

COMPARATIVE EVALUATION OF EVOLUTIONARY LEARNING FITNESS FUNCTIONS IN MODEL FITTING FOR HUMAN HEART RATE DURING TREADMILL EXERCISE

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With the prevalence of new wearable devices and personal sensing, model fitting from real-world human-generated data has become a topic of interest in the fields of bioengineering, sports science, and medical engineering. In this study we analyze a fitting procedure based on evolutionary learning for human heart rate during treadmill exercise. We propose a new fitness function for the genetic algorithm based on the Pearson correlation coefficient and the coefficient of determination. This study utilizes real-world experimental data collected for linearity analysis, baseline model fitting, and validation, and includes statistical analysis of validation data. Results show that compared with a classical fitness function based on the root mean of square error, the proposed function is suitable for model fitting.

Keywords: model fitting, evolutionary learning, human heart rate, fitness functions, genetic algorithms, model identification, experimental data

1. Introduction

Model fitting from real-world data is a crucial approach in the fields of sports and medicine that involves calibrating mathematical models to empirical data gathered from the performance of athletes, training regimens, and physiological responses. This process aids in optimizing training programs, injury prevention, and performance enhancement [1]. Similarly, in medicine, model fitting using real-world patient data informs the understanding of physiological processes, disease progression, and treatment responses, ultimately leading to personalized healthcare interventions and improved patient outcomes [2].

The increase in wearable technology is currently enabling new methods of symptom assessment through digital phenotyping, which is defined as the on-line quantification of physiological responses using sensors and mobile devices [3].

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This trend increased the need for both computationally fast models obtained from raw data suitable for implementation on wearables (e.g., smartwatches), as well as fast, reliable fitting methods to be applied in on-line contexts (e.g., during exercise).

For machine learning, this is a compelling challenge supported by recent interests of reducing dataset sizes required for learning. This research direction is called data-centric AI and advocates for data quality over quantity [4]. Ultimately, this aligns with the new ubiquitous computing individualized model fitting needs.

Few studies regarding data-driven analysis and modeling of human heart rate (HR) response to physical exercise have been carried out in recent years. In [5, 6] the authors modeled the HR response during exercise and recovery situations. The models contain feed-forward and feedback components and are trustworthy for short duration exercises. A non-switching, non-linear anti-windup integral control for the long duration heart rate response to treadmill exercise was developed in [7]. In [8], the authors show that HR will continue to increase during prolonged exercise, due to causes such as increased body temperature, dehydration, accumulation of metabolites.

The identification and control of Hammerstein systems with the purpose of achieving a desired HR profile by tracking performances for an automated treadmill system is realized in [9], where the authors found a first order process for the HR model. A nonlinear system that models the HR response during and after treadmill walking exercise is developed in [10], as an interconnected system which consists of components that describe the central and peripheral local responses to exercise and their interactions. The model parameters were identified experimentally from subjects walking on treadmill at different speeds. Some studies focus on the dynamics of the HR response during exercises [11], for instance in [12] the model is formed of two coupled ordinary differential equations, for the HR kinetics in response to exercise. More recently, first and second order linear models have been explored [13].

Because human HR response to exercise is highly heterogenous, widely varying across categories (age, health status, neurological disorders, nutrition, time of day, circadian rhythm, physical fitness levels, lifestyle, etc.) [14], model fitting over real-world data is a problem that must result in individualized models. In this study, we explore HR model fitting with evolutionary learning, more specifically genetic algorithms [15]. These meta-heuristic optimization methods have been used in data-driven modelling with promising results [16].

For genetic algorithms, the fitness function describes the problem to be solved, i.e., the criterion to be optimized. During the artificial evolution process, the potential solutions to the problem are tested against the fitness function and a level of their fit to solve the problem is calculated. For modelling, the fitness function returns a measure of how well the model output matches the real-world collected data. In [17], we presented several fitness functions as performance

indexes for real-coded genetic algorithms, which ascertain the level of fit based on the approximation error between the model output and the real-world data.

The aim of this study is to determine whether fitness functions based on the root mean of square error, the Pearson coefficient of correlation, and the coefficient of determination are feasible to use for model fitting of HR response.

The rest of the paper is organized as follows: Section 2 describes the method, Section 3 the results and discussion, and finally Section 4 the conclusions.

2. Method

Evolutionary learning model fitting. The basic principle of model fitting with evolutionary learning (Fig. 1) requires computing an approximation error ε between heart rate as experimental data y_{exp} and model output y_m for the model excited with the same treadmill speed as input u . The learning procedure (a genetic algorithm) optimizes criterion J to find a fit model M . The output of the genetic algorithm is a vector of model parameters M_P that are coded into chromosomes as potential solutions of the fitting problem.

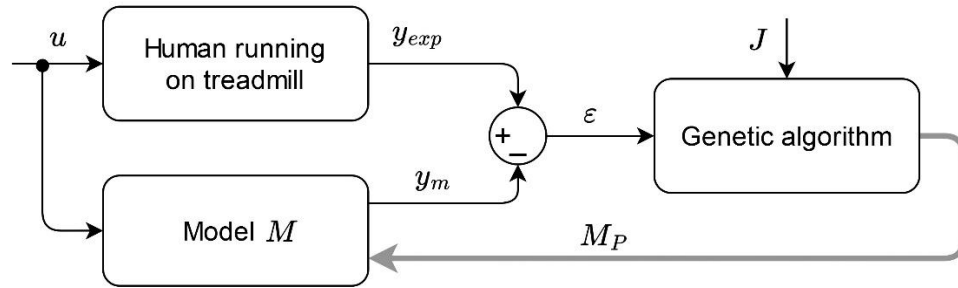


Fig. 1. Model fitting with evolutionary learning on experimental real-world data.

In this study, we employ a real-coded genetic algorithm that fits the data to a first order linear model given by:

$$M(s) = \frac{K}{Ts+1}, \quad (1)$$

where K is the model gain (adimensional) and T is the time constant (measured in seconds). Thus, the chromosome is given by:

$$[K \quad T]. \quad (2)$$

The evolution mechanisms are: uniform mutation, which replaces a randomly selected gene with a random value within the specified gene boundaries; arithmetic crossover, which generates two children based on a linear combination

of two parents as described in [18]; normalized ranking selection, introduced in [19], which is based on the probability of selecting an individual based on a ranked list. The termination is at either 100 generations or performance of $J = 0.0005$.

In this study, we propose a criterion J_{PR} based on the Pearson correlation coefficient ρ and the coefficient of determination R^2 [20, 21]:

$$\rho = \frac{\text{cov}(y_{exp}, y_m)}{\sigma_{y_{exp}} \sigma_{y_m}} \quad \text{and} \quad R^2 = 1 - \frac{\sum_{k=1}^n \varepsilon_k^2}{\sum_{k=1}^n (y_{exp_k} - \bar{y}_{exp_k})^2}, \quad (3)$$

where n is the number of data samples, $\varepsilon = y_{exp} - y_m$ is the approximation error, and \bar{y}_{exp} is the mean of the experimental data. Both ρ and R^2 measure the predictive power of the model and are defined over $[-1, 1]$, where positive values are for increasingly better fits (0 is worst and 1 is best), while the negative values are for opposing signal variations. The proposed fitness function becomes:

$$\begin{cases} \min_k J_{PR}(k) = \min_k \frac{\rho}{R^2} \\ 0 < \rho, R^2 \leq 1 \end{cases}. \quad (4)$$

We then use a fitness function criterion J_{RMSE} based on the classic root mean of square error (RMSE), defined as:

$$\min_k J_{RMSE}(k) = \min_k \sqrt{\frac{1}{n} \sum_{k=1}^n \varepsilon_k^2}. \quad (5)$$

We also test the fitness functions given by (4) and (5) against two fitness functions with a criterions equal to either ρ or R^2 and the limit conditions:

$$\begin{cases} \min_k J_P(k) = \min_k \rho \\ 0 < \rho, R^2 \leq 1 \end{cases} \quad \text{and} \quad \begin{cases} \min_k J_R(k) = \min_k R^2 \\ 0 < \rho, R^2 \leq 1 \end{cases}. \quad (6)$$

The search space of the model parameters has pre-defined boundaries given by: $K \in [K_{min}, K_{max}]$ and $T \in [T_{min}, T_{max}]$.

Experimental setup and data acquisition. The experiment consisted of a series of HR measurements using a portable heart rate sensor during running on a treadmill. The subject was a fit male adult (35 years) that has at least 12 hours of training during his weekly schedule. For this experiment we used a Kettler Boston XL treadmill (Fig. 2) for controlling the running speed and a Polar Wearlink Bluetooth heart rate monitor to record their output (Fig. 2). The signal from the HR monitor was fed into a Labview VI that records the heart rate and then saved as an Excel Worksheet file. The treadmill can generate speeds between 0 and 16 km/h,

while the sensor measures HR between 20 and 250 beats per minute (bpm).



Fig. 2. Equipment for data acquisition: treadmill (left) and sensor (right).

Two data collection experiments were performed: for linearity analysis and for dynamic model fitting.

Experiment 1: linearity analysis. The protocol for this experiment used a gradual increase in treadmill speed: 15 minutes warm-up; 7 minutes at 4 km/h; 7 minutes at 6 km/h; 5 minutes at 8 km/h; 5 minutes at 10 km/h; 5 minutes at 12 km/h; 5 minutes at 14 km/h; 15 minutes cool-down.

Experiment 2: model fitting. The protocol for this experiment was split into two stages spanning over 8 weeks, to allow for variations in the response of the exercise. First, a baseline measurement was collected at week 1. During week 8, six measurements were collected once a day. The measurement protocol is: 15 minutes warm-up; 5 minutes at 6 km/h; 5 minutes at 12 km/h; 10 minutes cool-down.

Analysis. Data from Experiment 1 informs the linearity analysis, through which we test the time invariance of the HR response for speeds in the [4, 14] km/h interval. Thus, we can determine the running speeds for which model M described in (1) will be valid. Data for Experiment 2 informs the model fitting with evolutionary learning. To obtain an adimensional model gain K , we first normalize input and output data to [0, 1]: a) conversion to percentages, b) vertical shift to initialize the response in 0, and c) scaling by step size. The normalization procedure is standard for data-driven dynamic model identification [17].

In our study, we conducted an analysis of variance (ANOVA) test to analyze the differences among the four fitness functions. To account for the increased risk of errors resulting from multiple comparisons, we applied a post-hoc Bonferroni test. This test adjusts the significance level for each individual comparison, ensuring a more rigorous threshold and reducing the likelihood of false positives.

All fitting, optimization, analyses, and visualizations are obtained using Matlab 16 and SPSS 17.0b. Results are considered significant for $p < 0.05$.

Implementation. The evolutionary learning procedure is available as a case study included in the MATLAB/Simulink GAOT-ECM (Genetic Algorithm

Optimization Toolbox: Extension for Control and Modeling) software package, which is available for download at [22] and includes test data. A detailed description of the toolbox extension is given in [17]. The toolbox is based on the GAOT implementation by [23]. The example for this study includes:

1. Level 1: inexperienced users can customize the demonstration script configuration using the "HRidentification.m" file:
 - name of a Simulink implementation (*.mdl file) of the model to be fitted; GAOT-ECM provides several model structures.
 - name of a *.mat file that is used in identification process, containing:
 - initial and general boundaries for each model parameter.
 - an array containing the input and output experimental data.
 - name of the fitness function (*.m file).
 - name of performance criterion (from [17]).
2. Level 2: allows more advanced configuration settings in the main function "GAOT ECM ModelIdentification.m" [23]:
 - evalFN: name of the *.m file that represents the fitness function.
 - termFNOptimalValue: scalar representing a termination constraint (satisfactory fitness chosen by user).
 - initBounds, varBounds: arrays containing the initial and general search domains for each parameter (one line per parameter).
 - populationSize: scalar number of individuals per generation.
3. Level 3: configurations of the GA itself in file "GAOT ECM ModelIdentificationGA.m": number of generation, accepted tolerance, selection methods, crossover and mutation operators [23].

Note: the GAOT-ECM package provides several configuration options for model identification. Depending on the model structure, the effects of these parameters can vary or be inconsistent over several runs of the algorithm.

Ethics approval is in accordance with ethical guidelines under Romanian Law No. 206 27/05/2004. The data collection took place at and was approved by the departmental review board of the Center for Interdisciplinary Research in Physical Education and Sport, at Babes-Bolyai University, Cluj-Napoca, Romania. The study participant gave written consent for participation and publication.

3. Results and discussion

Linearity analysis. The data from Experiment 1 is illustrated in Fig. 3 (left). The linearity interval is between [6-14] km/h, obtained by least-squares first-order polynomial fitting over sets of the data points (pairs of heart rate and input speed) and selecting the interval with best fit.

Model fitting. The baseline measurement of Experiment 2 (not normalized) is presented in Fig. 3 (right). The heart rate response to a step input from 6 to 12

km/h treadmill speed has a shape suitable for fitting over a first order linear model as described in (1).

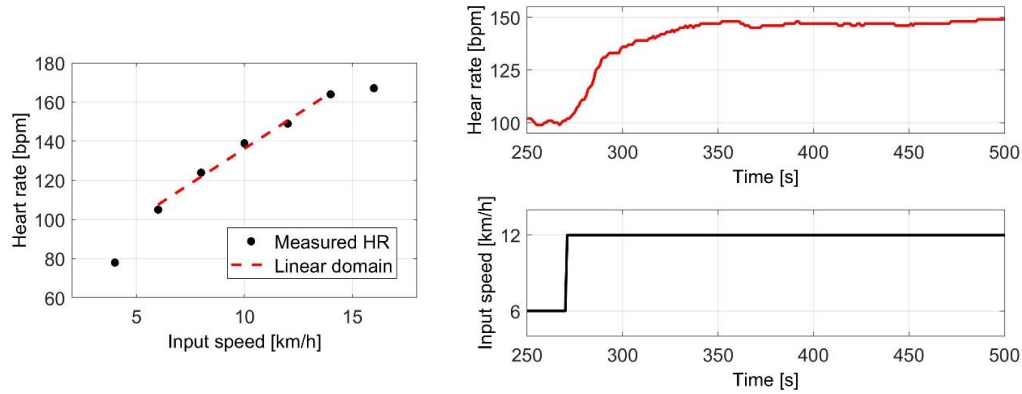


Fig. 3. Experiment 1 data (left) and Experiment 2 baseline data (right)

The results of the evolutionary learning for the four fitness functions are presented in Table 1 and Fig. 4. The genetic algorithm procedure was run 10 times for each fitting. All Pearson correlation calculations had $p \ll 0.001$ and all runs terminated at the 100 generations conditions. The mean elapsed time for each run was between 2 and 3 seconds (on a system equipped with an Intel Core i5 CPU @2.60 GHz and 16 GB of installed memory), which means that the evolutionary learning procedure, with proper code optimization, would be able to run on portable devices, such as smartwatches. This also means that individualized models can be fitted with low computing time expenses.

Results show that the performances of the proposed ρ/R^2 fitness function are comparable with the classic RMSE criterion, with small parameter variance and comparable, very high predictive power (both ρ and R^2 higher than 95%). The fitness function based on ρ alone produced the worst fit; this is because the Pearson correlation coefficient is a good indicator of shape, but not of scale. The fitness function based on the coefficient of determination produced satisfactory results, but the with low correlation and large parameter variation.

Table 1

Mean model parameters and fitting performances.

Fitness function	K [K_{min} ; K_{max}]	T [s] [T_{min} ; T_{max}]	RMSE	Pearson coefficient ρ	Coefficient of determination R^2	Elapsed time [s]
1. ρ/R^2	0.594 [0.586; 0.603]	24.12 [24.28; 28.01]	0.022	0.973	0.988	2.270
2. RMSE	0.592 [0.585; 0.598]	24.68 [23.26; 25.66]	0.021	0.974	0.990	2.138
3. ρ	0.518 [0.484; 0.566]	1.71 [1.04; 2.92]	0.124	0.144	0.473	2.649
4. R^2	0.6 [0.439; 0.824]	42.41 [4.78; 67.42]	0.133	0.011	0.911	2.557

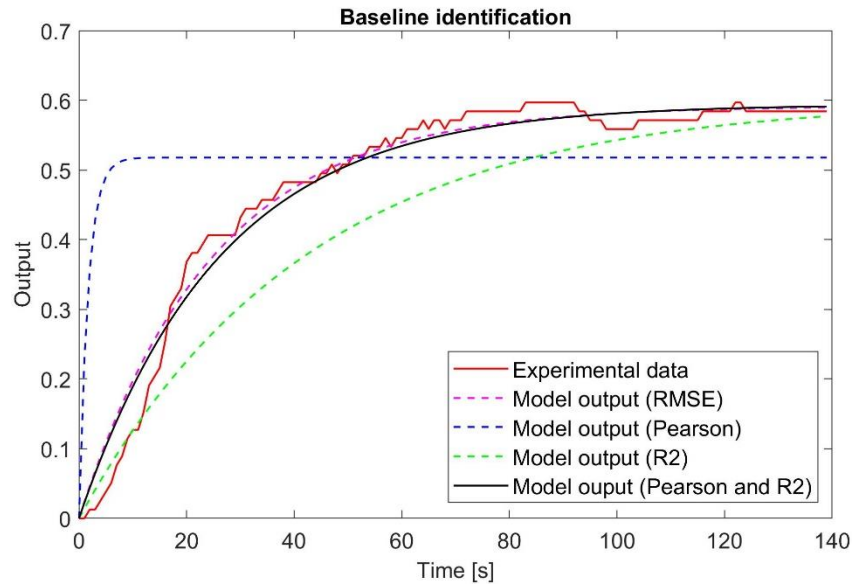


Fig. 4. Results of the evolutionary model fitting on Experiment 2 baseline data

We use the remaining six measurements of Experiment 2 for validation. Table 2 shows the performances of the four models against these measurements as means for RMSE, Pearson correlation coefficient ρ and coefficient of determination R^2 . Fig. 5 presents the response of the four models fitted on the baseline data overlaid onto each of the six measurements.

Table 2

Validation results means and standard deviations (SD) over six measurements.

Fitness function	K	T [s]	RMSE Mean (\pm SD)	Pearson coeff. ρ Mean (\pm SD)	Coeff. det. R^2 Mean (\pm SD)
1. ρ/R^2	0.594	24.12	0.047 (\pm 0.019)	0.990 (\pm 0.005)	0.901 (\pm 0.060)
2. RMSE	0.592	24.68	0.050 (\pm 0.017)	0.988 (\pm 0.006)	0.888 (\pm 0.067)
3. ρ	0.518	1.71	0.151 (\pm 0.015)	0.456 (\pm 0.039)	0.062 (\pm 0.111)
4. R^2	0.6	42.41	0.056 (\pm 0.044)	0.984 (\pm 0.009)	0.854 (\pm 0.204)

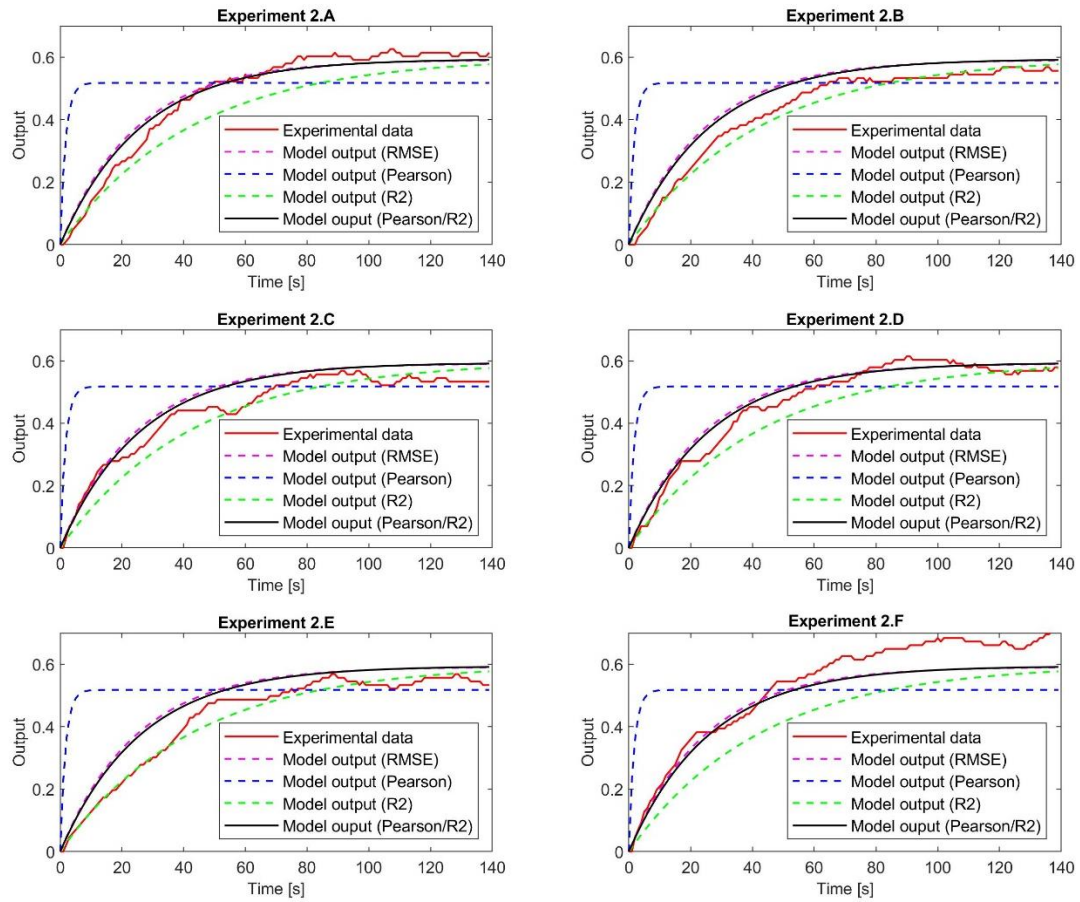


Fig. 5. Responses of the four fitted models against the six validation measurements

A one-way ANOVA between experiments was conducted to compare the model fitting outcomes calculated for the validation data (RMSE, coefficient of determination R^2 and Pearson correlation coefficient ρ) for the four heart rate models obtained with the four fitness functions: ρ/R^2 , ρ , RMSE and R^2 . We found a significant statistical difference ($p < 0.05$) for all three conditions: RMSE [$F(3, 20) = 3.19$, $p \ll 0.001$], coefficient of determination R^2 [$F(3, 20) = 109.35$, $p \ll 0.001$] and Pearson correlation coefficient ρ [$F(3, 20) = 1963.82$, $p \ll 0.001$].

Table 3

Results of the post-hoc Bonferroni test for the ANOVA test

Dependent Variable	Pair	Mean Difference	Standard Error	p-value
RMSE	ρ/R^2 vs. RMSE	-0.00322	0.01150	1
	ρ/R^2 vs. ρ	-0.10416*	0.01150	$\ll 0.001$
	ρ/R^2 vs. R^2	-0.00858	0.01150	1

	RMSE vs. ρ	-0.10094*	0.01150	$\ll 0.001$
	RMSE vs. R^2	-0.00536	0.01150	1
	ρ vs. R^2	0.09558*	0.01150	$\ll 0.001$
Coefficient of determination R^2	ρ/R^2 vs. RMSE	0.01257	0.05545	1
	ρ/R^2 vs. ρ	0.83873*	0.05545	$\ll 0.001$
	ρ/R^2 vs. R^2	0.04655	0.05545	1
	RMSE vs. ρ	0.82616*	0.05545	$\ll 0.001$
	RMSE vs. R^2	0.03398	0.05545	1
	ρ vs. R^2	-0.79218*	0.05545	$\ll 0.001$
Pearson correlation coefficient ρ	ρ/R^2 vs. RMSE	0.00216	0.00847	1
	ρ/R^2 vs. ρ	0.53355*	0.00847	$\ll 0.001$
	ρ/R^2 vs. R^2	0.00557	0.00847	1
	RMSE vs. ρ	0.53139*	0.00847	$\ll 0.001$
	RMSE vs. R^2	0.00341	0.00847	1
	ρ vs. R^2	-0.52798*	0.00847	$\ll 0.001$

Upon conducting post-hoc comparisons using the Bonferroni test (Table 3), we found that among the three outcomes (RMSE, coefficient of determination R^2 and Pearson correlation coefficient ρ), the third fitness function (ρ) demonstrated significant statistical differences. Conversely, no significant statistical differences were observed when comparing any combination of the remaining three fitness functions outcomes. These results highlight that the fitness function based on the Pearson coefficient alone does not provide consistent results, in the context of the evaluated outcomes, while emphasizing the consistency of the other three fitness functions in their statistical performance. A non-significant outcome from the Bonferroni test indicates that the performance of each fitness function in the comparison pair is practically indistinguishable. Accordingly, these favorable results support the adoption of our proposed fitness function (ρ/R^2) for model fitting of human heart rate during treadmill exercise.

4. Conclusions

In this paper we study human heart rate model fitting with genetic algorithms, in which we defined fitness functions based on the root mean of square error, the Pearson correlation coefficient, and the coefficient of determination. The purpose is to determine if these fitness functions are feasible to use for model fitting of heart rate response during treadmill exercise. The implemented genetic algorithm fits the data to a first order linear model. An example of model fitting with various fitness functions is available as part of the GAOT-ECM toolbox.

The proposed fitness functions are (1) minimization of the ratio ρ/R^2 between the Pearson correlation coefficient ρ and the coefficient of determination R^2 ; (2) minimization of the classic root mean of square error (RMSE); (3) minimization of the Pearson correlation coefficient ρ , and (4) minimization of the coefficient of determination R^2 .

We performed two data collection experiments, for linearity analysis and for dynamic model fitting. Results show that the performances of the proposed fitness function based on ρ/R^2 are comparable with the classic RMSE criterion. The fitness function based on ρ alone produced the worst fit, while the one based on R^2 produced satisfactory results, but with lower correlation.

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