

## EFFECT OF VERTICAL TRACK IRREGULARITIES ON THE VIBRATION OF RAILWAY BOGIE

Mădălina DUMITRIU<sup>1</sup>, Ioan Cristian CRUCEANU<sup>2</sup>

*The paper delves into the effect of the vertical irregularities of the track upon the vibrations in a bogie of a rail vehicle, based on the results derived from numerical simulations, which are later compared with experimental results. The vertical track irregularities are introduced into the numerical model of the vehicle-track system in the form of a pseudo-stochastic function obtained through an original synthesis method of the vertical alignment, depending on the geometric quality of the track and velocity. The results derived underline the amplification of the bogie vibrations during running on a low quality track, speed-independent.*

**Keywords:** railway bogie, vertical track irregularities, track quality, vibration.

### 1. Introduction

The evolution of the dynamic performance in the rail vehicle and identification of certain solutions to increase safety, ride quality and ride comfort and decrease the dynamic loads occurring in the track, firstly involves the knowledge of the influence that the vibration-generating disturbances have during running.

The track irregularities are the main explanation of the vibrations in the railway vehicle [1]. It is about, on the one hand, about the track geometric irregularities that mainly come from the construction imperfections, track exploitation, change in the infrastructure due to the action of the environment factors or soil movements [2] and, on the other hand, about the irregularities in the rolling surfaces of railway rails. To these, the discontinuities of the rails are added - joints, switches, crossings [3].

The track irregularities exhibit deviations from the design geometry, which are a consequence of the lateral and vertical deviations of each rail in the nominal position. Typically, these deviations are combined to give an alternative set of four independent irregularities: track alignment or lateral alignment, longitudinal level or vertical alignment, cant variation and gauge variation.

---

<sup>1</sup> Assoc. Prof., Department of Railway Vehicles, University POLITEHNICA of Bucharest, Romania, e-mail: madalinadumitriu@yahoo.com

<sup>2</sup> Phd. Student, University POLITEHNICA of Bucharest, Romania, e-mail: ioan.cruceanu@gmail.com

Rolling on a track with deviations from the ideal geometry triggers vibrations in an axle that are further transmitted to the suspended masses of the vehicle, thus generating and maintaining their vibrations [4, 5], so that the dynamic response of the entire vehicle is affected by the track irregularities [6, 7].

One of the major causes for the vertical vibrations in the railway vehicle is the vertical alignment. This represents the vertical movement of each rail along the track, with respect to its design configuration, as shown in Fig. 1, (a) [2]. Another definition of the longitudinal level is to be found in the UIC 518 Leaflet [8]. The longitudinal level is herein defined as the geometrical error in the vertical plane, represented by the difference between a point of the rail top in the running plane and the ideal mean line of the longitudinal profile. Very often, the vertical alignment ( $\eta$ ) is defined on the track axis as the average value of the vertical alignment of the two rails ( $\eta_{r1,2}$ ), (see. Fig. 1, (b)) [9 - 11].

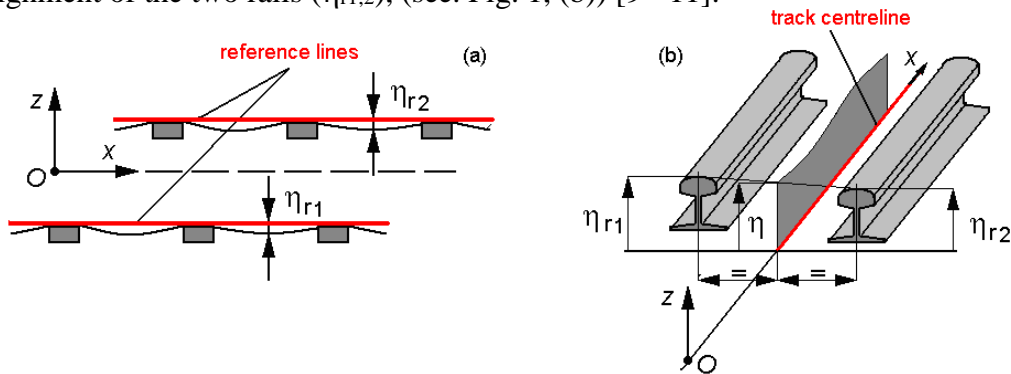


Fig. 1. Vertical alignment.

In order to study the influence of the track geometric deviations on the dynamic behaviour of the railway vehicle, within numerical models of the vehicle-track system, it is required to input as primary values the absolute values of the track irregularities defined as distance functions. These values are calculated by measuring the track geometry with specially fitted vehicles [12, 13]. Many times, this information is not available, hence the analytical description of the track irregularities can be used [14 - 16]. Since it is difficult to have such a detailed analytical description of the track geometry, a statistical representation is acceptable. This representation must contain data on the wavelengths, as well as the amplitudes of the track irregularities. A more complete description of the track geometry can be given by the power spectral densities of the measured track irregularities [11, 16 - 18]. A power spectral densities can be used either directly as an input for power spectra analysis or they can be retransformed into track irregularities as function of distance.

To study the effect upon the vertical vibrations in the railway bogie, the paper introduces the track vertical irregularities in the numerical model of the vehicle-track system in an original manner, i.e. in the form of a pseudo-stochastic

function that is conditioned by the track quality level and velocity [19]. This function is obtained by the synthesizing of the vertical alignment through a method relying on the power spectral density of the track irregularities, described as in ORE B176 [20] and the specifications included in the UIC 518 Leaflet [8] concerning the track geometric quality. On the one hand, the shape of the vertical track irregularities spectrum is being considered, and on the other hand, the standard deviations will be taken into account for vertical alignment and the peak values of the isolated track errors, which are used in the UIC 518 Leaflet to define the track quality levels QN1 (high level quality) and QN2 (low level quality).

The effect of the track vertical irregularities in correlation with the velocity upon the vertical vibrations in the railway bogie is analyzed in dependence on the RMS accelerations (root mean square) in the axles and bogie frame obtained from numerical simulations. The results of these simulations are later contrasted with the experimental results.

## 2. The mechanical model and the bogie-track system motion equations

To look into the impact of the track vertical irregularities upon the vertical vibrations in the bogie of the railway vehicle, the model in Fig. 2 is considered, as it features a two-axle bogie travelling at a constant speed  $V$  on a track with vertical irregularities in alignment and cross level. The model of the bogie includes 3 rigid bodies that help with modelling the bogie chassis and the two axles connected between them by Kelvin-Voigt type systems that model the primary suspension corresponding to each axle. The elastic element of the wheelset suspension has the constant  $2k_b$  and the damping elements has the constant  $2c_b$ .

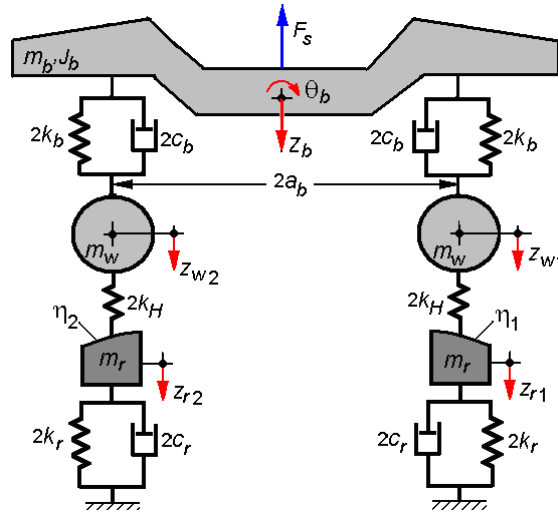


Fig. 2. The mechanical model of the bogie-track system.

The rigid vibration modes of the bogie in the vertical plan, namely bounce ( $z_b$ ) and pitch ( $\theta_b$ ) are taken into account. The bogie parameters are:  $m_b$  – bogie mass,  $2a_b$  – bogie wheelset,  $J_b = m_b i_b^2$  – inertia moment, with  $i_b$  – the gyration radius of the bogie. The axles of mass  $m_w$  operate a translation motion on the vertical direction ( $z_{w1,2}$ ).

While overlooking the coupling effects between the wheels due to the propagation of the bending wavelength, in the frequency domain specific to the vertical vibrations of the vehicle, an equivalent model with lumped parameters will be selected. Against each axle, the track is represented as an oscillating system with one degree of freedom that can travel on the vertical direction, with the displacement  $z_{r1,2}$ . The equivalent model of the track has mass  $m_r$ , stiffness  $2k_r$  and the damping coefficient  $2c_r$ .

The elasticity of the wheel-rail contact will be chosen by introducing certain elastic elements with a linear characteristic. The calculation of the stiffness in the contact elastic elements -  $2k_H$  for a wheel-rail pair - is done based on the theory of contact between two Hertz's elastic bodies by applying the linearization of the relation of contact deformation against the deformation corresponding to the static load on the wheel.

The track vertical irregularities are described against each axle via a pseudo-stochastic function [19]

$$\eta_{1,2}(x_{1,2}) = K_\eta f(x_{1,2}) \sum_{k=0}^N U_k \cos(\Omega_k x_{1,2} + \varphi_k), \text{ for } x_{1,2} > 0, \quad (1)$$

with  $x_1 = Vt$  and  $x_2 = Vt - 2a_b$ .

The amplitude  $U_k$  of the spectral component  $,k'$ , corresponding to the wavelength  $\Omega_k$ , is established with the help of the power spectral density of the track vertical irregularities, as defined in ORE B176

$$\Phi(\Omega) = \frac{A_{QN1,2} \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \quad (2)$$

where  $\Omega_c = 0,8246$  rad/m,  $\Omega_r = 0,0206$  rad/m, and  $A_{QN1,2}$  is a constant depending on the track quality; for the lag of the spectral component  $\varphi_k$  of the spectral component  $,k'$ , a uniform random distribution is selected.

The constant  $A_{QN1,2}$  is hence calculated that the standard deviation of the vertical alignment ( $\sigma_{\eta QN1,2}$ ) due to the components with the wavelength ranging from  $\Lambda_1 = 3$  m to  $\Lambda_2 = 25$  m correspond with the information in the UIC 518 Leaflet, subject to the track quality level, QN1 or QN2 (see Fig. 3)

$$A_{QN1,2} = 2\pi \frac{\sigma_{\eta QN1,2}^2}{\Omega_c^2 I_0}, \quad (3)$$

$$\text{with } I_0 = \int_{\Omega_2}^{\Omega_1} \frac{d\Omega}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \text{ for } \Omega_{1,2} = 2\pi/\Lambda_{1,2}. \quad (4)$$

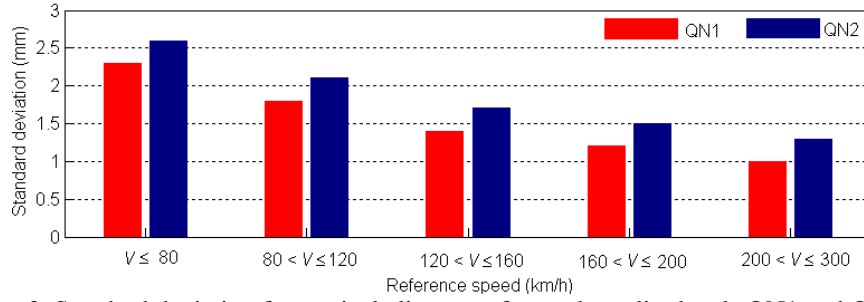


Fig. 3. Standard deviation for vertical alignment for track quality levels QN1 and QN2.

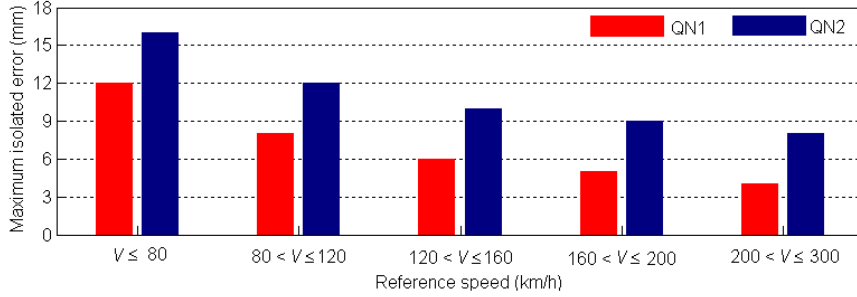


Fig. 4. The peak value for vertical alignment for track quality levels QN1 and QN2.

The coefficient  $K_\eta$  is a scaling coefficient of the amplitudes in the track vertical irregularities, a result of the ratio between the value of the admitted isolated defect  $\eta_{\text{admQN1,2}}$  as in the UIC 518 Leaflet (see Fig. 4) and the absolute maximum value of the vertical irregularities ( $\max|\eta(x_{1,2})|$ )

$$K_\eta = \frac{\eta_{\text{admQN1,2}}}{\max \left| f(x_{1,2}) \sum_{k=0}^N U_k \cos(\Omega_k x_{1,2} + \varphi_k) \right|}. \quad (5)$$

Function  $f(x_{1,2})$  is an adjustment function applied to the distance  $L_0$ , in the form of

$$f(x_{1,2}) = \left[ 6 \left( \frac{x_{1,2}}{L_0} \right)^5 - 15 \left( \frac{x_{1,2}}{L_0} \right)^4 + 10 \left( \frac{x_{1,2}}{L_0} \right)^3 \right] H(L_0 - x_{1,2}) + H(x_{1,2} - L_0), \quad (6)$$

where  $H(\cdot)$  is Heaviside step function.

The vertical motions of the bogie-track system are described by six motion equations, corresponding to the vibration modes of the bogie – bounce and pitch, the vertical displacements of the wheels and of the rails

$$m_b \ddot{z}_b + 2c_b[2\dot{z}_b - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_b[2z_b - (z_{w1} + z_{w2})] - F_s = 0 \quad (7)$$

$$J_b \ddot{\theta}_b + 2c_{b1}a_b[2a_b\dot{\theta}_b - (\dot{z}_{w1} - \dot{z}_{w2})] + 2k_ba_b[2a_b\theta_b - (z_{w1} - z_{w2})] = 0 \quad (8)$$

$$m_w \ddot{z}_{w1,2} + 2c_b(\dot{z}_{w1,2} - \dot{z}_b \mp a_b\dot{\theta}_b) + 2k_b(z_{w1,2} - z_b \mp a_b\theta_b) + 2k_H(z_{w1,2} - z_{r1,2} - \eta_{1,2}) = 0 \quad (9)$$

$$m_r \ddot{z}_{r1,2} + 2c_r\dot{z}_{r1,2} + 2k_s z_{r1,2} + 2k_H(z_{r1,2} - z_{w1,2} + \eta_{1,2}) = 0. \quad (10)$$

A system of six differential equations of second order was thus obtained, in which the variables of state, displacements and velocities are introduced, as such

$$q_{2k-1} = p_k, \quad q_{2k} = \dot{p}_k, \quad \text{for } k = 1 \dots 6, \quad (11)$$

where  $p_1 = z_b$ ,  $p_2 = \theta_b$ ,  $p_{3,4} = z_{w1,2}$ ,  $p_{5,6} = z_{r1,2}$ .

The result will be a system of 12 differential equations of first order that can be written in a matrix-like form,

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}, \quad (12)$$

where  $\mathbf{q}$  is the vector of the state variables,  $\mathbf{A}$  the matrix of the system and  $\mathbf{B}$  – the vector of the non-homogeneous terms. The system of equations (10) can be solved by a numeric integration, applying the Runge-Kutta algorithm.

### 3. The results of the numerical simulations

This section features the results of the numerical simulations concerning the bogie vibrations during running at a constant velocity on a track with vertical irregularities. The RMS accelerations of the axle and the bogie frame against the two axles are introduced, as calculated for different velocities on a good quality track (quality level QN1) and on a low quality track (quality level QN2).

Table 1

The parameters of the numerical model

Bogie mass	$m_b = 2700 \text{ kg}$
Axle mass	$m_w = 1400 \text{ kg}$
Rail mass (under the axle)	$M_r = 175 \text{ kg}$
Bogie wheelset	$2a_b = 2.5 \text{ m}$
Inertia moment	$J_b = 1.73 \cdot 10^3 \text{ kg} \cdot \text{m}^2$
Elastic constant of the suspension corresponding to a wheel	$k_b = 0.616 \text{ MN/m}$
Damping constant of the suspension corresponding to a wheel	$c_b = 9.05 \text{ kNs/m}$
Rail vertical stiffness	$k_r = 70 \text{ MN/m}$
Vertical damping of the track	$c_r = 20 \text{ kNs/m}$
Stiffness of the wheel-rail contact	$k_H = 1500 \text{ MN/m}$

As seen in Table 1, the parameters of the numerical model were specially provided for Minden-Deutz bogie, while the contribution of 300 spectral components with wavelengths between 3 and 120 m was considered to synthesize

the vertical irregularities in the track. The limit values of the interval of the wavelengths were thus calculated so that they are representative for the frequency interval of the bogie vertical vibrations.

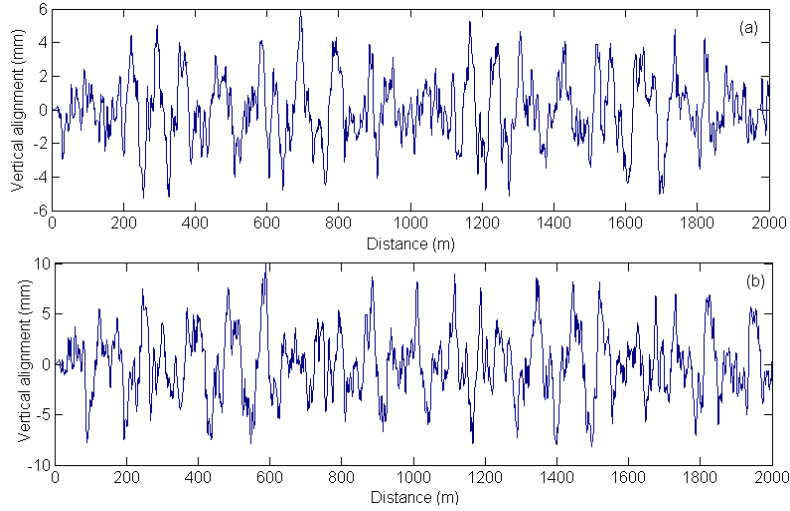


Fig. 5. The vertical track irregularities synthesis.

The track vertical irregularities were synthesized for a 2000 m distance, for a QN1 quality track and QN2, respectively (Fig. 5). The standard deviations for the vertical alignment and the peak values of the isolated defects were chosen for the maximum velocity of 140 km/h, namely  $\sigma_{\eta QN1} = 1.4$  mm;  $\sigma_{\eta QN2} = 1.7$  mm;  $\eta_{admQN1} = 6$  mm;  $\eta_{admQN2} = 10$  mm (see. Fig. 3 and Fig. 4).

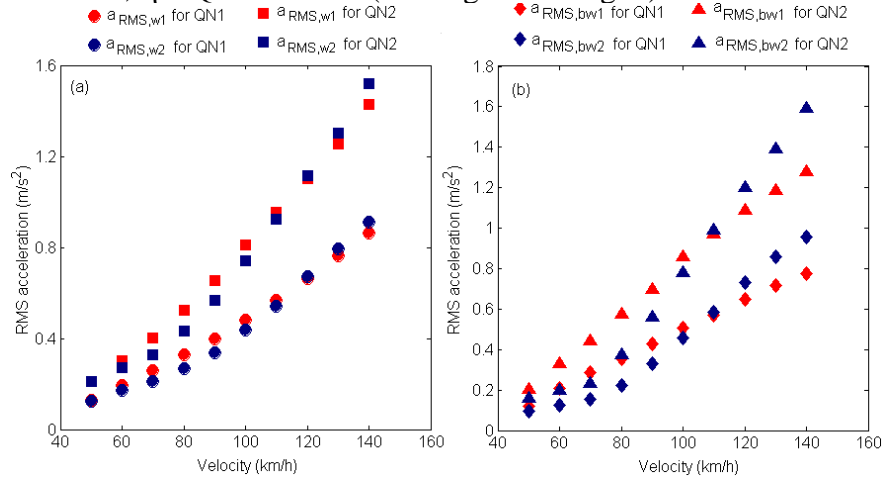


Fig. 6. RMS acceleration: (a) in axle 1 ( $a_{RMS,w1}$ ) and in axle 2 ( $a_{RMS,w2}$ );

(b) on the bogie frame above the axle 1 ( $a_{RMS,bw1}$ ) and above the axle 2 ( $a_{RMS,bw2}$ ).

The results in the diagrams of Fig. 6 show that the amplification of the bogie vertical vibrations are due to the increase in velocity. For instance, in axle 1, the

RMS acceleration during running on a QN1 quality track rises almost 7 times when velocity goes from 50 km/h to 140 km/h.

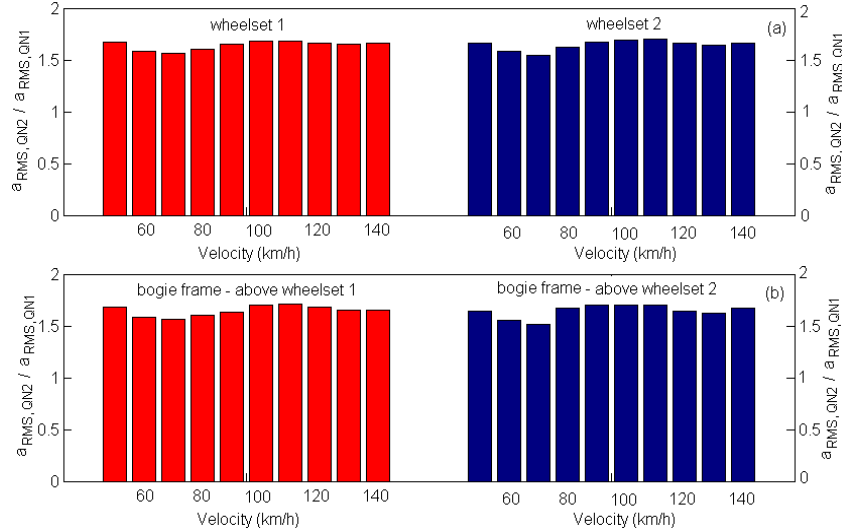


Fig. 7. Increase of the RMS acceleration in a QN2 quality track:  
(a) in axles; (b) on the bogie frame above the axles.

On the other hand, the amplification of the bogie vertical vibrations comes from the track geometric quality, speed independent. As seen in Fig. 7, the RMS accelerations of the axles and the bogie frame when running on a QN2 quality track is circa 6 times higher than the RMS accelerations in the QN1 quality track running situation.

Another thing to be noticed is that the RMS accelerations in the two axles are not equal, due to the phase displacement corresponding to the distance  $2a_b$  and the velocity. Similarly, the accelerations of the bogie frame against the two axles are not equal, either. For velocities under 120 km/h, the RMS acceleration is higher in the axle 1 and also on the bogie frame against axle 1. The accelerations of the axle 2 and of the bogie frame above the axle 2 are higher, along with an increase in velocity.

#### 4. Verification of the numerical results

To verify the results from the numerical simulations, the RMS accelerations of the axles and the RMS accelerations of the bogie above the two axles are compared with the measured RMS accelerations.

The experimental measurements were carried out using a measuring chain incorporating, on the one hand, the components of the measurement, acquisition and processing system for the vertical accelerations – namely four of 4514 Brüel & Kjær piezoelectric accelerometers and the set of the NI cDAQ-9174 chassis for data acquisition and the NI 9234 module for acquisition and synthesizing the data



from accelerometers, and, on the other hand, the NL-602U type GPS receiver for monitoring and recording the vehicle velocity.

The experimental measurements were made during a running on a current track, available for 160 km/h, on a track section in alignment and crosslevel. The measurements were done for the vertical accelerations of the axles and bogies on a passenger vehicle fitted with Minden-Deutz bogies, meant for the average and long lead traffic. The maximum speed of the vehicle is of 140 km/h.

The four accelerometers were installed on a side of the bogie, with one accelerometer on each axle-box and one accelerometer on the bogie frame against each axle (Fig. 8).

The accelerations measured at a constant velocity were recorded during 20-second sequences and the number of samples per second was 2048. The maximum velocity reached during measurements was 137 km/h.



Fig. 8. Mounting accelerometers on the bogie frame and the axle-boxes.

To compare experimental results with the simulated results, the measured accelerations are bandpass filtered in the frequency range corresponding to the wavelengths of the track irregularities between 3 and 120 m and to the velocity.

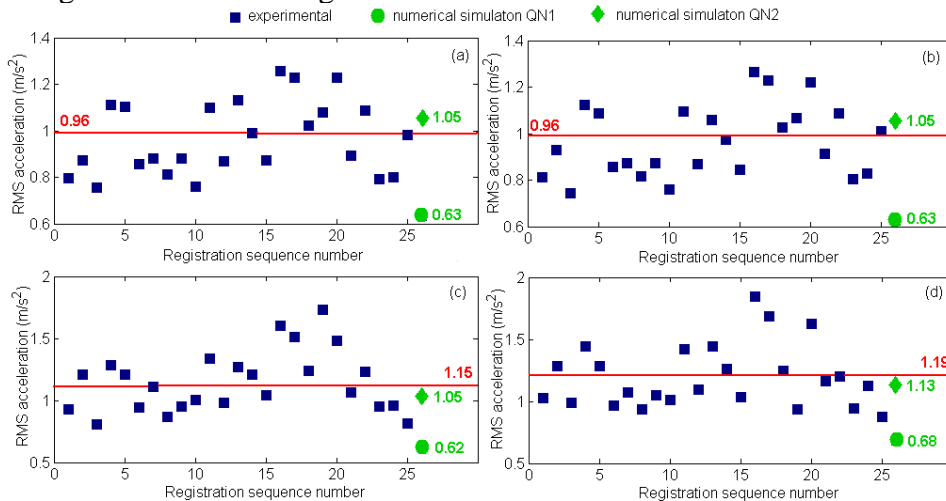


Fig. 9. Comparison of the RMS accelerations from numerical simulations to the RMS accelerations measured at the speed of 117 km/h: (a) in axle 1; (b) in axle 2;

(c) on the bogie frame above the axle 1; (d) on the bogie frame above the axle 2.

Fig. 9 shows the RMS accelerations for 25 measurement sequences at the constant speed of 117 km/h and the RMS accelerations derived from numerical simulations for a QN1 quality track and QN2 quality track, respectively. The measured accelerations were bandpass filtered in the range frequency of 0.27 – 10.83 Hz. Each measurement sequence is described by a certain value of the RMS acceleration, between  $0.75 \text{ m/s}^2$  and  $1.23 \text{ m/s}^2$  for axles, with a medium value of  $0.96 \text{ m/s}^2$ ; for the bogie frame, the values are between  $0.80 \text{ m/s}^2$  and  $1.84 \text{ m/s}^2$ , with an average value of  $1.15 \text{ m/s}^2$  and  $1.19 \text{ m/s}^2$ . This interval also contains the value of the RMS acceleration from numerical simulations for a QN2 quality track. In all the cases under study, this value is very close to the average value of the measured RMS accelerations. The difference between the RMS acceleration from numerical simulations and the average of the measured RMS accelerations does not exceed 10%. But instead, the RMS acceleration from numerical simulations for a QN1 quality track is outside the intervals of values corresponding to the measured RMS acceleration.

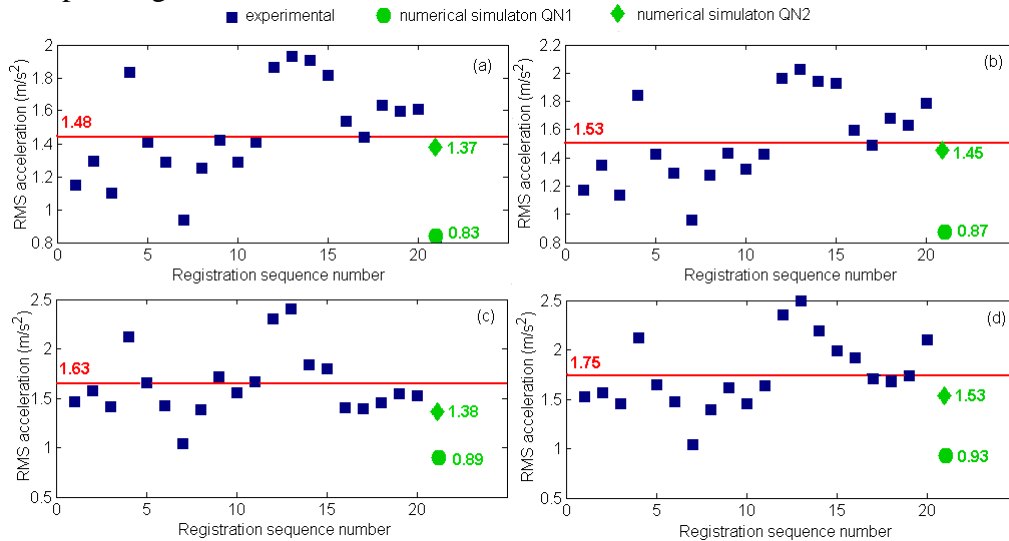


Fig. 10. Comparison of the RMS accelerations from numerical simulations to the RMS accelerations measured at the speed of 137 km/h: (a) in axle 1; (b) in axle 2; (c) on the bogie frame above the axle 1; (d) on the bogie frame above the axle 2.

Fig. 10 features the RMS acceleration for 20 measurement sequences at the constant speed of 137 km/h and the RMS accelerations from numerical simulations for a QN1 quality track and QN2 quality track. In this case, the measured accelerations were bandpass filtered in the 0.31 – 12.68 Hz frequency range. The values for the RMS accelerations in the axle 1 are between  $0.93 \text{ m/s}^2$  and  $1.93 \text{ m/s}^2$ , with an average of  $1.48 \text{ m/s}^2$ , while in the axle 2, the values are  $0.96 \text{ m/s}^2$  -  $2.02 \text{ m/s}^2$ , with an average of  $1.53 \text{ m/s}^2$ . The difference between the RMS acceleration from numerical simulations for a QN2 quality track and the

average value is of circa 8% in axle 1 and 5% for axle 2. The RMS accelerations measured on the bogie frame vary from  $1.04 \text{ m/s}^2$  to  $2.40 \text{ m/s}^2$  – above axle 1, with an average of  $1.63 \text{ m/s}^2$ , and from  $1.03 \text{ m/s}^2$  to  $2.49 \text{ m/s}^2$  – above axle 2, with an average of  $1.75 \text{ m/s}^2$ . Herein, the differences between the RMS acceleration from numerical simulations for a QN2 quality track and the average values are higher than 10%, namely 18% - above axle 1; 14% - above axle 2.

## 6. Conclusions

The effect of the track vertical irregularities upon the vibrations in the bogie of a passenger vehicle was examined as based on the results derived from numerical simulations, later compared to the experimental results.

The development of the numerical simulations relied on a bogie-track system model, where the track vertical irregularities were introduced in the form of a pseudo-stochastic function, which depends on the track geometric quality and on velocity. This function was derived via an original method, by which the track vertical irregularities are synthesized starting from the power spectral density recommended by ORE B176 for the average statistical properties of the European railways and the specifications in the UIC 518 Leaflet regarding the track geometric quality for the vertical alignment. An important advantage of the method herein consists in the fact that it enables the convenient establishment of the limits in the specific domain of the wavelengths in the track irregularities, so that it will be representative for the frequency range of the vertical vibrations in the railway vehicle. Another advantage is related to the idea that the method can be easily adjusted to any other form of power spectral density that describes the track irregularities and be also applied to synthesize the lateral alignment [21, 22].

The results above have validated that the bogie vibrations amplify along with the increase in speed and circulation on a low quality track. Independently from the velocity, the RMS accelerations during running on a low-quality track are of circa 1.6 times bigger than for a high quality track. The verification phase has shown that the accelerations from numerical simulations for a low-quality track are to be found in the dispersion interval of the measured RMS accelerations. The differences between the simulated RMS accelerations of the axle and the average value of the RMS accelerations measured in the axle-box do not exceed 10%. For the RMS accelerations in the bogie frame, such differences increased along with the velocity, being able to reach up to circa 18%.

## Acknowledgments

This work was supported by a grant of the Romanian National Authority for Science Research and Innovation, UEFISCDI, project number PN-III-P2-2.1-PED-2016-0212, within PNCDI III.

## REFERENCES

- [1]. *F. Cheli, R. Corradi*, On rail vehicle vibrations induced by track unevenness: Analysis of the excitation mechanism, *Journal of Sound and Vibration*, **vol. 330**, 2011, pp 3744–3765.

- [2]. *J. Pombo, J. Ambrósio*, An alternative method to include track irregularities in railway vehicle dynamic analyses, *Nonlinear Dynamics*, **vol. 68**, issue 1-2, 2012, pp 161-176.
- [3]. *M. Steenbergen*, Modelling of wheels and rail discontinuities in dynamic wheel-rail contact analysis, *Vehicle System Dynamics*, **vol. 44**, issue 10, 2006, pp. 763-787.
- [4]. *T. Mazilu*, Analysis of infinite structure response due to moving wheel in the presence of irregularities via Green's functions method, *Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science*, **10**, 2009, pp. 139-150.
- [5]. *T. Mazilu*, On the dynamic effects of wheel running on discretely supported rail, *Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science*, **vol. 10**, 2009, pp. 269-276
- [6]. *M. Dumitriu*, Analysis of the dynamic response in the railway vehicles to the track vertical irregularities. Part I: The theoretical model and the vehicle response functions, *Journal of Engineering Science and Technology Review*, **vol. 8**, issue 4, 2015, pp. 24 – 31.
- [7]. *M. Dumitriu*, Analysis of the dynamic response in the railway vehicles to the track vertical irregularities. Part II: The numerical analysis, *Journal of Engineering Science and Technology Review*, **vol. 8**, issue 4, 2015, pp. 32 – 39.
- [8]. UIC 518 Leaflet, Testing and approval of railway vehicles from the point of view of their dynamic behaviour – Safety – Track fatigue – Running behaviour, 2009.
- [9]. *H. Tsunashima, Y. Naganuma, T. Kobayash*, Track geometry estimation from car-body vibration, *Vehicle System Dynamics*, **vol. 52**, supplement, 2014, pp. 207-219.
- [10]. *V.K. Garg, R.V. Dukkipati*, *Dynamics of railway vehicle systems*, New York, 1984.
- [11]. *H. Claus, W. Schiehlen*, Modeling and simulation of railway bogie structural vibrations, *Vehicle System Dynamics*, **vol. 28**, Supplement, 1998, pp. 538-552.
- [12]. *S. Mohammadzadeh, H. Molatefi*, A novel method to estimate derailment probability due to track geometric irregularities using reliability techniques and advanced simulation methods, *Archive of Applied Mechanics*, **vol. 81**, 2011, pp. 1621–1637.
- [13]. *Y.Q. Sun, C. Cole, M. Spiriyagin*, Study on track dynamic forces due to rail short-wavelength dip defects using rail vehicle-track dynamics simulations, *Journal of Mechanical Science and Technology*, **vol. 27**, no. 3, 2013, pp. 629-640.
- [14]. *K. Karttunen, E. Kaho, A. Ekberg*, The influence of track geometry irregularities on rolling contact fatigue, *Wear*, **vol. 314**, iss. 1–2, 2014, pp. 78–86.
- [15]. *M. Zhu, X. Cheng, L. Miao*, Advanced stochastic modelling of railway track irregularities, *Advances in Mechanical Engineering*, Article ID 401637, 2013.
- [16]. *N. Demiridis, C. Pyrgidis*, Speed as a stand-alone indicator of the quality of the railway track, *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, **vol. 221**, 2007, pp. 419-428.
- [17]. *M.X.D. Li, E.G. Berggren, M. Berg, I. Persson*, Assessing track geometry quality based on wavelength spectra and track–vehicle dynamic interaction, *Vehicle System Dynamics*, **vol. 46**, 2008, pp. 261–276.
- [18]. *M. Dumitriu, I. Sebeşan*, Influence of the vertical track irregularities upon the wheel-rail dynamic forces, *Journal of Engineering Science and Technology Review*, **vol. 10**, issue 1, 2017, pp. 160-167.
- [19]. *M. Dumitriu*, Method to synthesize the track vertical irregularities, *Scientific Bulletin of the "Petru Maior" University of Tîrgu Mureş*, **vol. 11**, no. 2, 2014, pp. 17-24.
- [20]. ORE B 176. Bogies with steered or steering wheelsets, Report No. 1: Specifications and preliminary studies, 1989.
- [21]. *M. Dumitriu*, Numerical synthesis of the track alignment and applications. Part I: The synthesis method, *Transport Problems*, **vol. 11**, issue 1, 2016, pp. 19 – 28.
- [22]. *M. Dumitriu*, Numerical synthesis of the track alignment and applications. *Part II: The simulation of the dynamic behaviour in the railway vehicles*, *Transport Problems*, **vol. 11**, issue 2, 2016, pp. 5 – 16.