

## DELAY DIFFERENTIAL EQUATIONS MODELS FOR MECHANO AND ELECTROHYDRAULIC SERVOMECHANISMS

Daniela Enciu<sup>1</sup>, Andrei Halanay<sup>2</sup>, Ioan Ursu<sup>3</sup>

*The problem of modeling hydraulic servomechanisms as systems with time delay is less researched, although the input-output time delay is herein objective and, in fact, real in every system which involves load dynamics. The objective of this paper is to propose a few types of delay differential equations which characterize the dynamics of the hydraulic servomechanisms from the primary flight controls of an airplane. The hydraulic servomechanism (mechanical or electrical) is a feedback control system. Therefore three possible locations for the time delay are herein taken into account: on state variables, on measurement variables, and on control variables.*

**Keywords:** delay-differential equations, mecano and electrohydraulic servomechanisms, state delay, control delay, measurement delay

### 1. Introduction

Mechanohydraulic and electrohydraulic servomechanisms are widely used in those industries where heavy objects must be manipulated, or large forces and torques with high speeds, meaning fast time constants, must be exerted. The specific areas of interest are: civil engineering, machine tools, mobile equipment and robots, radar antenna, land vehicles, and first and foremost in the field of naval and aerospace systems.

The purpose of this paper is to introduce a very unusual theme in the recent literature of the field, but also a realistic one, namely the time delay in the dynamics of hydraulic servomechanisms. The theme of delayed differential equations was present in the years 1950-1975, and it is supported by a solid mathematical apparatus: [1]-[5]. We do not refer to more recent works, but

---

<sup>1</sup> National Institute for Aerospace Research Elie Carafoli-INCAS Bucharest,Romania, and Department of Mathematics and Informatics, University Politehnica of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania, e-mail: enciu.daniela@incas.ro

<sup>2</sup> Department of Mathematics and Informatics, University Politehnica of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania, e-mail: halanay@mathem.pub.ro

<sup>3</sup> National Institute for Aerospace Research Elie Carafoli-INCAS Bucharest,Romania, e-mail: ioan.ursu@incas.ro

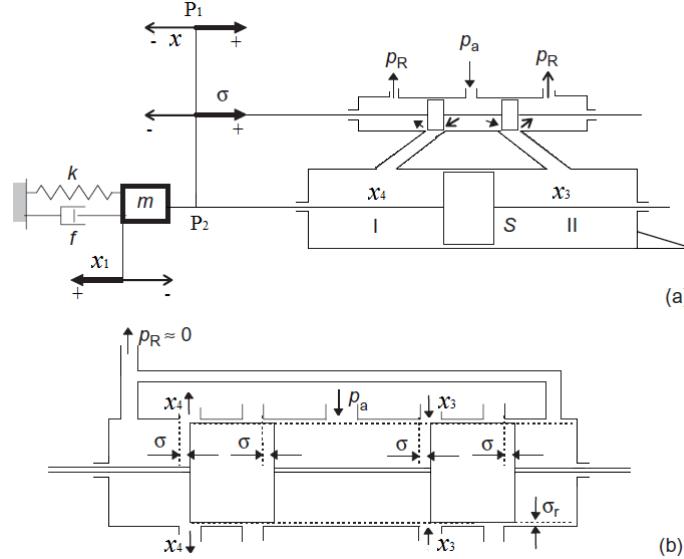


FIGURE 1. a. Schematic drawing of hydraulic servomechanism physical model; b. zoom on the spool type valve

we note that all these newer or older works, with one exception [5], are mathematical works and do not belong to the engineering of hydraulic automation systems. Explanations of the poverty of mathematical models with time delay in the literature of hydraulic servomechanisms can be multiple: the neglect of the matter by engineers, the non-involvement of a so-called technological delay in the block diagram of servomechanism. This is the situation defined in [6]: the technological delay occurs if it necessary to take into account the finiteness of the time needed to complete the technological process. Such a delay can be considered the time necessary for a liquid to enter or leave a column with recycling in a reactor. But, as shown in the present article, a state delay can be introduced to simplify for example the mathematical model of dry friction, or of overlapped spool valve, otherwise strongly non-linear. Other types of time delays (on control variable, on measurement, or on human pilot input) are inherent and are introduced as such.

## 2. Mathematical models with delay for mechanohydraulic and electrohydraulic servomechanisms

By analysing the dynamics of hydraulic servomechanisms and by studying the literature, several sources of delays can be distinguished in defining their mathematical models: inertia of moving components of the servomechanism and inertia of the controlled load, essential nonlinearities of the servomechanisms (mainly, the overlap in the spool valve), complex dry friction between moving and fixed parts of the hydrocilinder (see LuGre model [7]),

delay in reading signals from transducers taking into account sampling time, delays in the human pilot's reaction [8]. There is also a delay induced by the time required to compute the control variable in the computing unit. These aspects will be considered in the elaboration of the mathematical models with time delay described below, in the case of mechanohydraulic servomechanism (MHS) (Fig.1), but mainly of electrohydraulic servomechanism (EHS) (Fig. 2).

The notations in Fig. 1 and 2 refer to:  $x$  - the input, a mechanical displacement in the case of MHS or electrical signal in the case of EHS;  $x_2 = \dot{x}_1$ ;  $x_3$  and  $x_4$  - the pressures in the two chambers of the hydrocylinder (HC);  $x_5$  - the servovalve spool displacement;  $u$  - control variable;  $\sigma$  - the so-called error signal describing the opening of the distribution slots;  $\sigma_r$  - radial clearance spool-sleeve;  $p_a$  - supply pressure;  $p_R$  - tank pressure, around 1 bar (thus negligible);  $m$  - equivalent inertial load;  $f$  - equivalent viscous friction force coefficient;  $k$  - equivalent aerodynamic elastic force coefficient;  $S$  - servomechanism piston area (ring shaped); EHSV - electrohydraulic servovalve; TM- torque motor; CL - controller with an implemented control law; T - transducer.

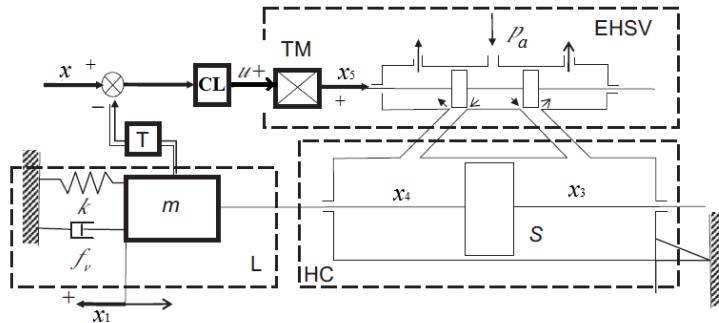


FIGURE 2. Physical model of EHS

In the classical books on hydraulic servomechanisms [9], [10], the mathematical model of the MHS and EHS without delay is built on the simplifying hypothesis  $x_3 + x_4 = p_a$ . The failing of this equation may indicate, in dynamic transient behavior, among other things, the presence of cavitation in system [11]. This allows us to consider a more realistic hydraulic servo model, with  $x_3 + x_4 \neq p_a$ , already introduced a long while ago (see [12], for example), and considered in more recent works (see [13], [14]). Thus, a four-state model of the MHS in the canonical form, including two pressures, is proposed and

explained in detail in [11], [15]-[17]

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{1}{m}[S(x_3 - x_4) - fx_2 - kx_1] \\ \dot{x}_3 &= \frac{B}{V + Sx_1} \left[ c_d w |\sigma| \operatorname{sgn}[p_a(1 + \operatorname{sgn}(\sigma)) - 2x_3] \sqrt{\frac{|p_a(1 + \operatorname{sgn}(\sigma)) - 2x_3|}{\rho}} - Sx_2 \right] \\ \dot{x}_4 &= \frac{B}{V - Sx_1} \left[ c_d w |\sigma| \operatorname{sgn}[p_a(1 - \operatorname{sgn}(\sigma)) - 2x_4] \sqrt{\frac{|p_a(1 - \operatorname{sgn}(\sigma)) - 2x_4|}{\rho}} + Sx_2 \right]\end{aligned}\quad (1)$$

$c_d$  is the discharge coefficient in spool valve,  $w$  is the valve ports width,  $\rho$  is the hydraulic oil density,  $B$  is the bulk modulus of hydraulic oil, and  $V$  is the cylinder half-volume. The feedback linkage equation is generally taken as an algebraic linear equation (kinematic equation) connecting the input variable  $x$ , the output variable  $x_1$  and the "error" variable  $\sigma$

$$\sigma = \lambda_k(x - x_1) \quad (2)$$

A five-state model of the EHS is given in [11], [18]-[22]

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{1}{m}(-kx_1 - fx_2 + Sx_3 - Sx_4) \\ \dot{x}_3 &= \frac{B}{V + Sx_1} [C|x_5| \operatorname{sgn}[p_a(1 + \operatorname{sgn}(x_5)) - 2x_3] \sqrt{|p_a(1 + \operatorname{sgn}(x_5)) - 2x_3|} - Sx_2] \\ \dot{x}_4 &= \frac{B}{V - Sx_1} [C|x_5| \operatorname{sgn}[p_a(1 + \operatorname{sgn}(x_5)) - 2x_4] \sqrt{|p_a(1 + \operatorname{sgn}(x_5)) - 2x_4|} + Sx_2] \\ \dot{x}_5 &= -\frac{1}{\tau_{SV}}x_5 + \frac{k_{SV}}{\tau_{SV}}u; \quad C := \frac{c_d w}{\sqrt{\rho}}\end{aligned}\quad (3)$$

EHS replaces the spool valve of the MHS with the electrohydraulic servovalve EHSV, see Fig. 2. Instead of the rigid feedback linkage of the MHS represented by the lever  $P_1P_2$  (Fig. 1), the comparison element between the electric input  $x$  (Volts) and the electric signal provided by a load transducer T (Volts) appears. The distinction of the mathematical model of EHS from the model of the MHS refers only to the replacement of the kinematic equation (2) with an equation of the comparison element generating the error signal  $\sigma := x - k_T x_1$ , where  $k_T$  is the gain of the transducer. The relative spool-sleeve displacement in EHSV, noted  $x_5$ , is generated by the control variable  $u$  which is given by the control synthesis law, CL. The synthesis is thus based on the error  $x - k_T x_1$ .  $\tau_{SV}$  is the EHSV time constant and  $k_{SV}$  is the associated input gain. It can be seen that the nonlinear model of the EHS differs from that of the MHS only by introducing a dynamic equation of the servovalve, the last equation in (3). Higher orders for the servovalve equation can be considered.

In the following, some mathematical models with delay will be proposed for MHS and EHS.

## 2.1. Primary mathematical models with time delay for EHS and MHS

For EHS and MHS we will present two primary mathematical models with time delay reducible to a single differential equation. The study of these models

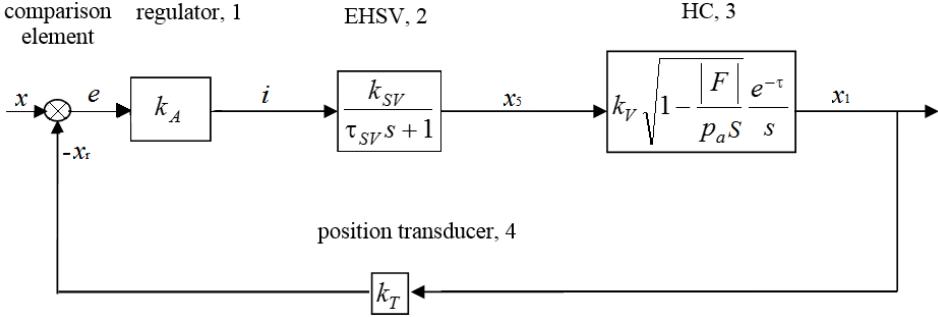


FIGURE 3. Transfer functions of the EHS system

will be useful for the preparation of the instruments and understanding the consequences of the time delay presence in the dynamic equations for hydraulic servomechanisms. An electrohydraulic servomechanism can be described by the following system of equations

$$\begin{aligned} e &= x(t) - x_r(t); \quad i(t) = k_A e; \quad x_5(t + \tau_e) = k_{SV} i(t) \\ \dot{x}_1(t) &= k_V \sqrt{1 - \frac{|F|}{p_a S}} x_5(t - \tau); \quad x_r = k_T x_1 \end{aligned} \quad (4)$$

The equations transcribe, successively, the components of the block diagram of the transfer functions from Fig. 3: the electrical comparison element; the voltage-current converter, in fact the controller; the EHSV; the HC rod displacement  $x_1$ , and the position transducer.  $F$  represents a maximum value of the load at the servomechanism rod,  $k_V$  is a velocity gain of the HC, and  $k_A$  is the "controller gain" given by the control law [11].  $\tau$  assimilates the influence of inertia  $\tau_i$ , and the influence of other parameters  $\tau_{pn}$

$$\tau = \tau_i + \tau_{pn} \quad (5)$$

$k_V$  and  $\tau_e$  are given by

$$k_V = \frac{c_d w}{S} \sqrt{\frac{p_a}{\rho}}; \quad \tau_e = \alpha \tau_{SV} \quad (6)$$

with  $\alpha$  a suitable coefficient.

**Proposition 1.** *A primary mathematical model with time delay of the EHS is described by the delay differential equation*

$$\dot{x}_1(t) + k x_1(t - \tau_0) = \frac{k}{k_T} x(t - \tau_0) \quad (7)$$

where  $\tau_0$  is an equivalent time delay containing inertia and viscous load effects in hydrocylinder and electrohydraulic servovalve,

$$\tau_0 = \tau + \tau_e \quad (8)$$

*Proof.* From (4), the following load movement equations are obtained

$$x(t + \tau_e) = k_{SV}k_A[x(t) - k_Tx_1(t)]; \quad \dot{x}_1(t) = k_V \sqrt{1 - \frac{|F|}{p_a S}}x(t - \tau); \quad (9)$$

From (8) and with the notation

$$k = k_T k_A k_{SV} k_V \sqrt{1 - \frac{|F|}{p_a S}}, \quad [k] = 1/\text{s} \quad (10)$$

system (9) can be rewritten as follows

$$\begin{aligned} x(t + \tau_e - \tau_0) &= k_{SV}k_A[x(t - \tau_0) - k_Tx_1(t - \tau_0)] \\ \dot{x}_1(t) &= k_V \sqrt{1 - \frac{|F|}{p_a S}}x(t + \tau_e - \tau_0) \end{aligned} \quad (11)$$

Now it is possible to eliminate the  $x(t + \tau_e - \tau_0)$  obtaining thus the delay differential equation of the EHS (7).  $\square$

With this simplified form, a case study of the stability can be considered.

**Proposition 2.** *The characteristic equation of the delay differential equation mathematical model of EHS is stable (i.e. has roots with negative real parts) if  $k\tau_0 < \frac{\pi}{2}$ .*

*Proof.* The characteristic equation associated to (7) is

$$\lambda + ke^{-\tau_0 \lambda} = 0 \quad (12)$$

With the notations

$$z = \tau_0 \lambda, \quad p = 0, \quad q = -k\tau_0$$

the equation (12) is rewritten as follows

$$\lambda - \frac{q}{\tau_0} e^{-z} = 0$$

or

$$\tau_0 \lambda e^z - q = 0$$

or

$$pe^z - ze^z + q = 0 \quad (13)$$

that is the equation (3) from [23]. According to the theorem 13.8 from [1], a necessary and sufficient condition of stability of the roots of the equation (13) is

- (1)  $p < 1$
- (2)  $p < -q < (\theta^2 + p^2)^{1/2}$

where  $\theta$  is the unique root of the transcendent equation  $\theta = p \tan \theta$ ,  $0 < \theta < \pi$ . For the case  $p = 0$ ,  $\theta = \frac{\pi}{2}$ . Thus, the condition of stability reduces on the fulfillment of the inequality  $\theta = \frac{\pi}{2} > k\tau_0$ .  $\square$

**Proposition 3.** *A primary mathematical model with time delay of the MHS is described by the time delay differential equation*

$$\dot{x}_1(t) + kx_1(t - \tau_0) = kx(t - \tau_0) \quad (14)$$

*Proof.* Starting from the mathematical model (4), a time delay mathematical model can also be given for the MHS. The equations are

$$\sigma(t) = \lambda_k(x(t) - x_1(t)); \quad \dot{x}_1 = k_V \sqrt{1 - \frac{|F|}{p_a S}} \sigma_r(t - \tau); \quad \sigma_r(t + \tau_e) = k_s \sigma(t) \quad (15)$$

with: 1) the tracking error evaluating the desired movement(input),  $x$ , compared to the obtained displacement (output),  $x_1$ , and 2) the equation of the hydrocylinder HC.  $\lambda_k$  is a kinematic factor (see equation 2). The error  $\sigma$  will be performed as  $\sigma_r$  realized with a time delay  $\tau_s$  due to: a) spool inertia and b) overlap in spool valve. The fifth equation from the system (4) is missing because the MHS is not equipped with transducers for position, speed, acceleration response, those being replaced by the rigid feedback,  $P_1P_2$ .

Thus, the MHS is described by

$$\dot{x}_1(t) + kx_1(t - \tau_0) = kx(t - \tau_0) \quad (16)$$

In conclusion, we have the same characteristic equation as in the EHS case:

$$\lambda e^{\tau_0 \lambda} + k = 0 \quad (17)$$

where

$$k = \lambda_k k_V k_S \sqrt{1 - \frac{|F|}{p_a S}} \quad (18)$$

□

## 2.2. Mathematical models with delay on control

The delay in the control variable of an automatic system is a real current problem because the determination of the control variable requires a certain amount of time that is associated with a delay. This problem is relevant to the electrohydraulic servomechanism. Let us notice that the mathematical models of the hydraulic servomechanisms imply a switch given by the signum function. In particular, a nonlinear switching model of the electrohydraulic servomechanism derives directly from the equations (3). The problem is to analyse the behaviour of the system in the presence of the delay  $\tau$  on the two sequences of the control variables  $u_1$  and  $u_2$ , and to develop some techniques to compensate the considered delay.

More precisely, for the case when  $x_5 \geq 0$ , we have the system of equations

$$\begin{aligned} \dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1} (Cx_5 \sqrt{p_a - x_3} - Sx_2) \\ \dot{x}_4 &= -\frac{B}{V_0 - Sx_1} (Cx_5 \sqrt{x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}} x_5 + \frac{k_{SV}}{\tau_{SV}} u_1 (X(t - \tau)) \end{aligned} \quad (19)$$

For  $x_5 < 0$ , a different system of equations is obtained

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1}(Cx_5\sqrt{x_3} - Sx_2) \\ \dot{x}_4 &= -\frac{B}{V_0 - Sx_1}(Cx_5\sqrt{p_a - x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5 + \frac{k_{SV}}{\tau_{SV}}u_2(X(t - \tau))\end{aligned}\quad (20)$$

In the coupled systems (19) and (20) it was considered the general case of control variable synthesis,  $u_1$  and  $u_2$ , as feedback from the state vector  $X = [x_1, x_2, x_3, x_4, x_5]^T$ . The real situation is that of a feedback by a measured output  $z = C_0X$ .  $C_0$  is a matrix of the appropriate size. In fact, there is a delay in the measurements coming from the transducers. In this case, we have the linear switching model (let it be the linear case)

$$\begin{aligned}\dot{X} &= A_1X + u_1(C_0X(t - \tau)), \quad x_5 \geq 0 \\ \dot{X} &= A_2X + u_2(C_0X(t - \tau)), \quad x_5 < 0\end{aligned}\quad (21)$$

$A_1, A_2$  are the Jacobian matrices derived from (19) and (20), see [21].

### 2.3. A mathematical model of the EHS with delay on the servovalve state

Let us consider the essential nonlinearities of the EHS spool valve, the saturation and the overlap. They can be assimilated with a delay on the state of the servovalve  $x_5$  [16], [19]. The models are obtained for the case when  $x_5 \geq 0$ ,

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1}(Cx_5(t - \tau)\sqrt{p_a - x_3} - Sx_2) \\ \dot{x}_4 &= \frac{B}{V_0 - Sx_1}(Cx_5(t - \tau)\sqrt{x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5(t - \tau) + \frac{k_{SV}}{\tau_{SV}}u_1(X)\end{aligned}\quad (22)$$

and also for the case  $x_5 < 0$ ,

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1}(Cx_5(t - \tau)\sqrt{x_3} - Sx_2) \\ \dot{x}_4 &= \frac{B}{V_0 - Sx_1}(Cx_5(t - \tau)\sqrt{p_a - x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5(t - \tau) + \frac{k_{SV}}{\tau_{SV}}u_2(X)\end{aligned}\quad (23)$$

### 2.4. A mathematical model of EHS with two delays

Friction is a phenomenon of interest in the study of systems. For the hydraulic servomechanism, the problem is focused on the piston-hydrocylinder friction, and, also, the friction in the transmission line from piston rod to the inertial mass can be considered. In the paper [7], the authors propose a relatively simplified form of the LuGre friction model [7], [24], described by a delay model called the „Armstrong mode”.

For the EHS it is proposed a mathematical model with delay on the servovalve state  $x_5$ , and on the speed state  $x_2$ , the latter being caused by the hydrocylinder-piston dry friction. The delay is included in the expression of the friction force  $F_f$ . Both delays are realistic. For  $x_5 \geq 0$  the following system of equation is obtained

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 - F_f + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1}(Cx_5(t - \tau)\sqrt{p_a - x_3} - Sx_2) \\ \dot{x}_4 &= \frac{B}{V_0 - Sx_1}(Cx_5(t - \tau)\sqrt{x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5(t - \tau) + \frac{k_{SV}}{\tau_{SV}}u_1(X)\end{aligned}\quad (24)$$

and for  $x_5 < 0$  we have

$$\begin{aligned}\dot{x}_1 &= x_2, \dot{x}_2 = \frac{-kx_1 - fx_2 - F_f + Sx_3 - Sx_4}{m}; \quad \dot{x}_3 = \frac{B}{V_0 + Sx_1} (Cx_5(t - \tau) \sqrt{x_3} - Sx_2) \\ \dot{x}_4 &= \frac{B}{V_0 - Sx_1} (Cx_5(t - \tau) \sqrt{p_a - x_4} - Sx_2); \quad \dot{x}_5 = -\frac{1}{\tau_{SV}} x_5(t - \tau) + \frac{k_{SV}}{\tau_{SV}} u_2(X)\end{aligned}\quad (25)$$

where  $F_f$  is given by

$$F_f = \left( F_c + F_s \frac{1}{1 + \left( \frac{x_2(t - \tau_f)}{v_s} \right)^2} \operatorname{sgn} x_2 + f_v x_2 \right) \quad (26)$$

with  $F_c$ ,  $F_s$  and the other variables to be numerically-experimentally determined.

### 3. Conclusions

The present paper continues the works of the authors published in national and international journals [15]-[22], see also [11], in the field of hydraulic servomechanisms analysis and synthesis. This time, a special attention is given to the development of mathematical models with time delay for these servomechanisms, very important in applications of all kinds, an aspect which is surprisingly relatively neglected in the domain literature. Therefore, this paper will be the starting point for the qualitative analysis of the proposed MHS and EHS mathematical models, as well for the synthesis of the control laws that compensate, in the case of the EHS, the effects of the time delay.

### REFERENCES

- [1] *R. Bellman, K. L. Cooke*, Differential-difference equation, Academic Press, 1963.
- [2] *A. Halanay*, Differential equations: stability, oscillations, time lags, Academic Press, 1966.
- [3] *L. E. Elsgolt, S. B. Norkin*, Introduction to the theory of differential equations with deviating arguments, Moskow, 1973.
- [4] *V. Răsvan*, Absolute stability of feedback control systems with delay (in Romanian), Bucharest, Publishing House of the Romanian Academy RSR, 1975.
- [5] *V. A. Khokhlov*, Electrohydraulic tracking systems (in Russian), Moscow, Nauka, 1971.
- [6] *V. Kolmanovskii, A. Myshkis*, Introduction to the theory and applications of the functional differential equations, Kluwer Academic Publishers, 1999.
- [7] *H. Olsson, K. J. Astrom, C. Canudas de Wit, M. Gafvert, P. Lischinsky*, Friction models and friction compensation, European Journal of Control, **18** (1998), No. 4, 65-80.
- [8] *A. Toader, I. Ursu*, Pilot modeling based on time delay synthesis, Proceedings of the Institution of Mechanical Engineers - Part G: Journal of Aerospace Engineering (Proc IME G J Aero Eng), **228** (2014), No. 5, 740-754.
- [9] *H. E. Merritt*, Hydraulic control systems, John Wiley & Sons, New York, 1967.
- [10] *M. Guillon*, L'asservissement hydraulique et électrohydraulique, Edition Dunod, Paris, 1972.
- [11] *I. Ursu, F. Ursu*, Active and semiactive control (in Romanian), Publishing House of the Romanian Academy, 2002.

- [12] *P. K. C. Wang*, Analytical design of electrohydraulic servomechanism with near time-optimal response, *IEEE Transactions on Automatic Control*, **8** (1963), No. 1, 15-27.
- [13] *E. Richard, R. Outbib*, Feedback stabilization of an electrohydraulic system, *Proceedings of 3rd European Control Conference*, Rome, Italy, 1995, 330-334.
- [14] *H. Hahn, A. Piepenbrink, K.-D. Leimbach*, Input/output linearization control of an electro-servo-hydraulic actuator, *Proceedings of 3rd IEEE Conference on Control Applications*, Glasgow, August, 1994, 9951000.
- [15] *A. Halanay, C. A. Safta, I. Ursu, F. Ursu*, Stability of equilibria in a four-dimensional nonlinear model of a hydraulic servomechanism, *Journal of Engineering Mathematics*, **49** (2004), No. 4, 391-40.
- [16] *A. Halanay, C. A. Safta, F. Ursu and I. Ursu*, Stability analysis for a nonlinear model of a hydraulic servomechanism in a servoelastic framework, *Nonlinear Analysis: Real World Applications*, **10** (2009), No. 2, 1197-1209.
- [17] *S. Balea, A. Halanay and I. Ursu*, Coordinates transformation and stabilization for switching models of actuators in servoelastic framework, *Applied Mathematical Sciences*, **4 (73)** (2010), 3625-3642.
- [18] *A. Halanay, C. A. Safta*, A critical case for stability of equilibria of delay differential equations and the study of a model for an electrohydraulic servomechanism, preprint, 2017.
- [19] *A. Halanay, C. A. Safta, C. Dragoi, V. F. Piraianu*, Stability analysis for a delay differential equations model of a hydraulic turbine speed governor, *Proceedings of the ICNPAA 2016 World Congress, AIP Conf. Proc.* 1798, 020134-1020134-9; doi: 10.1063/1.4972726, 2016.
- [20] *I. Ursu, A. Toader, S. Balea and A. Halanay*, New stabilization and tracking control laws for electrohydraulic servomechanisms, *European Journal of Control*, **19** (2013), No. 1, 65-80.
- [21] *A. Halanay and I. Ursu*, Stability of equilibria of some switched nonlinear systems with applications to control synthesis for electrohydraulic servomechanisms, *IMA Journal of Applied Mathematics*, **74** (2009), No. 3, 361-373.
- [22] *A. Halanay and I. Ursu*, Stability of equilibria in a model for electrohydraulic servomechanisms, *Mathematical Reports*, **9 (59)** (2007), No. 1, 47-54.
- [23] *K. L. Cooke and Z. Grossman*, Discrete Delay, Distributed Delay and Stability Switches, *Journal of Mathematical Analysis and Applications*, **86** (1982), 592-627.
- [24] *C. Canudas de Wit, H. Olsson, K. J. Astrom, and P. Lischinsky*, Dynamic friction models and control design, *Proceedings of the 1993 American Control Conference*, San Francisco, California, 1993, pages 19201926.