

## STRUCTURE OF IDEALISTIC FUZZY SOFT $\Gamma$ -NEAR-RING

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*The concept of idealistic fuzzy soft  $\Gamma$ -near-ring is introduced and some properties of idealistic fuzzy soft  $\Gamma$ -near-ring are given. Also, the definitions of homomorphic and inverse image of idealistic fuzzy soft  $\Gamma$ -near-ring are introduced and some theory of them is considered.*

**Keywords:**  $\Gamma$ -near-ring, idealistic fuzzy soft  $\Gamma$ -near-ring.

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### 1. Introduction

After the concept of soft sets has been introduced by Molodtsov [1] in 1999, soft sets theory has been extensively studied by many authors. Soft set theory has been applied to many different fields, such as function smoothness, Riemann integration, Perron integration, measurement theory, theory of groups, game theory, decision making. Maji et al. [20],[21] pointed out several directions of the applications of soft sets. They also studied several operations in the theory of soft sets. Chen et al. [2] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. Feng et al. [3] and Bhattacharya and Davvaz [4] studied the basic concept of soft set theory, and compared soft sets to fuzzy and rough sets, providing some examples to clarify their differences, also see [5].

The algebraic structure of soft set theory dealing with uncertainties has been studied by some authors. Aktas et al. [6] applied the notion of soft set to the group theory. Jun [7] introduced the notions of soft BCK/BCI-algebras, and then Jun et al. investigated their basic properties [8]. Öztürk et al. [9] discussed a new view of fuzzy  $\Gamma$ -rings. Yamak et al. [10] and Anvariyyeh et al. [11] studied soft algebraic hyperstructures.

It is well known that the concept of fuzzy sets, introduced by Zadeh [12], has been extensively applied to many scientific fields. In 1971, Rosenfeld [13] applied the concept to the theory of groupoids and groups. In 1982, Liu [14] defined and studied fuzzy subrings as well as fuzzy ideals. Since then, many papers concerning various fuzzy algebraic structures have appeared in the literature. The various constructions

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of fuzzy quotients rings and fuzzy isomorphisms have been investigated respectively by several researchers (see e.g.[15],[16],[17],[18],[19]).

Also, Maji et al. [20] expanded the soft set theory to fuzzy soft set theory introducing the definition of fuzzy soft set and presented some applications of this notion to the decision-making problems in [21]. İnan et al. [22] have already introduced the definition of fuzzy soft rings and studied some of their basic properties. Yao et al. proposed the concept of fuzzy sets and defined some operations on fuzzy soft sets [23].

$\Gamma$ -near-rings were defined by Satyanarayana [24], and the ideal theory in  $\Gamma$ -near-rings was studied by Satyanarayana [24] and Booth [25],[26]. Yun et al. [27],[28] considered the fuzzification of  $\Gamma$ -near-rings and their ideals.

In this paper, we attempt to study fuzzy soft  $\Gamma$ -near-ring theory by using fuzzy soft sets. We first introduce idealistic fuzzy soft  $\Gamma$ -near-rings generated by fuzzy soft sets and give properties of them. Consequently, we study definition of trivial and whole idealistic fuzzy soft  $\Gamma$ -near-ring and derive some results on them, respectively.

## 2. Preliminaries

In this section, for the sake of completeness, we first cite some useful definitions and results.

**Definition 2.1.** [12] *A fuzzy subset  $\mu$  in a set  $X$  is a function  $\mu : X \rightarrow [0, 1]$ .*

**Definition 2.2.** [1] *Let  $U$  be an initial universe and  $E$  a set of parameters. Let  $P(U)$  denotes the power set of  $U$  and  $A \subset E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F : A \rightarrow P(U)$ .*

In other words, a soft set over  $U$  is a parametrized family of subsets of the universe  $U$ .

**Definition 2.3.** [1] *Let  $U$  be an initial universe,  $E$  be a set of parameters and  $FS(U)$  denotes the fuzzy power set of  $U$  and  $A \subset E$ . A pair  $(F, A)$  is called a fuzzy soft set over  $U$ , where  $F$  is a mapping given by  $F : A \rightarrow FS(U)$ .*

*A fuzzy soft set is a parametrized family of fuzzy subsets of  $U$ .*

**Definition 2.4.** [1] *Let  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets over  $U$ . Then,  $(F, A)$  is said to be a fuzzy soft subset of  $(G, B)$  if*

- (1)  $A \subset B$
- (2)  $F(x)$  is a fuzzy subset of  $G(x)$ , for all  $x \in A$ .

*We denote the above inclusion relationship by  $(F, A) \tilde{\subset} (G, B)$ . Similarly,  $(F, A)$  is called a fuzzy soft superset of  $(G, B)$  if  $(G, B)$  is a fuzzy soft subset of  $(F, A)$ . We denote the above relationship by  $(F, A) \tilde{\supset} (G, B)$ .*

**Definition 2.5.** [6] *The intersection of two fuzzy soft sets  $(F, A)$  and  $(G, B)$  over  $U$  is the fuzzy soft set  $(H, C)$ , where  $C = A \cup B$  and  $x \in C$  defined by*

$$H(x) = \begin{cases} F(x) & \text{if } x \in A - B \\ G(x) & \text{if } x \in B - A \\ \min \{F(x), G(x)\} & \text{if } x \in A \cap B. \end{cases}$$

This is denoted by  $(F, A) \tilde{\cap} (G, B) = (H, C)$ .

**Definition 2.6.** [6] The union of two fuzzy soft sets  $(F, A)$  and  $(G, B)$  over  $U$  is the fuzzy soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $x \in C$ , defined by

$$H(x) = \begin{cases} F(x) & \text{if } x \in A - B \\ G(x) & \text{if } x \in B - A \\ \max \{F(x), G(x)\} & \text{otherwise.} \end{cases}$$

The above relationship is denoted by  $(F, A) \tilde{\cup} (G, B) = (H, C)$ .

**Definition 2.7.** [6] Let  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets. Then, we denote  $(F, A)$  AND  $(G, B)$  by  $(F, A) \tilde{\wedge} (G, B)$ . The fuzzy soft set  $(F, A) \tilde{\wedge} (G, B)$  is defined by  $(H, A \times B)$ , where  $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ , for all  $(\alpha, \beta) \in A \times B$ .

**Definition 2.8.** [6] Let  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets. Then,  $(F, A)$  OR  $(G, B)$  denoted by  $(F, A) \tilde{\vee} (G, B)$  is defined by  $(H, A \times B)$ , where  $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$ , for all  $(\alpha, \beta) \in A \times B$ .

In contrast with the above definitions of fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

**Definition 2.9.** [29] Let  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets over an universe  $U$  such that  $A \cap B \neq \emptyset$ . The bi-union of  $(F, A)$  and  $(G, B)$  is defined to be the fuzzy soft set  $(H, C)$ , where  $C = A \cap B$  and  $H(\varepsilon) = F(\varepsilon) \cup G(\varepsilon)$  for all  $\varepsilon \in C$ . This is denoted by  $(H, C) = (F, A) \tilde{\cup} (G, B)$ .

**Definition 2.10.** [29] Let  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets over a universe  $U$  such that  $A \cap B \neq \emptyset$ . The bi-intersection  $(F, A)$  and  $(G, B)$  is defined to be the fuzzy soft set  $(H, C)$ , where  $C = A \cap B$  and  $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$  for all  $\varepsilon \in C$ . This is denoted by  $(H, C) = (F, A) \tilde{\cap} (G, B)$ .

Now, we give the definition of near-ring and  $\Gamma$ -near-ring.

**Definition 2.11.** [30] A non-empty set  $R$  with two binary operations "+" (addition) and " $\cdot$ " (multiplication) is called a near-ring if it satisfies the following axioms:

- (1)  $(R, +)$  is a group,
- (2)  $(R, \cdot)$  is a semigroup,
- (3)  $(x + y) \cdot z = x \cdot z + y \cdot z$ , for all  $x, y, z \in R$ . It is a right near-ring because it satisfies the right distributive law.

**Definition 2.12.** [24] A  $\Gamma$ -near-ring is a triple  $(M, +, \Gamma)$  where

- (i)  $(M, +)$  is a group,
- (ii)  $\Gamma$  is a non-empty set of binary operators on  $M$  such that for each  $\alpha \in \Gamma$ ,  $(M, +, \alpha)$  is a near-ring.
- (iii)  $x\alpha(y\beta z) = (x\alpha y)\beta z$  for all  $x, y, z \in M$  and  $\alpha, \beta \in \Gamma$ .

**Definition 2.13.** [27] Let  $M$  and  $N$  be two  $\Gamma$ -near-rings and  $f$  be a mapping of  $M$  into  $N$ . Then,  $f$  is called a  $\Gamma$ -homomorphism if  $f(a + b) = f(a) + f(b)$  and  $f(a\alpha b) = f(a)\alpha f(b)$  for all  $a, b \in M, \alpha \in \Gamma$ .

**Definition 2.14.** [27] A fuzzy set  $\mu$  in  $\Gamma$ -near-ring  $M$  is called a fuzzy left (respectively right) ideal of  $M$  if the following requirements are met:

- (1)  $\mu(x - y) \geq \min \{\mu(x), \mu(y)\}$ ,
- (2)  $\mu(y + x - y) \geq \mu(x)$ , for all  $x, y \in M$
- (3)  $\mu(x\alpha u) \geq \mu(x)$  (respectively,  $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x)$ ) for all  $x, y, u, v \in M$  and  $\alpha \in \Gamma$ .

The concept of a support is defined for both fuzzy sets and formal power series in the literature. A similar notion for soft set  $(F, A)$  is defined, the set  $\text{Supp}(F, A) = \{x \in A \mid F(x) \neq \emptyset\}$  is called the support of the soft set  $(F, A)$ . The null soft set is a soft set with an empty support and a soft set  $(F, A)$  is non-null if  $\text{Supp}(F, A) \neq \emptyset$  [31].

**Definition 2.15.** [31] Let  $(F, A)$  be a non-null soft set over a near-ring  $M$ . Then,  $(F, A)$  is called a soft near-ring over  $M$ , if  $F(x)$  is a sub-near-ring of  $M$  for all  $x \in \text{Supp}(F, A)$

### 3. Some Operations Applied to Idealistic Fuzzy Soft $\Gamma$ -Near-ring

In this section, we introduce the notion of fuzzy soft  $\Gamma$ -near-ring and ideal of fuzzy soft  $\Gamma$ -near-ring. Throughout this paper, let  $(R, +, \Gamma)$  be  $\Gamma$ -near-ring,  $E$  be a parameter set and  $N \subset E$ . For the sake of simplicity, we will denote the  $\Gamma$ -near-ring  $(R, +, \Gamma)$  simply as  $R$ .

**Definition 3.1.** Let  $(f, N)$  be a non-null fuzzy soft set over  $R$ . Then,  $(f, N)$  is called an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if  $f_\mu(x)$  is a fuzzy ideal of  $R$  for all  $x \in \text{Supp}(f, N)$ . That is for all  $\mu \in N \subset E$  and for all  $x, y, u \in \text{Supp}(f, N)$ ,  $\alpha \in \Gamma$ , the following conditions hold:

- i.  $f_\mu(x - y) \geq \min \{f_\mu(x), f_\mu(y)\}$
- ii.  $f_\mu(y + x - y) \geq f_\mu(x)$
- iii.  $f_\mu(x\alpha u) \geq f_\mu(x)$  (respectively,  $f_\mu(u\alpha(x + v) - u\alpha v) \geq f_\mu(x)$ )

It may be noted that every idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  is a fuzzy soft  $\Gamma$ -near-ring of  $R$ , but the converse is not true in general.

**Example 3.1.** Let  $R = \{0, a, b, c\}$  and  $\Gamma$  the non-empty set of binary operations such that  $\alpha, \beta \in \Gamma$  are defined below:

$+$	0	a	b	c	$\alpha$	0	a	b	c	$\beta$	0	a	b	c
0	0	a	b	c	0	0	0	0	0	0	0	0	0	0
a	a	0	c	b	a	0	b	0	b	a	0	a	0	0
b	b	c	0	a	b	0	0	0	0	b	0	0	b	0
c	c	b	a	0	c	0	b	0	b	c	0	0	0	c

Clearly,  $(R, +, \Gamma)$  is a  $\Gamma$ -near-ring. Let  $A = \{a, b\}$ ,  $F : A \rightarrow FS(U)$  be a set-valued function defined by  $F(a) = \{(0, 0.8), (a, 0.2), (b, 0.2), (c, 0.2)\}$ ,  $F(b) = \{(0, 1), (a, 0.5), (b, 0.5), (c, 0.5)\}$ . Obviously,  $(F, A)$  is a fuzzy soft set over  $R$ . Also, we see that  $F(t)$  is a fuzzy ideal of  $R$  for all  $t \in A$ . Thus,  $(F, A)$  is a fuzzy idealistic soft  $\Gamma$ -near-ring over  $R$ .

**Example 3.2.** Let  $X$  and  $G$  be a non-empty set and an additive group respectively. Let  $N = M(X, G)$  and let  $\Gamma = M(G, X)$ , where  $M(A, B)$  denotes the set of all mappings from  $A$  into  $B$ . Then,  $N$  is a  $\Gamma$ -near-ring with the operations pointwise addition and composition of mappings.

Let  $(F, A)$  be a fuzzy soft set over  $N$ , where  $A = \{a, b\}$ ,  $F : A \rightarrow FS(N)$  be a set-valued function defined by  $F(a) = \{(0_N, 0.5), (f \neq 0_N, 0.3)\}$ ,  $F(b) = \{(0_N, 1), (f \neq 0_N, 0.2)\}$ . It can be easily checked that  $(F, A)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $N$ .

**Theorem 3.1.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then,  $(f, N) \tilde{\wedge} (g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if it is non-null.

*Proof.* Let  $(f, N) \tilde{\wedge} (g, M) = (h, S)$ , where  $S = N \times M$ ,  $\forall (a, b) \in N \times M$  and  $\forall x \in Supp(h, S)$ . We get  $h_{(a,b)}(x) = h_a(x) \cap g_b(x)$ . Since  $(f, N)$  and  $(g, M)$  are both non-null, the basic intersection between them must be non-null too. Therefore,  $\forall (a, b) \in N \times M$  and  $\forall x, y, u \in Supp(h, S)$  and  $\alpha \in \Gamma$  we have

$$\begin{aligned}
h_{(a,b)}(x - y) &= f_a(x - y) \cap g_b(x - y) \\
&\geq \min \{f_a(x), f_a(y)\} \cap \min \{g_b(x), g_b(y)\} \\
&= \min \{(f_a(x) \cap g_b(x)), (f_a(y) \cap g_b(y))\} \\
&= \min \{(f_a \cap g_b)(x), (f_a \cap g_b)(y)\} \\
&= \min \{h_{(a,b)}(x), h_{(a,b)}(y)\} \\
h_{(a,b)}(x - y) &\geq \min \{h_{(a,b)}(x), h_{(a,b)}(y)\} \\
h_{(a,b)}(y + x - y) &= f_a(y + x - y) \cap g_b(y + x - y) \\
&\geq f_a(x) \cap g_b(x) = (f_a \cap g_b)(x) \\
h_{(a,b)}(y + x - y) &\geq h_{(a,b)}(x) \\
h_{(a,b)}(x\alpha u) &= f_a(x\alpha u) \cap g_b(x\alpha u) \\
&\geq f_a(x) \cap g_b(x) = (f_a \cap g_b)(x) \\
h_{(a,b)}(x\alpha u) &\geq h_{(a,b)}(x).
\end{aligned}$$

Thus,  $(f, N) \tilde{\wedge} (g, M) = (h, S)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$   $\square$

**Remark 3.1.** If  $(f, N)$  and  $(g, M)$  are two non-null idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N) \tilde{\sqcap} (g, M)$  is also an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Remark 3.2.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then,  $(f, N) \tilde{\vee} (g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if it is non-null.

**Remark 3.3.** If  $(f, N)$  and  $(g, M)$  are two non-null idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is also an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Theorem 3.2.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then,  $(f, N)\tilde{\cap}(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if it is non-null.

*Proof.* We can write the intersection of two non-null idealistic fuzzy soft  $\Gamma$ -near-rings  $(f, N)$  and  $(g, M)$  over  $R$  denoted by  $(f, N)\tilde{\cap}(g, M)$  and defined by  $(f, N)\tilde{\cap}(g, M) = (h, C)$ , where  $C = N \cup M$  and  $\forall c \in C, \forall x, y, u \in R, \forall \alpha \in \Gamma$

$$h_c = \begin{cases} f_c & \text{if } c \in N - M \\ g_c & \text{if } c \in M - N \\ \min\{f_c, g_c\} & \text{if } c \in N \cap M \end{cases}$$

**Case 1:**  $c \in N - M$ . Then,  $h_c(x) = f_c(x)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  since  $(f, N)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Case 2:**  $c \in M - N$ . Then,  $h_c(x) = g_c(x)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  since  $(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Case 3:**  $c \in N \cap M$ . Then,  $h_c(x) = f_c(x) \cap g_c(x)$ . Therefore,

$$\begin{aligned} h_c(x - y) &= f_c(x - y) \cap g_c(x - y) \\ &\geq \min\{f_c(x), f_c(y)\} \cap \min\{g_c(x), g_c(y)\} \\ &= \min\{(f_c(x) \cap g_c(x)), (f_c(y) \cap g_c(y))\} \\ &= \min\{(f_c \cap g_c(x)), (f_c \cap g_c(y))\} \\ h_c(x - y) &= \min\{h_c(x), h_c(y)\} \\ h_c(y + x - y) &= f_c(y + x - y) \cap g_c(y + x - y) \\ &\geq f_c(x) \cap g_c(x) = f_c \cap g_c(x) \\ h_c(y + x - y) &\geq h_c(x) \\ h_c(x\alpha u) &= f_c(x\alpha u) \cap g_c(x\alpha u) \\ &\geq f_c(x) \cap g_c(x) = f_c \cap g_c(x) \\ h_c(x\alpha u) &\geq h_c(x) \end{aligned}$$

Thus,  $(f, N)\tilde{\cap}(g, M) = (h, C)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .  $\square$

**Remark 3.4.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then,  $(f, N)\tilde{\cup}(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if  $N$  and  $M$  are disjoint.

**Definition 3.2.** Let  $(f, N)$  be a non-null idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ . Then, we have the following:

- (1)  $(f, N)$  is said to be a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if  $f_\mu(x) = \{0\}$  for all  $x \in \text{Supp}(f, N)$  and for all  $\mu \in N$ .
- (2)  $(f, N)$  is said to be a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  if  $f_\mu(x) = R$  for all  $x \in \text{Supp}(f, N)$  and for all  $\mu \in N$ .

**Theorem 3.3.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then, the following hold:

- (1) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cap}(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (2) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cap}(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (3) If  $(f, N)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\cap}(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

*Proof.* By definition of the intersection of two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , we can write  $(f, N)\tilde{\cap}(g, M) = (h, C)$ , where  $C = N \cup M$  and for all  $c \in C$  and  $x \in \text{Supp}(h, C)$ ,

$$h_c(x) = \begin{cases} f_c(x) & \text{if } c \in N - M \\ g_c(x) & \text{if } c \in M - N \\ \min\{f_c, g_c\} & \text{if } c \in N \cap M, \end{cases}$$

- (1) Since  $C = N \cup M \neq \emptyset$ , it follows that  $c \in N - M, c \in M - N$  and  $c \in N \cap M$

**Case 1:**  $c \in N - M$ . Then,  $h_c(x) = f_c(x)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , since  $(f, N)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Case 2:**  $c \in M - N$ . Then,  $h_c(x) = g_c(x)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , since  $(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Case 3:**  $c \in N \cap M$ . Then,  $h_c(x) = f_c(x) \cap g_c(x)$ . Since both  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ ,  $f_c(x) = g_c(x) = \{0\}$  for all  $c \in C$  and for all  $x \in \text{Supp}(h, C)$ . Therefore,  $f_c(x) \cap g_c(x) = \{0\} \cap \{0\} = \{0\} = h_c(x)$ . Thus,  $(f, N)\tilde{\cap}(g, M) = (h, C)$  is also a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

The other conditions 2 and 3 can be easily obtained similarly.  $\square$

**Remark 3.5.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then, the following hold:

- (1) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (2) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (3) If  $(f, N)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Remark 3.6.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then, the following hold:

- (1) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\wedge}(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (2) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\wedge}(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

(3) If  $(f, N)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N) \tilde{\wedge} (g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**Remark 3.7.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then, the following hold:

- (1) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N) \tilde{\vee} (g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (2) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ , then  $(f, N) \tilde{\vee} (g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .
- (3) If  $(f, N)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, N) \tilde{\vee} (g, M)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

#### 4. Idealistic Fuzzy Soft sub- $\Gamma$ -near-ring

**Definition 4.1.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . The idealistic fuzzy soft  $\Gamma$ -near-ring  $(g, M)$  is called an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$ , if it satisfies

- i.  $M \subset N$
- ii.  $g_c$  is idealistic sub- $\Gamma$ -near-ring of  $f_c$  for all  $c \in M$ .

**Proposition 4.1.** Let  $(f, N)$  be a non-null fuzzy soft set over  $R$  and  $M \subset N$ . If  $(f, N)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(f, M)$  is also an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  provided it is non-null.

*Proof.* Since  $(f, N)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ ,  $f_c(x)$  is a fuzzy soft ideal of  $R$  for all  $x \in \text{Supp}(f, N)$ . Due to  $M \subset N$ ,  $(f, M)$  is a fuzzy soft  $\Gamma$ -subring of  $(f, N)$ . Therefore,  $f_c(y)$  is fuzzy soft ideal of  $(f, N)$  for all  $y \in \text{Supp}(f, M)$ . This implies that  $f_c(y)$  is a fuzzy ideal of  $R$  for all  $y \in \text{Supp}(f, M)$  since  $(f, M)$  is a fuzzy soft  $\Gamma$ -subring of  $(f, N)$ . Hence,  $(f, M)$  is also an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .  $\square$

**Definition 4.2.** Let  $(f_i, N_i)_{i \in I}$  be non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then, the intersection of these non-null idealistic fuzzy soft  $\Gamma$ -near-rings is defined to be non-null the idealistic fuzzy soft  $\Gamma$ -near-ring  $(g, M)$  satisfying the following conditions

- (1)  $M = \bigcap_{i \in I} A_i$
- (2) for all  $c \in M$ , there exists an  $i_0 \in I$  such that  $g_c = f_{i_0}(c)$ . In this case, we write  $\bigcap_{i \in I} (f_i, N_i) = (g, M)$ .

**Definition 4.3.** Let  $(f_i, N_i)_{i \in I}$  be non-null idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . Then,

- (1)  $(g, M) = \widetilde{\wedge}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring such that  $M = \prod_{i \in I} N_i$  and  $g_\mu = \bigcap_{i \in I} f_{i, \mu_i}$  for all  $\mu = (\mu_i)_{i \in I} \in M$

(2)  $(g, M) = \widetilde{\bigvee}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring such that  $M = \prod_{i \in I} N_i$  and  $g_\mu = \bigcup_{i \in I} f_{i, \mu_i}$  for all  $\mu = (\mu_i)_{i \in I} \in M$ .

**Theorem 4.1.** Let  $(f, N)$  be a non-null idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ . If  $\{(f_i, N_i) \mid i \in I\}$  is a non-empty family of idealistic fuzzy soft sub- $\Gamma$ -near-rings of  $(f, N)$ , where  $I$  is an index set, then:

- (1)  $\widetilde{\bigcap}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$
- (2)  $\widetilde{\bigwedge}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$
- (3)  $\widetilde{\bigvee}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$ , where  $N_i \cap N_j = \emptyset$  for all  $i, j \in I$ .

*Proof.* Using Definition 4.2 and since  $(f_i, N_i)$  is an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$  for all  $i \in I$ , we have  $g : M \rightarrow P(R)$  by  $g_\mu = f_{i, \mu}$  for all  $\mu \in M = \bigcap_{i \in I} N_i \subset N$  and  $i \in I$ . In this case  $f_{i, \mu}$  is an idealistic sub- $\Gamma$ -near-ring of  $R$ , for all  $\mu \in M$  and  $i \in I$ , so  $\bigcap_{i \in I} f_{i, \mu}$  is an idealistic sub- $\Gamma$ -near-ring of  $R$ . Thus,  $(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ . Hence,  $(g, M) = \widetilde{\bigcap}_{i \in I} (f_i, N_i)$  is an idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$  by the Definition 4.2.

The proofs of (2) and (3) can be done similarly.  $\square$

**Definition 4.4.** Let  $(f, N)$  and  $(g, M)$  be two trivial idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . The trivial idealistic fuzzy soft  $\Gamma$ -near-ring  $(g, M)$  is called a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$ , if it satisfies

- i.  $M \subset N$
- ii.  $g_c$  is trivial idealistic sub- $\Gamma$ -near-ring of  $f_c$  for all  $c \in M$ .

**Definition 4.5.** Let  $(f, N)$  and  $(g, M)$  be two whole idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$ . The whole idealistic fuzzy soft  $\Gamma$ -near-ring  $(g, M)$  is called a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring of  $(f, N)$ , if it satisfies

- i.  $M \subset N$
- ii.  $g_c$  is whole idealistic sub- $\Gamma$ -near-ring of  $f_c$  for all  $c \in M$ .

**Remark 4.1.** Let  $(f, N)$  and  $(g, M)$  be two non-null idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ . Then, the following hold:

- (1) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N) \widetilde{\vee} (g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (2) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N) \widetilde{\vee} (g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (3) If  $(f, N)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ , then  $(f, N) \widetilde{\vee} (g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (4) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N) \widetilde{\wedge} (g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (5) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N) \widetilde{\wedge} (g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .

- (6) If  $(f, N)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\wedge}(g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (7) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (8) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cup}(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (9) If  $(f, N)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\sqcup}(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (10) If  $(f, N)$  and  $(g, M)$  are trivial idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cap}(g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (11) If  $(f, N)$  and  $(g, M)$  are whole idealistic fuzzy soft sub- $\Gamma$ -near-rings over  $R$ , then  $(f, N)\tilde{\cap}(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .
- (12) If  $(f, N)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$  and  $(g, M)$  is a whole idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ , then  $(f, N)\tilde{\sqcap}(g, M)$  is a trivial idealistic fuzzy soft sub- $\Gamma$ -near-ring over  $R$ .

## 5. Homomorphic and inverse image of Idealistic Fuzzy Soft $\Gamma$ -Near-ring

**Definition 5.1.** Let  $(f, N)$  and  $(g, M)$  be two idealistic fuzzy soft  $\Gamma$ -near-rings over  $A$  and  $B$ , respectively. Let  $F : A \rightarrow B$  and  $G : N \rightarrow M$  be two functions. Then, the pair  $(F, G)$  is called an idealistic fuzzy soft  $\Gamma$ -near-ring homomorphism if it satisfies the following conditions

- i.  $F$  is an onto near-ring homomorphism;
- ii.  $G$  is an onto near-ring homomorphism;
- iii.  $F(f_c) = (g_c)G$  for all  $c \in N$ .

If there exists a fuzzy soft  $\Gamma$ -near-ring homomorphism between  $(f, N)$  and  $(g, M)$ , we say that  $(f, N)$  is idealistic fuzzy soft homomorphic to  $(g, M)$ , and it is denoted by  $(f, N) \sim_{\Gamma} (g, M)$ . Moreover, if  $F$  is an isomorphism and  $(f, N)$  is fuzzy soft homomorphic to  $(g, M)$ , it is denoted by  $(f, N) \simeq_{\Gamma} (g, M)$

Now, we show that the homomorphic image and preimage of an idealistic fuzzy soft  $\Gamma$ -near-ring are also idealistic fuzzy soft  $\Gamma$ -near-rings.

**Definition 5.2.** Let  $\varphi : X \rightarrow Y$  and  $\psi : N \rightarrow M$  be two functions, where  $N$  and  $M$  are parameter sets for fuzzy soft  $\Gamma$ -sets  $X$  and  $Y$ , respectively. Then, the pair  $(\varphi, \psi)$  is called a fuzzy soft  $\Gamma$ -function from  $X$  to  $Y$ .

**Definition 5.3.** Let  $(f, N)$  and  $(g, M)$  be two fuzzy soft  $\Gamma$ -near-ring over  $X$  and  $Y$ . Let  $(\varphi, \psi)$  be fuzzy soft  $\Gamma$ -function from  $X$  to  $Y$ .

- (1) The image of  $(f, N)$  under the fuzzy soft  $\Gamma$ -function  $(\varphi, \psi)$ , denoted by  $(\varphi, \psi)(f, N)$ , is the idealistic fuzzy soft  $\Gamma$ -near-ring over  $Y$  defined by  $(\varphi, \psi)(f, N) = (\varphi(f), \psi(N))$ , where

$$\varphi(f)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\varphi(a)=k} f_a(x) & \text{if } x \in \varphi^{-1}(y) \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in \varphi(N), \forall y \in Y$$

2. The preimage of  $(g, M)$  under the fuzzy soft  $\Gamma$ -function  $(\varphi, \psi)$ , denoted by  $(\varphi, \psi)^{-1}(g, M)$ , is the idealistic fuzzy soft  $\Gamma$ -near-ring over  $X$  defined by  $(\varphi, \psi)^{-1}(g, M) = (\varphi^{-1}(g), \psi^{-1}(M))$ , where  $\varphi^{-1}(g)_a(x) = g_{\psi(a)}(\varphi(x))$ ,  $\forall a \in \psi^{-1}(M)$ ,  $\forall x \in X$ .

If  $\varphi$  and  $\psi$  is injective(surjective), then  $(\varphi, \psi)$  is said to be injective(surjective).

**Definition 5.4.** Let  $(\varphi, \psi)$  be an idealistic fuzzy soft  $\Gamma$ -function from  $X$  to  $Y$ . If  $\varphi$  is a homomorphism function from  $X$  to  $Y$ , then  $(\varphi, \psi)$  is said to be idealistic fuzzy soft  $\Gamma$ -homomorphism. If  $\varphi$  is an isomorphism from  $X$  to  $Y$  and  $\psi$  is one to one mapping from  $N$  onto  $M$ , then  $(\varphi, \psi)$  is said to be idealistic fuzzy soft  $\Gamma$ -isomorphism.

**Theorem 5.1.** Let  $(f, N)$  be an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$  and  $(\varphi, \psi)$  be an idealistic fuzzy soft  $\Gamma$ -homomorphism from  $R$  to  $S$ . Then,  $(\varphi, \psi)(f, N)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .

*Proof.* Let  $k \in \psi(N)$  and  $y_1, y_2 \in Y, u \in S$ . If  $\varphi^{-1}(y_1) = \emptyset$  or  $\varphi^{-1}(y_2) = \emptyset$ , the proof is clear. Let us assume that  $\varphi(x_1) = y_1, \varphi(x_2) = y_2$  for some  $x_1, x_2 \in X$ . Then, we have

$$\begin{aligned} \varphi(f)_k(y_1 - y_2) &= \bigvee_{\varphi(x_1 - x_2) = y_1 - y_2} \bigvee_{\psi(a) = k} f_a(x_1 - x_2) \geq \bigvee_{\psi(a) = k} f_a(x_1 - x_2) \\ &\geq \bigvee_{\psi(a) = k} \min \{f_a(x_1), f_a(x_2)\} \\ &= \min \left\{ \bigvee_{\psi(a) = k} f_a(x_1), \bigvee_{\psi(a) = k} f_a(x_2) \right\} \end{aligned}$$

Then, we have

$$\begin{aligned} \varphi(f)_k(y_1 - y_2) &\geq \min \left\{ \left( \bigvee_{\varphi(x_1) = y_1} \bigvee_{\psi(a) = k} f_a(x_1) \right), \left( \bigvee_{\varphi(x_2) = y_2} \bigvee_{\psi(a) = k} f_a(x_2) \right) \right\} \\ &= \min \{ \varphi(f)_k(y_1), \varphi(f)_k(y_2) \} \end{aligned}$$

and similarly, for  $\alpha \in \Gamma$ , we have

$$\varphi(f)_k(y + x - y) \geq \varphi(f)_k(x) \text{ and } \varphi(f)_k(y_1 \alpha u) \geq \varphi(f)_k(y_1).$$

Hence,  $(\varphi, \psi)(f, N)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .  $\square$

**Theorem 5.2.** Let  $(g, M)$  be an idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$  and  $(\varphi, \psi)$  be a fuzzy soft  $\Gamma$ -homomorphism from  $R$  to  $S$ . Then,  $(\varphi, \psi)^{-1}(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

*Proof.* Let  $a \in \psi^{-1}(B)$  and  $x_1, x_2 \in X, \alpha \in \Gamma, u \in S$ .

$$\begin{aligned}\varphi^{-1}(g)_a(x_1 - x_2) &= g_{\psi(a)}(\varphi(x_1 - x_2)) \\ &\geq \min \{g_{\psi(a)}(\varphi(x_1)), g_{\psi(a)}(\varphi(x_2))\} \\ &\geq \min \{\varphi^{-1}(g)_a(x_1), \varphi^{-1}(g)_a(x_2)\} \\ \varphi^{-1}(g)_a(x_1\alpha u) &= g_{\psi(a)}(\varphi(x_1\alpha u)) = g_{\psi(a)}(\varphi(x_1)\alpha\varphi(u)) \\ &\geq g_{\psi(a)}(\varphi(x_1)) = \varphi^{-1}(g)_a(x_1)\end{aligned}$$

and similarly we have  $\varphi^{-1}(g)_a(x_1 + u - x_2) \geq \varphi^{-1}(g)_a(u)$ . Hence,  $(\varphi, \psi)^{-1}(g, M)$  is an idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .  $\square$

**Theorem 5.3.** *Let  $(f, N)$  and  $(g, M)$  be two idealistic fuzzy soft  $\Gamma$ -near-rings over  $R$  and  $S$ , respectively. Let  $(\varphi, \psi)$  be an idealistic fuzzy soft  $\Gamma$ -homomorphism from  $(f, N)$  to  $(g, M)$ . Then, the following hold true:*

- (1) *If  $f_c(x) = \text{Ker } (\varphi)$  for all  $x \in \text{Supp}(f, N)$ , then  $(\varphi(f), \psi(N))$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .*
- (2) *If  $(f, N)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ , then  $(\varphi(f), \psi(N))$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .*
- (3) *If  $(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ , then  $(\varphi^{-1}(g), \psi^{-1}(M))$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .*
- (4) *If  $g_c(y) = \varphi(R)$  for all  $y \in \text{Supp}(g, M)$ , then  $(\varphi^{-1}(g), \psi^{-1}(M))$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .*

**Proof. 1.** Suppose  $(\varphi, \psi)$  be an idealistic fuzzy soft  $\Gamma$ -homomorphism from  $(f, N)$  to  $(g, M)$ . Then,  $\psi$  is a mapping from  $N$  onto  $M$  and therefore  $\psi(N) = M$ . Also,  $\varphi$  is an idealistic near-ring  $\Gamma$ -homomorphism from  $R$  to  $S$ . Thus, the kernel of  $\varphi$  is given by  $\text{Ker } (\varphi) = \{x \in R : \varphi(x) = e_S\}$ , where  $e_S$  the identity element of  $S$ . Now, suppose that  $f_c(x) = \text{Ker } (\varphi)$ ,  $\forall x \in \text{Supp}(f, N)$ . Then,  $\forall c \in N$  and  $\forall x \in \text{Supp}(f, N)$ ,  $\varphi(f_c)(x) = \varphi(f_c(x)) = \varphi(x) = e_S$ . Hence,  $(\varphi(f), \psi(N))$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .

**2.** Suppose that  $(f, N)$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ . Then,  $f_c(x) = R$ ,  $\forall c \in N$  and  $\forall x \in \text{Supp}(f, N)$ . Therefore, it follows that  $\varphi(f_c)(x) = \varphi(f_c(x)) = \varphi(R) = S$ ,  $\forall c \in N$  and  $\forall x \in \text{Supp}(f, N)$ . Hence,  $(\varphi(f), \psi(N))$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ .

**3.** Since  $(\varphi, \psi)$  be an idealistic fuzzy soft  $\Gamma$ -homomorphism from  $(f, N)$  to  $(g, M)$ ,  $\psi : N \rightarrow M$  and therefore  $\psi^{-1}(M) = N$ . Also,  $\varphi$  is an idealistic near-ring  $\Gamma$ -homomorphism from  $R$  to  $S$  and therefore  $\varphi^{-1}(e_S) = e_R$ , where  $e_R$  and  $e_S$  are the identity elements of  $R$  and  $S$ , respectively. Now suppose that  $(g, M)$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $S$ . Then,  $g_c(y) = e_S$ ,  $\forall c \in M$  and  $\forall y \in \text{Supp}(g, M)$ . Therefore, we get  $\varphi^{-1}(g_c)(y) = \varphi^{-1}(g_c(y)) = \varphi^{-1}(e_S) = e_R$ ,  $\forall c \in M$  and  $\forall y \in \text{Supp}(g, M)$ . Hence,  $(\varphi^{-1}(g), \psi^{-1}(M))$  is a trivial idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .

**4.** Suppose that  $g_c(y) = \varphi(R)$  for all  $y \in \text{Supp}(g, M)$ . Since  $\varphi$  is an idealistic near-ring  $\Gamma$ -homomorphism from  $R$  to  $S$  and therefore  $\varphi(R) = S$ . Then,  $g_c(y) = \varphi(R) = S$  for all  $y \in \text{Supp}(g, M)$ . Because  $(g, M)$  is a whole idealistic fuzzy soft

$\Gamma$ -near-ring over  $S$ , it follows that  $\varphi^{-1}(g_c)(y) = \varphi^{-1}(g_c(y)) = \varphi^{-1}(\varphi(R)) = R$ . Hence,  $(\varphi^{-1}(g), \psi^{-1}(M))$  is a whole idealistic fuzzy soft  $\Gamma$ -near-ring over  $R$ .  $\square$

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