

## PARETO OPTIMIZATION APPROACH AND THE INHERENT SUB-OPTIMALITY OF THE RESULTED SEMI- CONTINUOUS WATER NETWORK

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*În prezent, numeroase studii abordează reducerea consumului de apă proaspătă în rețelele de ape industriale, considerând condiții complexe de operare și de topologie (contaminanți multipli, rețele de apă integrate, etc.) și utilizând pentru optimizare funcții obiectiv simple sau multiple. Studiul de față prezintă o abordare de tip Pareto, utilizată pentru optimizarea unei rețele de apă semi-continue, având un program de funcționare dat și contaminanți multipli. Cele două funcții obiectiv sunt antagonice, optimizarea lor generând fronturi Pareto (PF) pentru fiecare dintre intervalele de timp prestabilite. Procedura de optimizare a rețelei pentru intervalul de timp curent implică alegerea unui punct din frontul Pareto obținut anterior. Această lucrare prezintă o analiză a dependenței efectelor induse asupra topologiei și asupra condițiilor de operare la nivelul întregului proces discontinuu de modalitatea de selecție a punctului din PF utilizat pentru trecerea la intervalul următor de timp.*

*Many studies currently address the reduction of freshwater consumption at industrial level, under complex operating and topologic conditions (multiple contaminants, integrated water networks, etc.), while using single or multiple objectives. The present study offers insights on a Pareto approach used to optimize a semi-continuous water network handling multiple contaminants with a given schedule. The two objective functions are antagonistic by nature, the optimization generating Pareto fronts (PF) for each of the pre-established time intervals. The procedure of optimizing the network for the current time interval involves choosing a point from the previously obtained PF. This paper presents an analysis of the effects induced upon the topology and the operating conditions at the overall batch level by the way of the selection of the continuation point from the PF.*

**Keywords:** semi-continuous water network, batch processes, Pareto optimization approach, Pareto Front, freshwater minimization

### List of notations

$F_i$  = necessary flow of fresh feed, t/hr

$C_{kj}$ ,  $C_{ki}$  = concentration of contaminant  $k$  exiting unit  $j$ , respectively  $i$ , ppm

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- $C_{k,ST}$  = concentration of contaminant  $k$  exiting ST, ppm  
 $C_k^F$  = concentration of contaminant  $k$  in feed, ppm  
 $C_k^S$  = concentration of contaminant  $k$  in ST, ppm  
 $C_{ki}^{in}$  = concentration of contaminant  $k$  in the inlet stream of unit  $i$ , ppm  
 $C_{ki}^{out}$  = concentration of contaminant  $k$  in the outlet stream of unit  $i$ , ppm  
 $C_{ki}^{in,max}$  = maximum allowable inlet concentration of contaminant  $k$  for unit  $i$ , ppm  
 $C_{ki}^{out,max}$  = maximum allowable outlet concentration of contaminant  $k$  for unit  $i$ , ppm  
 $D_r$  = reference diameter  
 $D_{ij}$  = optimum economic diameter of the pipe linking the WU  $i$  and  $j$ ,  
 $\Delta \dot{m}_i$  = total load of contaminant provided by each unit, kg/hr  
 $\Delta \dot{m}_{ki}$  = load of contaminant  $k$  for water-using unit  $i$ , kg/hr  
 $F$  = Fanning friction factor  
 $F_{iq}$  = freshwater flow feeding WU $i$  in the time interval  $\tau_q$   
 $G_w$  = global fresh water quantity, t  
 $i, j$  = indexes for batch WUOs  
 $k$  = index for contaminants  
 $K$  = number of contaminants  
 $m$  = mass of wastewater in the storage tank, t  
 $N^*$  = number of overlapping WUs in a time interval  
 $N$  = number of overall semicontinuous WUs  
 $Q$  = number of time intervals  
 $q$  = indexes of time intervals  
 $S_i^{in}$  = flow coming from the ST and supplying unit  $i$ , t/hr  
 $S_i^{out}$  = flow directed from unit  $i$  to ST, t/hr  
 $SWN$  = semi-continuous water network  
 $\tau_q$  = time interval  
 $u$  = water flowrate t/hr  
 $W_i$  = stream of unit  $i$  directed to treatment network, t/hr  
 $W_{ST}$  = stream of ST directed to treatment network, t/hr  
 $X_{ji}, X_{ij}$  = internal flow from unit  $j$  to unit  $i$ , respectively from unit  $i$  to unit  $j$ , t/hr

## 1. Introduction

The increased population growth [1] determined a high consumption of earth resources. Therefore the scientific studies emphasized an escalating interest in complex optimization of resource utilization in industrial, domestic and agricultural processes. Water belongs to the most important utilities in the process

industries being used as reactant, thermal or cleaning agent. As large quantities of wastewater are produced by the continuous processes, they were the first to be subjected to optimization [2]. The increased interest in the batch processes appeared due to their inherent flexibility and adaptability, being suitable for the production of high-value chemicals like pharmaceuticals, paints, flavors and to the high contamination degree of the effluent [3-8]. Therefore, parallel work in batch water networks was developed in the last decade.

The two major classes of methods used for the optimization of water network (WN) systems are insight based (mainly pinch analysis and derived methods) and mathematical modeling techniques. Comprehensive reviews of these approaches are given by Foo [2], Bagajewicz [9] and Jeżowski [10].

The optimization of semi-continuous water networks (SWN) is performed with the methods already used for continuous WNs, but applied according to the discreteness of the tasks specific to SWN. Consequently, the difference between continuous and batch processes is given by the time constraints: wastewater of one water-using unit (WU) can be reused only by a different WU which starts immediately after the former WU ceased its processing time, considering that the concentration restrictions are met. This time constraint is partially bypassed using storage tanks (STs), which can collect the wastewater of a WU for it to be later reused.

The first insight based technique extended from continuous processes to SWNs is the one of Wang and Smith [11]. Usually, the insight based methods start from a predefined schedule trying to minimize the freshwater intake at the expense of the wastewater reuse increase. Concentration intervals are built so that the water is cascaded from the lowest concentration to the highest concentration interval. The excess water is stored for reuse from an interval to the other, and when required fresh water is added. From the same category of insight-based approaches, several algebraic techniques were developed, like water cascade analysis, time dependent concentration interval analysis [12-14]. They are based on the same concept of cascading water to a WU having higher inlet concentration limits.

When multiple contaminants are involved, the problem's complexity raises and the insight-based techniques lose their efficiency. Therefore the solution for such complex problems is represented by the mathematical modeling techniques for water minimization. They involve a superstructure, the pending mathematical model describing the constraints and one or more objective functions to be used in the optimization stage. The latter may start from a predefined schedule or it may include the scheduling part as well [15-17]. The optimizations are usually performed using deterministic algorithms, but evolutionary algorithms are progressively applied in SWN [18-20], although not as frequently as in continuous WN [10]. Adaptive random search [18], [21] and genetic algorithms [22-26] are

two representative examples of stochastic algorithms used in WN optimization. Tsai and Chang [27] were the first to use stochastic algorithms to optimize a WN. In SWN problems, the first to use a stochastic based genetic algorithm (GA) for overcoming the inherent nonlinearity of the problem were Zhou et al. [16].

Dogaru and Lavric [22] used GA to optimize a SWN dealing with multiple contaminants. Starting from a pre-established schedule, the WUs are ranked with respect to their maximum outlet concentration of contaminants and with respect to the process schedule (see Fig.1). Time intervals are delimited by events. An event represents the moment a WU starts or finishes its processing task. The wastewater is internally reused from WUs with lower maximum allowable outlet concentrations to WUs with higher outlet restrictions, synchronous within a time interval. The optimization of the SWN is performed on each time interval (defined according to the description above) targeting the best topology resulted when searching for the minimum freshwater consumption. The method is applicable to SWN with fixed loads and defined schedule, and can be easily extended to processes with water losses and/or gains. The network topology is dynamic as the overlapping WUs are different from one time interval to another. The method was tested on a case study composed of six WUs, three contaminants and one ST.

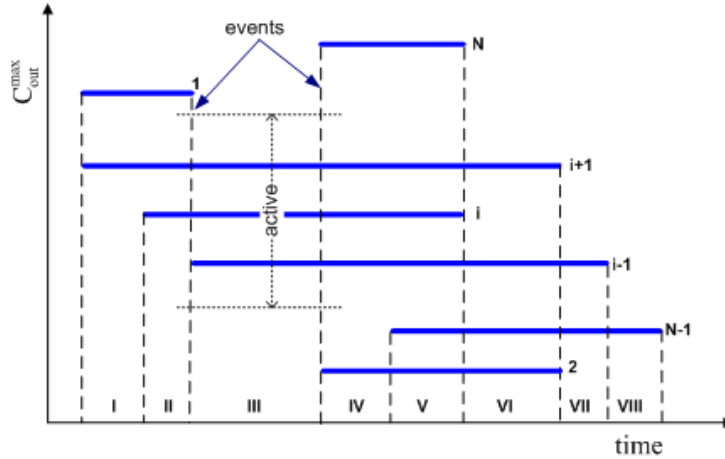


Fig.1. The Gantt diagram with the WUs ranked according to their schedule (primary ranking) and outlet maximum allowed concentration (secondary ranking) Dogaru and Lavric [24]

Furthermore, the work was extended by the addition of a second objective function: investments and operating costs [24]. The purpose of the optimization is to find the best succession of topologies that minimize simultaneously freshwater consumption and investments and operating costs. The optimization is performed using the Pareto approach with a vector-objective function: freshwater consumption and total investment and operating costs of the network pipes,

designed for optimum diameters. The same case study is analyzed together with the effects produced by introducing a regeneration unit to the network.

As a further development, Dogaru and Lavric [23] used a two level optimization strategy allowing simultaneous optimization of the dynamic network topology and the schedule. In the outer level, the schedule of the WUs is optimized according to their windows of opportunity [22] with respect to the freshwater consumption, while in the inner level, a dual-objective function is used to optimize the topology of the semi-continuous WN: the freshwater consumption and the combined investment and operating cost of the pipe network, designed for the optimum diameter. The optimum schedule should ensure a maximum reuse of wastewater between the discontinuous WUs and a minimum quantity of wastewater stored.

Currently, multi-objective optimization is used when several criteria must be met simultaneously when performing optimization. If the nature of the objectives is conflicting, the optimizer will deliver a set of equally optimal solutions (PF) from which one solution could be chosen using criteria which were not between the components of the vector objective function or are non-quantifiable.

The present study analyzes the Pareto approach for the SWN optimization and the inherent sub-optimality of the resulted semi-continuous water network, together with the effects produced by selecting one of the extreme points (corresponding to minimum values of the first objective function and of the second objective function, respectively) or the middle point on each PF obtained for a time interval, as starting point for the next. The consequences concerning freshwater consumption and total cost as well as effects on topology are discussed on a case study.

## **2. Problem statement**

There is given a SWN with a pre-established schedule, discontinuous with respect to raw-material transformation, but continuous with respect to the water/wastewater (seen here as utility) flow through WUs. The network of processing units could be abstracted as  $N$  WUs dealing with  $K$  contaminants. The timeline is formed by time intervals delimited by events (the moment a WU starts or finishes its task - with respect to raw-material transformation/water usage). The WUs functioning simultaneously on a time interval form a SWN supplied with freshwater and when possible with wastewater from different sources. All WUs are ranked according to their maximum outlet concentration of contaminants [22-24], in order to establish the rule for internal reuse of wastewater: it can be cascaded from the WUs having the lowest maximum outlet concentration of contaminants to the ones having a higher outlet limit. The outlet wastewater

which is not directly reused by other WUs is sent to the ST, which becomes a secondary source of wastewater for the SWN. It is assumed that the ST has unlimited capacity – this is useful for design purposes, when seeking for finding the ST capacity to account for its cost. The following time interval of the schedule is characterized also by a new topology as the new SWN changes according to the WUs working together and the associated operating conditions.

The optimization seeks the dynamics of the water/wastewater network topology during the whole schedule, with the associated wastewater internal reuse (treating the ST as a simultaneous sink and source) targeting the minimization of both the freshwater quantity and the investment and operating costs. The only difference between the possible topologies, at the whole batch scale, is represented by the network of pipes, as the operational costs related to the WUs are constant. Therefore, we safely assumed that the investments and operating costs are given by the pipes belonging to the network, which should have optimum diameters, and by the flows through these pipes.

Due to the dichotomy of the objective functions, for each time interval it is obtained a PF of equally optimal points. Rigorously, a huge number of runs are necessary to cover the entire optimality spectra associated to this basic SWN, rendering the problem unsolvable in a decent amount of time. Dogaru and Lavric [24] selected the points with values lower than one for the dimensionless freshwater consumption objective function. From them, the point situated in the middle of each PF was chosen as starting point for the optimization on the next time interval.

In this paper the results obtained by Dogaru and Lavric [24] are compared to the ones obtained when the points chosen from the PF correspond to the extreme values obtained on that time interval. The effects are analyzed and discussed based on a case study.

### 3. Network model

#### 3.1. Model description

The SWN is represented as an oriented graph having as nodes the active WUs and the ST and as arches the unidirectional flow connections [22-24]. The graph has a dynamic structure, as its configuration modifies whenever an event occurs. Within a time interval, the topology of the SWN does not change and the water/wastewater flows continuously throughout the overlapping WUs, permitting the use of some techniques from continuous WN optimization [22]. As illustrated in Fig. 2a WU can be supplied with freshwater ( $F_i$ ), with wastewater reused from a WU having a lower maximum outlet concentration of contaminants ( $X_{ji}$ ,  $j=1, i-1$ ) and/or with wastewater stored in the buffer tank ( $S_i^{in}$ ). The outlet wastewater

Fig.2. Schematic representation of the flows around the generic  $WU_i$

### 3.2. Simplifying assumptions

The simplifying assumptions regarding the WUs and the ST [22]:

- The freshwater is contaminant free:  $C_k^F = 0, \forall k = 1, K$
- The contaminant load for each WU,  $\Delta \dot{m}_{ki}, k = 1, K; i = 1, N^*$ , is constant over the entire operating period of the WU
- The internal and supply flows,  $X_{ji}, j = 1, i - 1; X_{ij}, j = i + 1, N^*$ ;  $S_i^{in}, i = 1, N^*$ , are constant over each time interval.

### 3.3. Mathematical model

The mathematical model consists of total and partial mass balance equations for each WU and for the ST [22].

a) Mass balances over the generic  $WU_i$  with  $1 \leq i \leq N^*$ , for a typical time interval (for brevity, the index of the time interval is disregarded)

- overall mass balance

$$F_i + \sum_{j=1}^{i-1} X_{ji} + S_i^{in} + \Delta \dot{m}_i = \sum_{j=i+1}^{N^*} X_{ij} + S_i^{out} \quad (1)$$

- partial mass balance

$$\sum_{j=1}^{i-1} (X_{ji} \cdot C_{kj}) + S_i^{in} \cdot C_k^S + \Delta \dot{m}_{ki} = \left( \sum_{j=i+1}^{N^*} X_{ij} + S_i^{out} \right) \cdot C_{ki} \quad (2)$$

If the *forbidden unit* concept applies  $S_{N^*}^{out}$  becomes  $W_{N^*}$  and the destination changes – TN instead of ST, while its flow rate and composition are unaffected.

#### b) Constraints for WUs

Each WU is characterized by maximum allowable values for their inlet and outlet concentrations of contaminants. The optimum solution is given by having both equalities satisfied in equations (3) and (4) for at least one contaminant.



- restrictions at the inlet of  $WU_i$

$$C_{ki}^{in} = \frac{\sum_{j=1}^{i-1} (X_{ji} \cdot C_{kj}) + S_i^{in} \cdot C_k^S}{F_i + \sum_{j=1}^{i-1} X_{ji} + S_i^{in}} \leq C_{ki}^{in,max} \quad (3)$$

- restrictions at the outlet of  $WU_i$

$$C_{ki}^{out} = \frac{\sum_{j=1}^{i-1} (X_{ji} \cdot C_{kj}) + S_i^{in} \cdot C_k^S + \Delta \dot{m}_{ki}}{\sum_{j=i+1}^{N^*} X_{ij} + S_i^{out}} \leq C_{ki}^{out,max} \quad (4)$$

c) Mass balances over the ST

- overall mass balance

$$\frac{dm}{dt} = \sum_{i=1}^{N^*} (S_i^{out} - S_i^{in}) \quad (5)$$

- partial mass balance

$$\frac{dC_k^S}{dt} = \frac{\sum_{i=1}^{N^*} S_i^{out} \cdot (C_{ki} - C_k^S)}{m} \quad (6)$$

Eq (6) illustrates that regardless the possible constant value of  $C_{ki}$  during a time interval, the contaminants' concentrations are time dependent due to the accumulation of mass  $m$  in the ST.

d) Double objective function

The optimization of the SWN is performed using a dual-objective function. The first objective targets the minimum freshwater consumption  $G_w$ , eq (7) by exploiting the internal reuse opportunities, disregarding the network complexity.

$$G_w = \sum_{i=1}^N \sum_{q=1}^Q (F_{iq} \cdot \tau_q) \quad (7)$$

where  $F_{iq}$  corresponds to the fresh water flow supplied to  $WU_i$  during the time interval  $\tau_q$ .

The second objective function comprises only the investment and operating costs of the pipes' network, considering the on-field geometry of the network, through the distances between WUs themselves, WUs and both freshwater source and ST. [24]. The resulted topology of the cost-based optimization should reveal a simplified network, regardless the freshwater consumption. In order to strengthen the solution optimality, every pipe of the wastewater system should have an optimum economic diameter, computed such as to minimize the friction losses due to the fluid velocity [28]. Choosing one year as the time basis, the unit length costs related to a pipe having optimum diameter [28] is:

$$C_{ij}^*(u) = \left\{ \left[ C^*(u) \right]_{pumping} + \left[ C^*(u) \right]_{pipe} \right\}_{ij} = \left[ \frac{\chi \cdot q_{ij}^a \cdot \rho^\beta \cdot \mu^\gamma \cdot K \cdot (1+J) \cdot H_y}{D_{ij}^\delta(u)} + B \right]_{pumping} + \left[ (1+u) \cdot X \cdot \left( \frac{D_{ij}(u)}{D_r} \right) \cdot K_F \right]_{pipe} \quad (8)$$

The exponents of the pumping term in eq (8) are determined by the flow regime, as implied by the flow rate  $u$ , and by the value of the Fanning friction factor.  $D_{ij}(u)$  is the optimum economic diameter of the pipe connecting the units  $i$  and  $j$ , and  $D_r$  represents the reference diameter [25], [28], [29].

The topology of the network, represented by the grid of active pipes (non-zero throughput flows) gives the dimensionless total cost of the network,  $\Gamma$ :

$$\Gamma = \sum_{q=1}^Q \left[ \frac{\underbrace{\sum_{j=1}^N C_{0,j}^* \cdot l_{0,j}^*}_{\text{supply pipes}} + \underbrace{\sum_{i=1}^N \sum_{j=i+1}^N C_{i,j}^* \cdot l_{i,j}^*}_{\text{internal pipes}}}{\underbrace{\sum_{j=1}^N C_{0,j}^* \cdot l_{0,j} + \sum_{i=1}^N \sum_{j=i+1}^N C_{i,j}^* \cdot l_{i,j}}_{\text{active and inactive/virtual}}} \right]_q \quad (9)$$

For computing the denominator in eq (9) it is used the largest diameter value corresponding to the highest freshwater inflow for all the pipes of the WN, despite they are active or not (no active pipe means no water flowing through it).

The two objective functions: FOB1 - freshwater consumption eq (7) and FOB2 - the total cost of the active pipes eq (9) compose the vector of the dimensionless dual-objective function eq (10):

$$fob = \begin{Bmatrix} G_w \\ \Gamma \end{Bmatrix} \quad (10)$$

The dichotomy of the components of the objective function eq (10) (a low value for  $G_w$  implies a high value for  $\Gamma$ , and the other way around) determined the use of the Pareto approach in the optimization process.

#### 4. Solving algorithm

The minimization of this dual-objective function, eq (10), is performed using the GA routine *gamultiobj* which implements in Matlab™ (MathWorks, Inc., USA) the Pareto approach. The GA was chosen since the complete mathematical model and the objective function are fully non-linear.

At the beginning of every time interval, by means of GA the internal flows,  $X_{ij}, i=1, N^*; j=i+1, N^*$  and the streams from the ST supplying the WUs,  $S_i^{in}, i=1, N^*$  are found and maintained constant over that time interval. Using the internal flows, the freshwater flow,  $F_i$ , and the stream directed from each WU to the ST,  $S_i^{out}$ , are computed together with the outlet contaminants concentrations of every WU,  $C_{ki}$ . The next step is integration of the differential algebraic equations (DAE) system (eq (1) to eq (7)), the length of the integration steps being pre-established, to obtain the values of contaminants concentrations in the ST. After every integration step, the computation of  $F_i$  and  $S_i^{out}$  is repeated and adjustments are made if the critical inlet/outlet concentration values are violated [22]. The detailed algorithm can be found in Dogaru and Lavric [22].

The Pareto approach targets simultaneous minimization of the dual-objective function, therefore for each time interval the result is a set of equally optimal points comprised in the PF. As the objectives are antagonistic, the points on the PF corresponding to low FOB1 values imply high costs on the axis representing FOB2. To continue the optimization on the next time interval, a point on the PF should be chosen. The rigorous procedure would imply analyzing the results obtained by continuing the optimization for each point in the PF, but the computational time will increase exponentially. For instance in the case study presented in Dogaru and Lavric [24] there are nine time intervals. In Table 1 the number of points in the PF corresponding to each time interval is listed. It should be emphasized that these values correspond to the continuation point from PF, namely from the middle of the points with FOB1 less than one [24].

Table 1

Number of points in the PF corresponding to each time interval is listed [24]

Time interval	I	II	III	IV	V	VI	VII	VIII	IX
No. of points on PF	12	44	5	24	23	23	11	64	2

For each of the 12 points present in the PF of the first time interval, the optimization should be continued on the second time interval, where 44 points are obtained; for the sake of simplicity, we assume that each of the 12 points will give the same number of equally optimal points on PF, namely 44. All of them should then be tested to find the next optimum and therefore for each of these 44 points we assume to obtain PFs with 5 points only (interval III). In the end, the number of routes to be investigated increases in a fractal like manner leading to a total number of about  $4.72 \cdot 10^{10}$  possibilities. Assuming each run takes one second, it would take around 1475 years to complete the analysis.

Dogaru and Lavric [24] proposed choosing the middle points of each of the PF corresponding to the time intervals, making a compromise between the two objective functions.

The present study takes a step further and analyzes by comparison the effects determined by choosing also the extreme points (corresponding to minimum values of FOB1, respectively FOB2) on each PF related to a time interval.

## 5. Results and discussions

This work presents a methodology for SWN optimization applied on a synthetic case study, as the literature does not offer such complex examples. The present study is designed to anticipate different challenges that can be met in practice, therefore it can be applied for any industrial case based on the same characteristics and given data.

### *Case study*

The case study chosen for the present analysis is the same used in Dogaru and Lavric [22-24]. Consider a network formed by six WUs, one ST of unlimited capacity, dealing with three contaminants. The inlet and outlet restrictions with respect to contaminants concentrations and the load of each contaminant released in the WUs are given together the specifications related to the schedule (see Tables 2 and 3).

Table 2

**Operating and Restriction Data for the WUs**

WUs	Loads, kg/h			Inlet Restrictions, ppm			Outlet Restrictions, ppm		
	$\Delta \dot{m}_{1i}$	$\Delta \dot{m}_{2i}$	$\Delta \dot{m}_{3i}$	$C_{1i}^{in,max}$	$C_{2i}^{in,max}$	$C_{3i}^{in,max}$	$C_{1i}^{out,max}$	$C_{2i}^{out,max}$	$C_{3i}^{out,max}$
WU <sub>1</sub>	0.35	0.25	0.35	0	0	0	35	45	55
WU <sub>2</sub>	0.15	0.35	0.25	15	20	25	70	115	110
WU <sub>3</sub>	0.55	0.45	0.15	15	35	0	75	95	85
WU <sub>4</sub>	0.45	0.15	0.45	25	45	45	105	85	100
WU <sub>5</sub>	0.25	0.65	0.65	45	35	55	90	105	120
WU <sub>6</sub>	0.65	0.55	0.85	35	20	25	85	110	95

Table 3

Schedule of the SWN

WUs	WU <sub>1</sub>	WU <sub>2</sub>	WU <sub>3</sub>	WU <sub>4</sub>	WU <sub>5</sub>	WU <sub>6</sub>
$t_{beg}$ , h	5	10	0	35	20	0
$t_{end}$ , h	50	80	25	100	75	70

WU1 and WU3 can be supplied with freshwater only as at least one of their maximum inlet concentrations of contaminants is zero.

For brevity the following terms will be used to designate the three options for the continuation of the analysis from a time interval to the next:

- lowF - the optimization is continued choosing the points on the PF of each time interval corresponding to the minimum value of FOB1 ( $FOB2 < 1$ )
- lowC - the optimization is continued choosing the points on the PF of each time interval corresponding to the minimum value of FOB2 ( $FOB1 < 1$ )
- middle - corresponding to the optimization performed choosing the middle points on the PF of each time interval (both FOB1 and FOB2 should be less than one)

It is worth mentioning that the points belonging to the PF are equally optimal; therefore another criterion is needed to be able to discriminate between the results best suited to the problem at hand.

#### *Freshwater consumption*

The results of optimization in terms of freshwater consumption are illustrated in Fig. 3.

The obtained results for freshwater consumption are intuitive: the lowest value is obtained when using lowF approach and it is increasing as we choose points on the PF corresponding to higher values for FOB1.

#### *Total cost*

Regarding the total cost, the results are unpredictable. Even if the highest value is obtained with the lowF approach, surprisingly, the lowest value among the three approaches belongs to the middle point (see Fig. 4).

The justification for this behavior is straightforward: when the main objective is minimizing FOB2, the optimization strategy is to simplify the complexity of the SWN, hence eliminating some pipe connections. In the present case study the connections having preponderantly longer lengths, link the WUs to the ST, therefore their removal will be preferred.

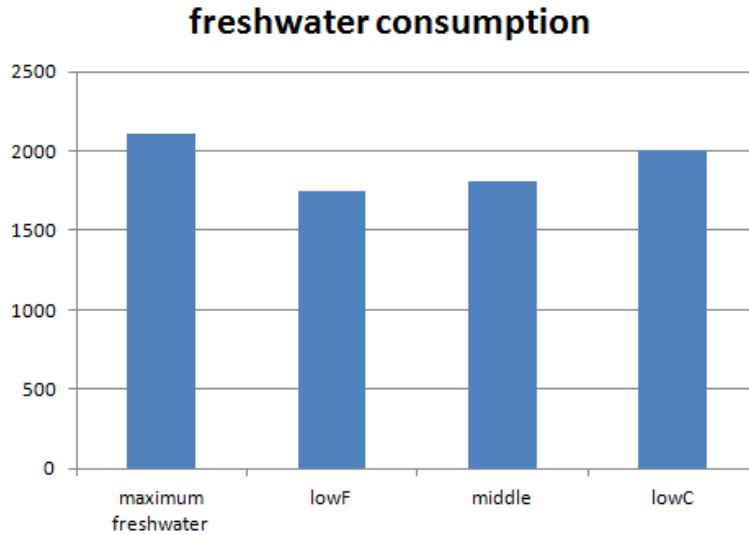


Fig.3. Freshwater consumption obtained after performing the Pareto optimization using the three approaches (lowF, lowC and middle point of the PF). The values are compared to the case when the WN uses only freshwater (maximum freshwater)

The figures Fig.5-7 present the topology of the operating WUs on time interval VII (50-70h) for the approaches: low F, middle point of PF and lowC, respectively. The connection between WU4 and the ST is suppressed in lowC case (see Fig. 7) determining higher internal flow rates ( $X_{46}$  and  $X_{45}$ ) and related diameters. For the other WUs of the network the flow rates directed to the ST are diminished ( $S_6^{out}$ ,  $S_2^{out}$  and  $S_5^{out}$ ), following the optimization strategy for simplifying the network. The increased internal flows supplying WU5 determined a significant requirement of freshwater that involved a larger diameter feeding pipe. Furthermore, the increased inlet flows for WU5 generated a higher outlet flow accompanied by a proportional diameter.

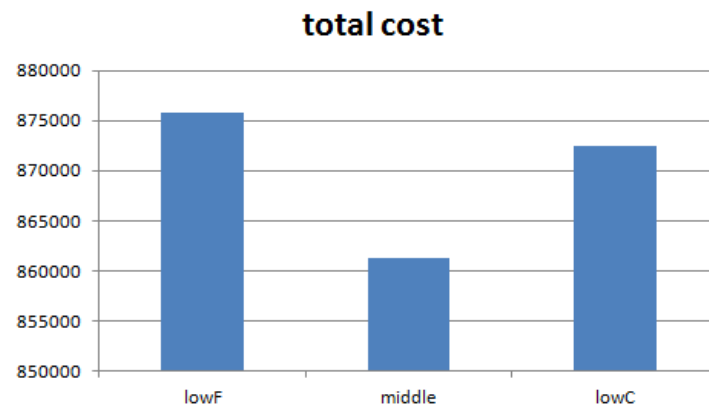


Fig.4. Total cost obtained after performing the Pareto optimization using the three approaches (lowF, lowC and middle point of the PF).

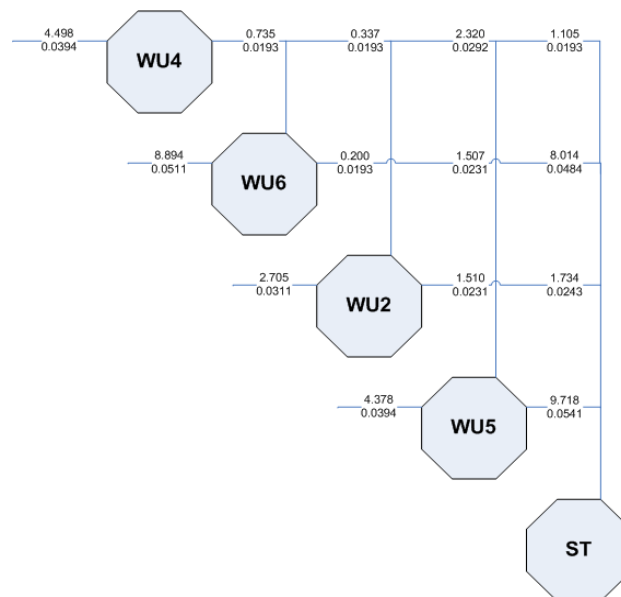


Fig.5. Network topology for time interval VII, resulted when optimization was performed using the lowF approach. Each pipe connection is characterized by a flow rate and its corresponding diameter (below the flow rate value) All the WUs have zero mean availability, thus the network is truly optimal

In this respect, as a connection to the ST is suppressed (see Fig. 6 and Fig. 7), the first consequence is the increase of the internal flows together with their corresponding pipe diameters. Moreover, by raising the internal flows, some

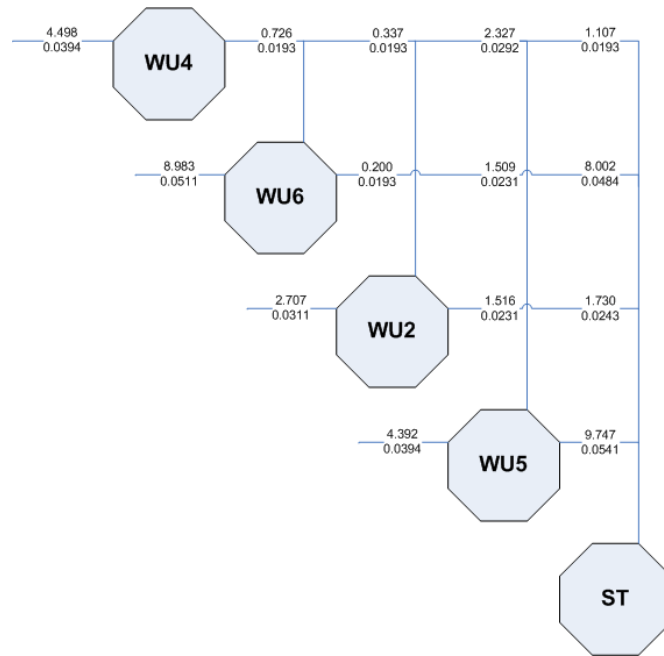


Fig. 6. Network topology for time interval VII, resulted when optimization was performed using the middle point approach. Each pipe connection is characterized by a flow rate and its corresponding diameter (below the flow rate value).

inlet restrictions for the WUs are violated requiring an increased freshwater quantity. Although the freshwater consumption is not added to the cost function, its increased flow would generate higher diameters for the pipes supplying the WUs with freshwater, leading to increased total cost. Higher flow rates decrease friction and lower the pumping costs, but this effect is overcome by the increased investment costs.

When making the compromise through the middle point approach, the SWN still has the tendency towards simplicity and cost reduction, but the equal magnitude of the first objective function (FOB1) determines a lower total cost. Consequently, the internal reuse may rise (Fig. 8) but within reasonable limits so that the freshwater consumption to be kept as low as possible. As a result the pipes' diameters are lower than in the lowC approach and so are the total costs.

The runs have demonstrated that there are optimum intervals of values for the operating parameters leading to numerous optimality conditions (flows - see Table 4) and several topologies. Due to the lack of space, we presented the data only for the freshwater consumption in Table 4.



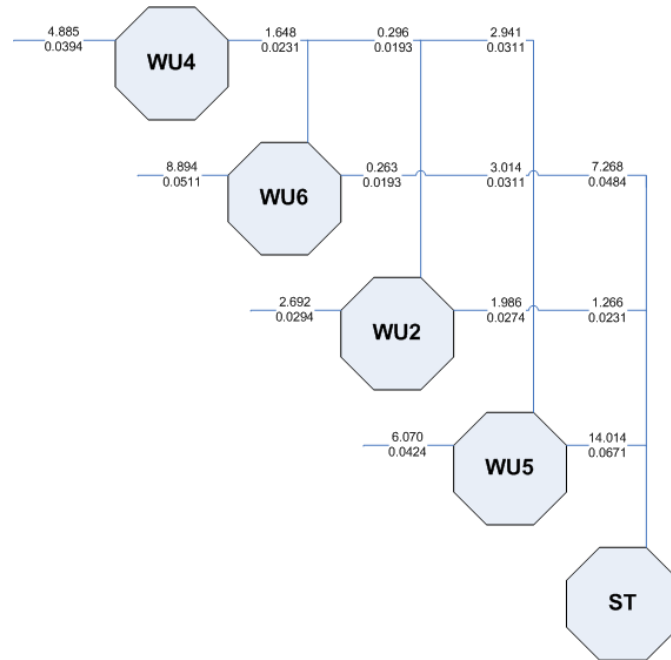


Fig.7. Network topology for time interval VII, resulted when optimization was performed using the lowC approach. Each pipe connection is characterized by a flow rate and its corresponding diameter (below the flow rate value).

Table 4

**Freshwater flow rates for each WU on each time interval. The first value in each cell corresponds to lowF approach, the second to middle and the third to lowC approach (if identical, only one value for the flow rate appears)**

Time interval	I	II	III	IV	V	VI	VII	VIII	IX	X
F1		9.999			10.033 10.916 10.112	10.768 10.768 13.666				
F2			2.214 1.979 2.181	2.262 2.267 2.842	2.069 2.089 2.402	2.344 2.344 5.735	2.705 2.707 2.692	2.713 2.675 2.659	2.703 2.855 2.855	
F3	7.332									
F4						1.390 1.390 2.607	4.498 4.498 4.885	4.498 4.769 4.973	4.498	5.380 5.256 4.915
F5				2.300 2.990 7.299	1.355 3.524 4.570	2.910 2.910 0	4.378 4.392 6.070	4.461 4.875 5.122		
F6	7.233 7.339 7.565	3.398 3.617 8.037	5.240 8.945 8.945	5.913 5.769 5.888	6.736 6.555 6.713	6.929 6.929 9.058	8.984 8.983 8.894			

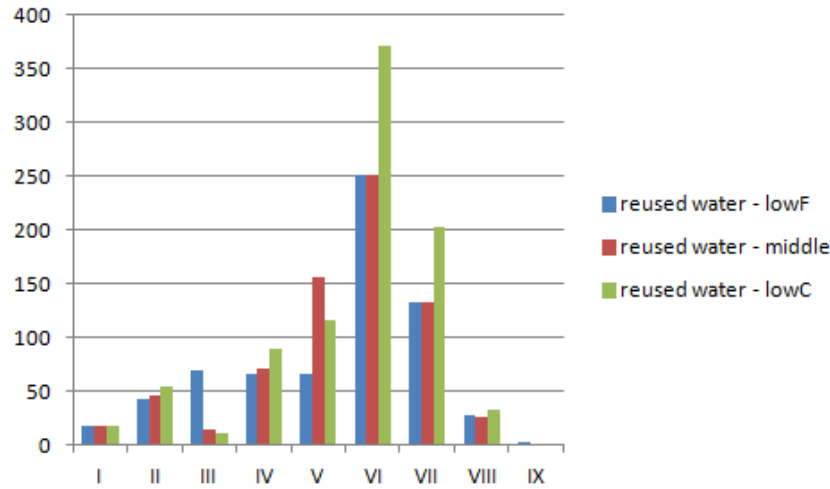


Fig.8. Internally reused wastewater within the batch cycle corresponding to three approaches

## 6. Conclusions

Pareto optimization approach searches for the best succession of topologies that ensures simultaneous minimization of freshwater consumption and total costs [24]. For each time interval, a set of equally optimal points are obtained, forming the Pareto front (PF). Advancing to the next time interval implies choosing only one point on the PF in order for the optimization to be performed in a decent amount of time. Dogaru and Lavric presented the results obtained when choosing the middle point of each PF corresponding to each time interval, as a compromise solution.

The present study analyzes by comparison the effects determined by choosing also the extreme points (corresponding to minimum values of FOB1, respectively FOB2) on each PF related to a time interval. Unexpectedly, the middle point approach not only offers balanced results with respect to freshwater consumption, but also the lowest total cost (among the three approaches: lowF, lowC and middle point).

One conclusion of the present analysis is that the results for the subsequent time interval and the overall optimization, respectively, are conditioned by the selection of the continuation point from the previously obtained PF. This aspect

reveals another conclusion: it was observed that, irrespectively of the selected point on the PF, the PF for the subsequent time interval is unchanged. Therefore, there is not an optimal topology having fixed values for the operating parameters.

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### REFERENCES

- [1] *I. M. L. Chew, S.L. Thillaivarrna, R. R. Tan, D. C. Y. Foo*, Analysis of inter-plant water integration with indirect integration schemes through game theory approach, *Clean Techn Environ Policy*, **13**, 2011, pp. 49-62
- [2] *D. C. Y. Foo*, A state-of-the-art review of pinch analysis techniques for water network synthesis, *Ind. Eng. Chem. Res.*, **48**, 2009, pp. 5125-5159
- [3] *J. F. Gouw, T. Majoz, D. C. Y. Foo, C.-L. Chen, J.-Y. Lee*, Water Minimisation Techniques for Batch Processes, *Ind. Eng. Chem. Res.*, **49**, 2010, pp. 8877-8893
- [4] *C.-L. Chen, C.-Y. Chang, J.-Y. Lee*, Resource-Task Network Approach to Simultaneous Scheduling and Water Minimization of Batch Plants, *Ind. Eng. Chem. Res.*, **50**, 2011, pp. 3660-3674
- [5] *E. Ahmetović, I.E. Grossmann*, Global Superstructure Optimization for the Design of Integrated Process Water Networks, *AIChE Journal*, **57**, 2011, pp. 434-457
- [6] *I. Halim, R. Srinivasan*, Sequential methodology for integrated optimization of energy and water use during batch process scheduling, *Computers and Chemical Engineering*, **35**, 2011, pp. 1575- 1597
- [7] *J.-K. Kim*, Design of discontinuous water-using systems with a graphical method, *Chemical Engineering Journal*, **172**, 2011, pp. 799- 810
- [8] *H. Tokos, Z. N. Pintarič, Y. Yang, Z. Kravanja*, Multilevel strategies for the retrofit of large-scale industrial water system-A brewery case study, *AIChE J.*, **58**, 2012, pp. 884-898
- [9] *M. Bagajewicz*, A Review of Recent Design Procedures for Water Networks in Refineries and Process Plants, *Comput. Chem. Eng.*, **24**, 2000, pp. 2093-2113
- [10] *J. Jeżowski*, Review of Water Network Design Methods with Literature Annotations. *Ind. Eng. Chem. Res.*, **49**, 2010, pp. 4475-4516
- [11] *Y. P. Wang, R. Smith*, Time pinch analysis, *Chem. Eng. Res. Des.*, **73**, 1995, pp. 905- 913
- [12] *Z. A. Manan, Y. L. Tan, D. C. Y. Foo*, Targeting the Minimum Water Flow rate Using Water Cascade Analysis Technique, *AIChE J.*, **50**, 2004, pp. 3169-3183
- [13] *D. C. Y. Foo, Z. A. Manan, Y. L. Tan*, Synthesis of maximum water recovery network for batch process systems, *J. Clean. Prod.*, **13**, 2005, pp. 1381-1394
- [14] *Y. Liu, X. Yuan, Y. Luo*, Synthesis of Water Utilization System Using Concentration Interval Analysis Method (I) Non-Mass-Transfer-Based Operation, *Chin. J. Chem. Eng.*, **15**, 2007, pp. 361-368
- [15] *C.-L. Chen, J.-Y. Lee, D. K. S. Ng, D. C. Y. Foo*, A Unified Model of Property Integration for Batch and Continuous Processes, *AIChE J.*, **56**, 2010, pp. 1845-1858

- 
- [16] R.-J. Zhou, L.-J. Li, W. Xiao, H.-G. Dong, Simultaneous optimization of batch process schedules and water-allocation network, *Comput. Chem. Eng.*, **33**, 2009, pp. 1153-1168
- [17] K. F. Cheng, C. T. Chang, Integrated water network designs for batch processes, *Ind. Eng. Chem. Res.*, **46**, 2007, pp. 1241-1253
- [18] J. Jeżowski, R. Bochenek, G. Poplewski, On application of stochastic optimization techniques to designing heat exchanger- and water networks, *Chem. Eng. Process.*, **46**, 2007, pp. 1160-1174
- [19] R. Karuppiiah, I. E. Grossmann, Global optimization of multiscenario mixed integer nonlinear programming models arising in the synthesis of integrated water networks under uncertainty, *Comput. Chem. Eng.*, **32**, 2008, pp. 145-160
- [20] G. Poplewski, J. M. Jeżowski, Stochastic optimization based approach for designing cost optimal water networks, *Comput.-Aided Chem. Eng.*, **20**, 2005, pp. 727-732
- [21] G. Poplewski, J. Jeżowski, A simultaneous approach for designing optimal wastewater treatment network, *Chem. Eng. Trans.*, **12**, 2007, pp. 321-326
- [22] E.-L. Dogaru, V. Lavric, Dynamic Water Network Topology Optimization of Batch Processes, *Ind. Eng. Chem. Res.*, **50**, 2011, 3636-3652
- [23] E.-L. Dogaru, V. Lavric, Multi-Objective Optimization of Semi-Continuous Water Networks, *Chem. Eng. Trans.*, **25**, 2011, 623-625
- [24] E.-L. Dogaru, V. Lavric, Pareto approach in designing optimal semi-continuous water networks, sent for the second revision at *Ind. Eng. Chem. Res.*, 2012
- [25] V. Lavric, P. Iancu, V. Plesu, Genetic algorithm optimization of water consumption and wastewater network topology, *J. Clean. Prod.*, **13**, 2005, pp. 1405-1415
- [26] R. Tudor, V. Lavric, Dual-objective optimization of integrated water/wastewater networks, *Comp. Chem. Eng.*, **35**, 2011, pp. 2853-2866
- [27] M.-J. Tsai, C.-T. Chang, Water usage and treatment network design using genetic algorithms, *Ind. Eng. Chem. Res.*, **40**, 2001, pp. 4874-4888
- [28] M.S. Peters, K.D. Timmerhaus, R.E. West, *Plant design and economics for chemical engineers*; McGraw Hill: Boston, 2003
- [29] X. Feng, K.H. Chu, Cost optimisation of industrial wastewater reuse systems, *Trans. IChemE. Part B Process Saf. Environ. Prot.*, **82**, 2004, 249-255