\hat{\mathbf{V}} \cdot \nabla$  is the convective term, the next term in (1) is the dissipative term and the last one is the dispersive term.  $D_F$  is the fractal dimension of the movement curve,  $\lambda$  is the spatial resolution scale,  $\tau$  is the temporal resolution scale and  $\lambda^2/\tau \equiv D$  is the Nottale coefficient specific to fractal – non-fractal transition [23,24]. In the case of fractal dimension  $D_F$ , we can use any definition (the Hausdorff – Besikovici fractal dimension, the Kolmogorov fractal dimension etc. [20,22,24]), but once such definition accepted, it has to be constant over the entire complex fluid dynamics analysis.

Applying the fractal operator (1) to the complex speed (2) and accepting a generalized inertial principle (a generalization of Nottale's principle of scale covariance [19,26]), we obtain the geodesics equation:

$$\frac{\hat{d}\hat{\mathbf{V}}}{dt} = \frac{\partial \hat{\mathbf{V}}}{\partial t} + (\hat{\mathbf{V}} \nabla) \hat{\mathbf{V}} - i \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \Delta \hat{\mathbf{V}} + \frac{\sqrt{2}}{3} \frac{\lambda^3}{\tau} \left( \frac{dt}{\tau} \right)^{(3/D_F)-1} \nabla^3 \hat{\mathbf{V}} = 0 \quad (3)$$

Equation (3) shows that at any point of a fractal path, local acceleration,  $\partial_t \hat{\mathbf{V}}$ , convection,  $(\hat{\mathbf{V}} \nabla) \hat{\mathbf{V}}$ , the dissipation,  $\lambda^2/\tau (dt/\tau)^{(2/D_F)-1} \Delta \hat{\mathbf{V}}$ , and dispersion,  $\lambda^3/\tau (dt/\tau)^{(3/D_F)-1} \nabla^3 \hat{\mathbf{V}}$  are in equilibrium. The presence of the dispersive term in (3) generalizes the results from [23,24], so that all implications from [19,26] (the behaviours of the complex fluids are viscoelastic or hysteretic) and not only, can be here extended.

Since the movement of the complex fluid lacks interaction, we practically make use of self-convection, self-dissipation and self-dispersion type

mechanisms. Then, the geodesics equations are identified with “stream lines” of the complex fluid.

### 3. Separation of flow dynamics regimes in complex fluids

Separating in geodesics equation (3) the real part from the imaginary one of velocity field (2), for the differentiable scale resolution, we obtain:

$$\begin{aligned} \frac{\hat{d}\mathbf{V}_D}{dt} &= \frac{\partial \mathbf{V}_D}{\partial t} + (\mathbf{V}_D \cdot \nabla) \mathbf{V}_D - (\mathbf{V}_F \cdot \nabla) \mathbf{V}_F - \\ &- \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \Delta \mathbf{V}_F + \frac{\sqrt{2}}{3} \frac{\lambda^3}{\tau} \left( \frac{dt}{\tau} \right)^{(3/D_F)-1} \nabla^3 \mathbf{V}_D = 0 \end{aligned} \quad (4)$$

while for the fractal one:

$$\begin{aligned} \frac{\hat{d}\mathbf{V}_F}{dt} &= \frac{\partial \mathbf{V}_F}{\partial t} + (\mathbf{V}_F \cdot \nabla) \mathbf{V}_D + (\mathbf{V}_D \cdot \nabla) \mathbf{V}_F + \\ &+ \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \Delta \mathbf{V}_D + \frac{\sqrt{2}}{3} \frac{\lambda^3}{\tau} \left( \frac{dt}{\tau} \right)^{(3/D_F)-1} \nabla^3 \mathbf{V}_F = 0 \end{aligned} \quad (5)$$

For irrotational motions of the velocity field (2),

$$\nabla \times \hat{\mathbf{V}} = 0, \nabla \times \mathbf{V}_D = 0, \nabla \times \mathbf{V}_F = 0 \quad (6a-c)$$

the following form can be chosen:

$$\hat{\mathbf{V}} = -2i \left( \frac{\lambda^2}{\tau} \right) \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \nabla \ln \psi \quad (7)$$

or explicitly, with  $\psi = \sqrt{\rho} \exp(iS)$ ,

$$\hat{\mathbf{V}} = 2 \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \nabla S - i \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \nabla \ln \rho$$

$$\mathbf{V}_D = 2 \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \nabla S \quad (8a-c)$$

$$\mathbf{V}_F = \frac{\lambda^2}{\tau} \left( \frac{dt}{\tau} \right)^{(2/D_F)-1} \nabla \ln \rho$$

where  $\sqrt{\rho}$  is an amplitude and  $S$  a phase. In such a context, if the dissipative effects are negligible compared to the convective and dispersive ones, then the equations (4) and (5) take the form (for details of the method see [18,25,26]):

$$\hat{\frac{d}\{V_D}{dt}} = \frac{\partial V_D}{\partial t} + (V_D \cdot \nabla) V_D + \frac{\sqrt{2}}{3} \frac{\lambda^3}{\tau} \left( \frac{dt}{\tau} \right)^{(3/D_F)-1} \nabla^3 V_D = 0 \quad (9)$$

An explicit form of the velocity field  $V_D$  is obtained for the one-dimensional case. In dimensionless variables

$$\omega t = \tau, kx = \xi, \theta = \xi - M\tau, \frac{V_D}{V_{D_0}} = \phi \quad (10\text{a-d})$$

the solution of equation (9) using the method from [19,26] becomes:

$$\Phi = \bar{\Phi} + 2a \left[ \frac{E(s)}{K(s)} - 1 \right] + 2acn^2 [\alpha(\theta - \theta_0); s] \quad (11)$$

where  $K(s)$ ,  $E(s)$  are the complete elliptical integrals of first and second kind of modulus  $s$ ,  $cn$  is the Jacobi elliptical function of argument  $\alpha(\theta - \theta_0)$  and modulus  $s$  [27],  $a$  is an amplitude and  $\bar{\Phi}$  is an average value of the states density. Details on defining parameters  $s$ ,  $a$  and  $\bar{\Phi}$  can be found in [4,6,28]. Moreover, in previous relations (10a-d),  $\omega$  is a specific pulsation,  $k$  is the inverse of a specific length,  $V_{D_0}$  is a specific velocity and  $M$  is an equivalent of Mach number. These parameters are specific to the complex fluid characterizing both structure and dynamic types [4-6]. Therefore the flow dynamics of the complex fluids is achieved through space-time cnoidal oscillation modes of the velocity field (Fig.1). The oscillation modes are explained through modulus  $s$  of the elliptical function  $cn$ , non-linearity parameter depending both on structure and dynamic types of the complex fluids. Moreover, the oscillation modes are self-similar via the parameter  $s$  (Fig.2a,b) which specifies the fractal character of the flow processes in complex fluids.

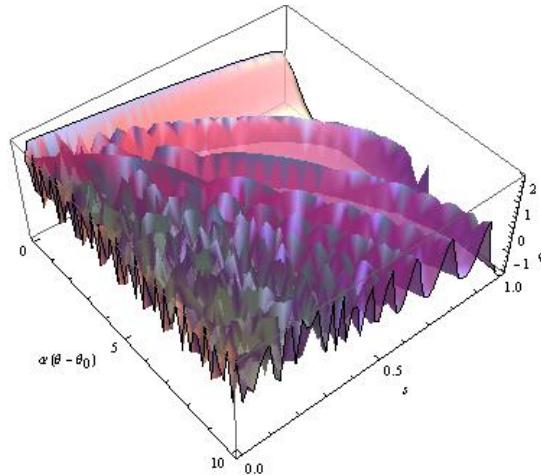


Fig. 1. Three-dimensional dependence of cnoidal oscillation modes of velocity field

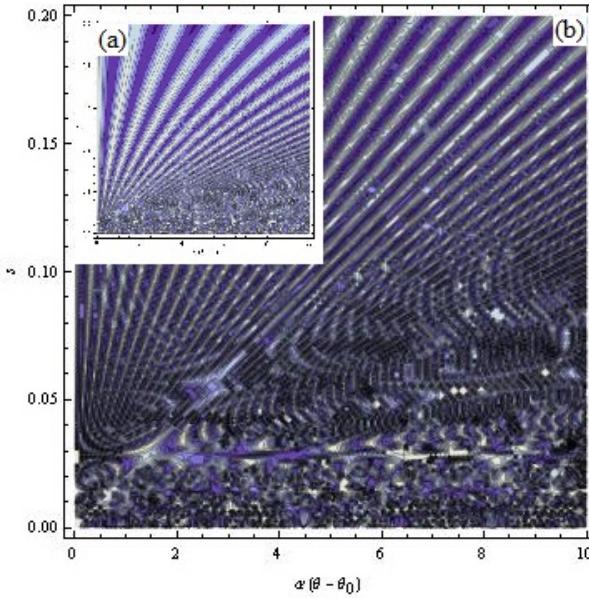


Fig. 2. Self-similar contour curves of the velocity field versus non-linear parameter  $s$ : a)  $0 \leq s \leq 0.5$ , b)  $0 \leq s \leq 0.2$

The self-similarity of the cnoidal modes specifies the existence of some “cloning” mechanisms (full and fractional velocity function – a function which evolves in time to a state describable as a collection of spatially distributed sub-velocity-functions that each closely reproduces the initial velocity-function shape) [29]. All these show a direct connection between the fractal structure of the flow dynamics of complex fluid and holographic behaviours [30].

The space-type cnoidal oscillation modes have the following characteristic parameters:

i) Wave number

$$k = \frac{\pi a^{1/2}}{s K(s)} \quad (12)$$

ii) Phase velocity

$$U = 6\bar{\Phi} + 4a \left[ \frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2} \right] \quad (13)$$

iii) Period (see Figure 3a,b)

$$\bar{T} = 1 / \left\{ \frac{3\bar{\Phi}a^{1/2}}{sK(s)} + \frac{2a^{3/2}}{sK(s)} \left[ \frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2} \right] \right\} \quad (14)$$

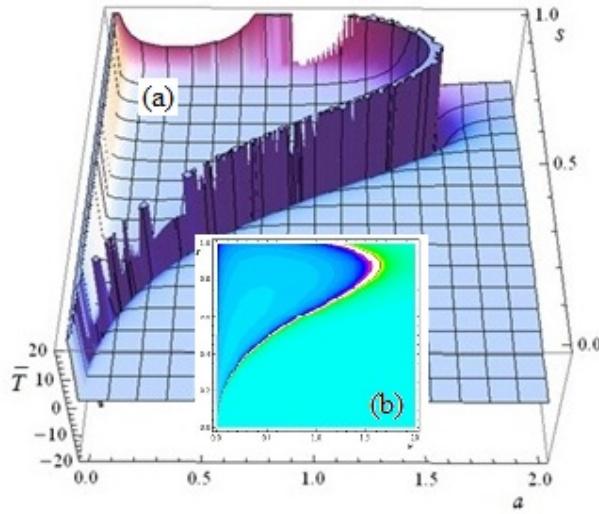


Fig. 3. The period of the cnoidal oscillation modes versus  $a$  and  $s$  (a); two dimensional contour of the period (b)

For  $s \rightarrow 0$  (11) it is reduced to a harmonic package type sequence, while for  $s \rightarrow 1$ , (11) it is reduced to a soliton package type sequence. For  $s = 0$  (11) it is reduced to a harmonic type sequence and for  $s = 1$  (11) it is reduced to soliton type sequence.

Through non-linearity  $s$  two distinct “flow” regimes of the complex fluid appears: non-quasi-autonomous regime (by harmonic type sequences, harmonic package type sequence), and quasi-autonomous respectively (by soliton type sequences, soliton package type sequences) - see Fig.3a, b.

In such context, we can consider that the nonlinear and chaotic dynamics generated by space charge configurations or other localized particles structures in an electrical plasma discharge can be explained and modelled by the complex fluid flow dynamics. Particularly, in the case of plasma ablation [31,32] the two plasma structures experimentally observed by optical and electrical measurements (structures that correspond to two types of electrons, cold and hot) can be generated by means of two flow regimes of the complex fluid previously mentioned. Indeed, if the temperature of the hot electrons is  $k_B T_e \approx 8$  eV and the temperature of the cold electrons is  $k_B T_e \approx 2$  eV [31-35] then, according with SRT, by means of the relation  $D = k_B T_e / m v_{es}$ , where  $m$  is the rest mass of the entities from the plasma ablation,  $v_{es}$  the collision frequency, we can define two fractal-nonfractal “diffusion coefficients” corresponding to the flow regimes. *i.e.*  $D_h \approx 4 \cdot 10^2$  m<sup>2</sup>/s and  $D_c \approx 30$  m<sup>2</sup>/s (for details see [35]). Considering that the two ablation plasma structures are associated with double layers [36] with the

dimensions  $L_h \approx 10^{-2}$  m and  $L_c \approx 1.5 \cdot 10^{-2}$  m (see the evolution of the visible emission from the aluminum plasma plume recorded using camera with gating time 20 ns [35]), it result the characteristic velocities  $V_h = D_h/L_h \approx 4 \cdot 10^4$  m/s<sup>2</sup> and  $V_c = D_c/L_c \approx 2 \cdot 10^3$  m/s. These values are close to those obtained by means of the expansion velocities of the experimental first (hot) and second (cold) plasma structures (for details on the method see [35]). We note that the complex fluid methods [18,19,25,26] give reliable results and can be used together with the methods of non-linear dynamics and chaos [7,8,11-13], to the analyses of the dynamics experimentally observed in electrical discharge plasmas.

#### 4. Conclusions

The main conclusions of the present paper are the following:

- Supposing that the entities of a complex fluid are moving on continuous and non-differentiable curves (fractal curves) the geodesics are obtained. These geodesics are identified with the complex fluid stream lines;
- If the dissipative effects are negligible compared to the convective and dispersive ones, for one dimensional case the solution of motion equation is established. In such context the flow regimes of the complex fluid are controlled by means of cnoidal oscillation modes of the velocity field;
- The flow regime types are explained by means of cnoidal elliptical function modulus. So, non-quasi-autonomous regime (by harmonic type sequences, harmonic package type sequence), and quasi-autonomous one (by soliton type sequences, soliton package type sequences) are obtained;
- The self-similarity of the cnoidal modes specifies the existence of some “cloning” mechanisms. All these show a direct connection between the fractal structure of the flow dynamics of complex fluid and holographic behaviours;
- Possible validations of the model by means of plasma ablation experimental data are presented.

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