

## OPTIMIZED CONTROL METHOD FOR VEHICLES AT UNSIGNALIZED INTERSECTIONS IN A HUMAN-MACHINE MIXED-DRIVING ENVIRONMENT

Hui SHEN<sup>1,\*</sup>, Weiming WANG<sup>2</sup>, Shangjiang YANG<sup>3</sup>, Yuqi YANG<sup>4</sup>

*In addressing the optimization control issue of vehicles at intersections within a human-machine mixed-driving traffic environment, the concepts of controllable and uncontrollable vehicle platoons are first proposed. Considering the impact of multi-lane vehicles moving in the same direction and the upstream and downstream conflict areas on the internal conflict process at intersections, an algorithm for handling conflicts between different types of vehicle platoons has been designed. Based on this, by considering the interaction between vehicles in the internal conflict zones of intersections and applying the scanline method, the actual conflict points are decoupled in a closed loop. Furthermore, based on the analysis of the speed-time space domain of the lead vehicle in the controllable platoon reaching the stop line, with the objectives of minimizing the number of actual conflict points and reducing average vehicle delay, a two-stage optimization model for vehicle scheduling at intersections in a human-machine mixed traffic flow environment is constructed. The model's optimized solution further facilitates the reverse optimization of lane-changing trajectories for autonomous vehicles.*

**Keywords:** human-machine mixed driving, unsignalized intersections, conflict decoupling, optimized control

### 1. Introduction

Autonomous driving technology has become a research hotspot in the field of intelligent transportation. With the continuous maturity of autonomous driving technologies, optimized control of vehicles at unsignalized intersections will become feasible. Currently, most research on this issue is targeted towards a fully autonomous driving environment. However, for a considerable period, there will exist a mixed-driving scenario where manually driven and autonomous vehicles coexist.

<sup>1\*</sup> Electrical and Control Engineering College, North China University of Technology, Beijing, China, \*Corresponding author, e-mail: shenhui2455@163.com

<sup>2</sup> Electrical and Control Engineering College, North China University of Technology, Beijing, China, e-mail: wwqntu@163.com

<sup>3</sup> Electrical and Control Engineering College, North China University of Technology, Beijing, China, e-mail: yangshangjiang123@126.com

<sup>4</sup> Electrical and Control Engineering College, North China University of Technology, Beijing, China, e-mail: wellyuqi@163.com

Unlike traditional intersections solely comprised of manually driven vehicles, the predictability and controllability of autonomous vehicles have significantly altered the control paradigms at intersections in a human-machine mixed-driving environment. Beyond the traditional traffic light control, the operation trajectories of autonomous vehicles can be planned in this environment. This planning can change the distribution of traffic flow and enhance the efficiency of intersection traffic. As the penetration rate of autonomous vehicles increases, studying the effects of optimized control at unsignalized intersections under various penetration rates will provide support for implementing unsignalized control strategies, thus bearing greater significance.

Based on this context, this research focuses on the optimized control of vehicles at intersections in a human-machine mixed-driving traffic environment. After analyzing the interaction between vehicles in the internal conflict zones at intersections, a two-stage optimization model for vehicle scheduling at intersections in a human-machine mixed traffic flow environment is constructed. The second part of this article provides a literature review of previous research. The third part describes the definition of the platform and the assumptions for input and output, the fourth part describes the model construction, including lane changing model, speed guidance model, conflict model, optimization objective model, etc., and the fifth part is the conclusion.

## 2. Literature review

In the context of signal-controlled scenarios, research on autonomous vehicle trajectory planning includes trajectory planning for single-lane autonomous vehicles and multi-lane autonomous vehicles. Wang et al. [1] focused on the car-following issue for autonomous vehicles and proposed a control structure based on rolling optimization. F. Zhou et al. [2], considering a mixed platoon consisting of autonomous and manually driven vehicles, extended the trajectory planning model for single-lane autonomous vehicles to accommodate system state measurements' uncertainties and established a stochastic optimal control model.

In addition to the aforementioned model-based methods, recent studies have begun exploring the potential of utilizing machine learning techniques for autonomous vehicle trajectory planning. In this regard, M. Zhou et al. [3] developed a car-following model based on reinforcement learning (RL) for autonomous vehicles at single-lane intersections, aiming to improve overall traffic efficiency, fuel consumption, and traffic safety. Shi et al. [4] used a traditional (non-deep) Q-learning approach to develop effective driving strategies for autonomous vehicles approaching signalized intersections. Mousa et al. [5] employed deep Q-learning with prioritized experience replay, target networks, and double learning to train RL agents, which allow autonomous vehicles to approach and leave signalized

intersections efficiently without interference from other vehicles. Wang et al. [6] focused on the control issues of autonomous vehicles within mixed traffic flows at signalized intersections, considering oscillations caused by manually driven vehicles, using deep reinforcement learning models to predict the trajectories of manually driven vehicles and control autonomous vehicles.

Yao et al. [7] considered the interaction between autonomous vehicle trajectory planning and manually driven vehicle lane changes and designed a human-machine mixed-traffic trajectory planning framework for multi-lane signalized intersections. Ma et al. [8] aimed to optimize the longitudinal and lateral trajectories of individual autonomous vehicles, and with signal timing and surrounding vehicle trajectory information provided, established a discrete-time bi-level optimization model with the objectives of minimizing vehicle delay, fuel consumption, and lane-changing costs. Bai et al. [9] developed a hybrid eco-driving strategy based on reinforcement learning for human-machine mixed traffic flows at signalized intersections. Xu et al. [10] proposed a two-step strategy for this joint optimal control problem. Guo et al. [11] also utilized a two-step method to separately optimize traffic signal timing and autonomous vehicle trajectories. Yu et al. [12] modeled the problem as a MILP (Mixed-integer linear programming) problem, treating the sequence of signal phases at the intersection. Liu et al. [13] also developed a MILP model for the joint optimization problem.

The computational complexity of the aforementioned joint optimization modeling methods is relatively high, making real-time computation challenging to ensure. To enhance computational efficiency, some research has simplified individual vehicle control to the overall control of a platoon by establishing mixed-platoon configurations, thereby reducing problem scale [14] [15]. In terms of distributed control, Naumann et al. [16] proposed a distributed control strategy for vehicles at intersections. Wu et al. [17] developed a distributed exclusion algorithm for intersection vehicles, where vehicles issuing travel requests compete with other vehicles to determine if they gain priority passage permission. Zhang et al. [18] introduced a distributed optimal control framework that ensures each vehicle obtains the optimal acceleration/deceleration at any given moment. Simulations demonstrated that this strategy not only avoids congestion and ensures safety but also reduces average fuel consumption under different levels of autonomous vehicle penetration. Quang et al. [19] proposed a vehicle scheduling method based on deep reinforcement learning. Li et al. [20] discussed optimal control strategies for mixed platoons with different formations, including cases where the lead vehicle is manually driven.

Overall, in the current research on vehicle control optimization at unsignalized intersections in a human-machine mixed-driving environment, although some results consider lane allocation or platoon reformation, the impact of reformation on intersection conflicts and conflict avoidance optimization is still

lacking. Additionally, when multiple lanes in the same direction are present at the entry points, coordination among platoons within these lanes also needs consideration.

### 3. Platoon definition

Assumptions and Inputs/Outputs:

- 1) During the analysis of the conflict areas at the intersection, only cross-conflict points are considered, ignoring conflict points related to diverging and merging traffic flows.
- 2) Communication delays related to V2V (Vehicle-to-Vehicle) and V2X (Vehicle-to-Infrastructure) technologies are not considered.

The model's inputs include the geometric parameters of the intersection, such as the number of entry and exit lanes from each direction, lane widths, coordinates of the stop line endpoints on each lane, coordinates of the inner boundaries of zebra crossings, and coordinates of the conflict area boundaries. Inputs also include vehicle attributes, such as autonomous and manually driven vehicles; additionally, real-time position and speed information of vehicles at the decision-making moment are included. The model's outputs are the guided speeds along the vehicle paths, vehicle entry and exit times at the intersection, and the delay time experienced by each vehicle.

In this paper, the non-lane-changing section upstream of the intersection is treated as the control segment, which generally consists of the guided lanes. To enhance the optimization effect when employing the unsignalized control method proposed in this paper, the length of the guided lanes can be extended. Given the randomness in the driving behaviors of manually driven vehicles, individual vehicle-based control often results in reduced control precision (due to vehicle speed estimation errors, leading to significant deviations between the analysis process and actual control outcomes), and tends to cause frequent acceleration and deceleration of vehicles within the intersection. Additionally, considering a platoon led by a Connected and Autonomous Vehicle (CAV) as the research subject can further reduce computational complexity. Therefore, this paper first defines a platoon and subsequently focuses on platoon-based optimization of control strategies. With the control section of the intersection area already defined, the platoon in this paper is required to meet the following conditions:

- (a) The platoon consists of vehicles within the range of the guided lanes.
- (b) Vehicles in the same platoon come from the same lane; vehicles from different lanes cannot be in the same platoon.
- (c) The time gap between vehicles within the platoon should be less than or equal to a set value.

Given the different compositions of vehicles in platoons from various directions at the intersection, this chapter categorizes platoons into controllable and uncontrollable types. A controllable platoon refers to a platoon led by a CAV whereas an uncontrollable platoon consists entirely of HDVs (Human-Driven Vehicles), as shown in Fig. 1(a). The platoon is modeled to include the type of platoon, number of vehicles in the platoon, and real-time attributes of the vehicles (such as vehicle type, position, speed, and acceleration), as shown in equation (1). As time progresses, the original attributes of the platoon may change, as illustrated in Fig. 1(b) and (c):

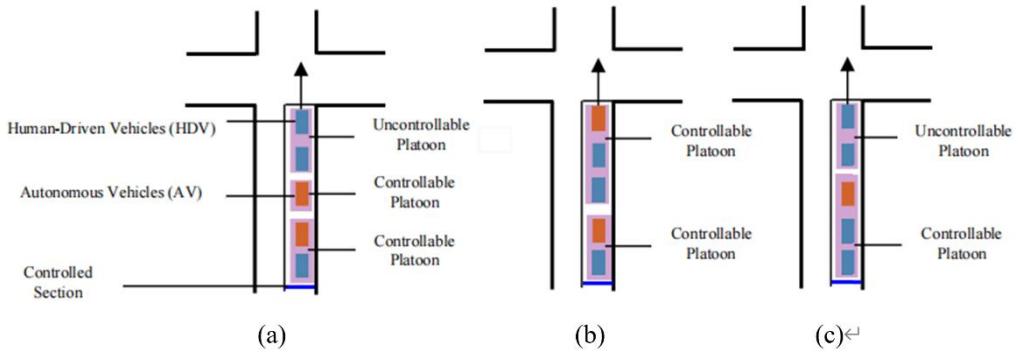


Fig.1 Schematic Diagram of Platoon Composition

$$\begin{aligned}
 f_{i,j}^{dir}(t) &= \{type_{i,j}^{dir}(t), num_{i,j}^{dir}(t), k_{i,j}^{dir}(t)\} \\
 k_{i,j}^{dir}(t) &= \{k_{i,j,1}^{dir}(t), k_{i,j,2}^{dir}(t), \dots, k_{i,j,n}^{dir}(t)\} \\
 k_{i,j,n}^{dir}(t) &= \{p_{i,j,n}^{dir}(t), d_{i,j,n}^{dir}(t), v_{i,j,n}^{dir}(t), a_{i,j,n}^{dir}(t)\}
 \end{aligned} \quad (1)$$

In equation (1),  $f_{i,j}^{dir}(t)$  represents the attributes of the  $j$ -th platoon on lane  $i$  in direction  $dir$  (approach direction) at time  $t$ ,  $type_{i,j}^{dir}(t)$  indicates the type of the  $j$ -th platoon on lane  $i$  in direction  $dir$  at time  $t$ , with  $type_{i,j}^{dir}(t)=1$  denoting a controllable platoon, and  $type_{i,j}^{dir}(t)=0$  indicating an uncontrollable platoon.  $num_{i,j}^{dir}(t)$  denotes the number of vehicles in the  $j$ -th platoon on lane  $i$  in direction  $dir$  at time  $t$ .  $k_{i,j}^{dir}(t)$  represents the real-time attributes of the vehicles in the  $j$ -th platoon on lane  $i$  in direction  $dir$ , and  $k_{i,j,n}^{dir}(t)$  specifies the real-time attributes of the  $n$ -th vehicle in the  $k_{i,j}^{dir}(t)$  platoon on lane  $i$  in direction  $dir$ .  $p_{i,j,n}^{dir}(t)$  indicates the type of the  $n$ -th vehicle in the  $j$ -th platoon on lane  $i$  in direction  $dir$ , where  $p_{i,j,n}^{dir}(t)=1$  signifies the vehicle is a CAV, and  $p_{i,j,n}^{dir}(t)=0$  means the vehicle is an HDV.  $d_{i,j,n}^{dir}(t)$  represents the real-time position of the  $n$ -th vehicle in the  $j$ -th

platoon on lane  $i$  in direction  $dir$ ,  $v_{i,j,n}^{dir}(t)$  denotes the real-time speed of the  $n$ -th vehicle in the  $j$ -th platoon on lane  $i$  in direction  $dir$ , and  $a_{i,j,n}^{dir}(t)$  is the real-time acceleration of the  $n$ -th vehicle in the  $j$ -th platoon on lane  $j$  in direction  $dir$ .

Controlled platoons manage the formation by adjusting the speed of the leading CAV. When the speed of the lead CAV changes, following HDVs calculate their trajectories using the IDM (Intelligent Driver Model). Under centralized comprehensive data collection and processing, the lead CAV in a controlled platoon can access real-time attributes  $f_{i,j}^{dir}(t)$  of other platoons currently. In the control strategies discussed in this chapter, if the conflict area involves only controlled platoons, the speed of the lead vehicle in controlled platoons can be managed to allow the entire platoon to pass through the conflict direction smoothly. However, if the conflict involves a controlled platoon and an uncontrolled platoon or between two uncontrolled platoons, conflict area management strategies are used to determine the process of vehicle passage. This means, in conflicts between controlled and uncontrolled platoons, HDVs may cut through the platoon.

When optimizing the speed of the lead CAV in a controllable platoon, it is essential first to ascertain the number of HDVs within the same platoon. If subsequent HDVs enter the guided lane, the conditions for these vehicles to join the preceding platoon include:

- (1) The time gap between the vehicle and the last vehicle of the preceding platoon must meet the platoon criterion (c).
- (2) Upon entry of the vehicle into the platoon, the vehicles in the direction of conflict must be able to pass through the conflict area safely as a consolidated platoon

$$\cdot \frac{v_{i,j,1}^{dir_2,b}(t_0) - \sqrt{(v_{i,j,1}^{dir_2,b}(t_0))^2 - 2a_{\max} l_{i,j,1}^{dir_2,m}(t_0)}}{a_{\max}} < t_{i,j,n}^{dir_1,m}(t_0) \quad (2)$$

In this context,  $t_{i,j,n}^{dir_1,m}(t_0)$  represents the time required for the newly added vehicle  $n$  in the  $j$ -th controllable platoon on lane  $i$  in direction  $dir_1$  to pass through the conflict area  $m$  starting from time  $t_0$ . Conversely,  $l_{i,j,1}^{dir_2,m}(t_0)$  denotes the distance from the lead vehicle of the  $j$ -th controllable platoon on lane  $i$  in direction  $dir_2$  to the conflict area at time  $t_0$ , and  $v_{i,j,1}^{dir_2,b}(t_0)$  represents the speed of the lead vehicle in the  $j$ -th controllable platoon on lane  $i$  in direction  $dir_2$  at time  $t_0$ . The variable  $a_{\max}$  corresponds to the maximum deceleration of the vehicle.

Considering the interactions between controllable platoons in conflict areas, the approach of allowing entire platoons to yield may sometimes result in infeasible situations. Therefore, it is necessary to address such infeasible scenarios as follows:

(1) If an infeasible situation arises during initialization, randomly select one platoon from the conflicting controllable platoons to reduce by one vehicle at the end, and recalculate until a solution is found.

(2) If an infeasible situation occurs during the optimization process, remove the newly added vehicle in the target platoon.

#### 4. Control model construction

##### 4.1 Lane change constraints

Based on the above analysis, it is obvious that when the direction of the conflict is a controllable fleet, the controllable space can be increased and the flexibility of vehicle trajectory control can be improved, therefore, the problem that needs to be solved by lane change control is how to increase the number of multi-lane controllable fleets in the same direction on the basis of not affecting the operation of the original controllable fleet. Based on this, this paper stipulates that when one of the two CAVs in the controllable fleet of the lane can change lanes and can change lanes to the front of the uncontrollable fleet in the adjacent lane, the CAV vehicle is controlled for lane change planning, and the applicable scenarios include two scenarios: single-rear vehicle lane change control for the target lane and lane change control for the target lane with dual rear vehicles, as shown in Fig. 2.

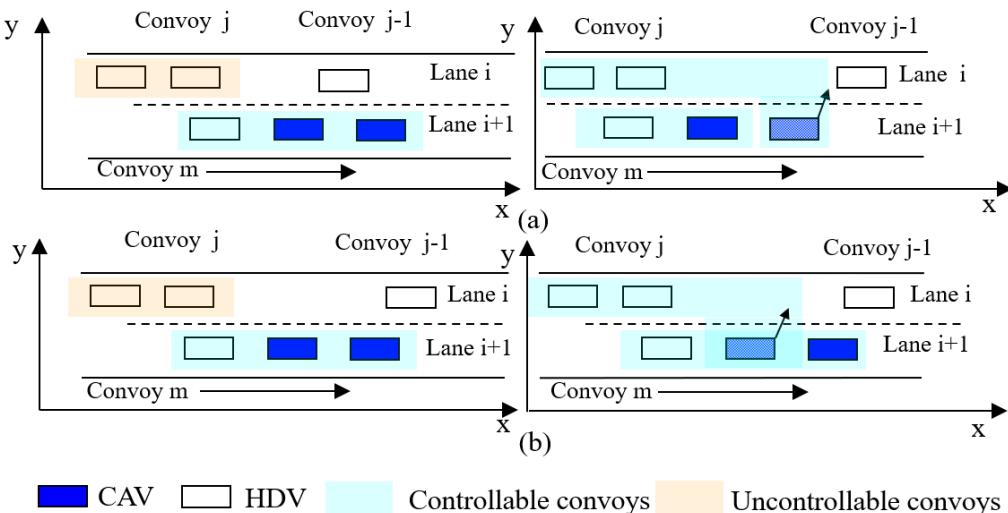


Fig. 2 (a) Applicable scenario of deceleration and lane change of double rear vehicles facing the target lane, (b) Applicable scenario of lane changing of single rear vehicle facing the target lane

(1) Compatible with single rear vehicles facing the target lane

Compared with conventional autonomous lane change, because it is a controlled lane change at this time, the lane change conditions only need to meet

the safety conditions, and at the same time, considering the fleet reorganization goal, the lane change conditions are as follows:

$$\begin{cases} Gap_{i,j-1,n}(t_{change,AV}) > Gap_{AV}^{safe}(t_{change,AV}) \\ Gap_{i,j,1}(t_{change,AV}) > Gap_{i,j,1}^{safe}(t_{change,AV}) \\ p_{i,j,1}^{dir}(t) = 0 \text{ and } d_{i,j,1}^{dir}(t_{change,AV}) < d_{i+1,m,p}^{dir}(t_{change,AV}) \text{ and } \\ d_{i,j-1,n}^{dir}(t_{change,AV}) > d_{i+1,m,p}^{dir}(t_{change,AV}) \text{ and } p_{i+1,m,p}^{dir}(t_{change,AV}) = 1 \\ p_{i+1,m,p}^{dir}(t_{change,AV}) = 1, p = 1, 2, \dots, k, k \geq 2 \end{cases} \quad (3)$$

Wherein,  $Gap_{i,j-1,n}(t_{change,AV})$  the distance  $t_{change,AV}$  between  $i+1$  the first  $m$  CAV of  $P$  the time lane team and  $i$  the last vehicle of the  $j-1$  adjacent lane  $n$  team  $Gap_{i,j,1}(t_{change,AV})$  is the time  $t_{change,AV}$  adjacent lane  $i$  team  $j$  The distance between the lead car and the CAV to be changed.

## (2) Compatible with dual rear vehicles facing the target lane

This paper stipulates that the lane change CAV of the double rear vehicle facing the target lane should be the head vehicle of the controllable team, and the vehicle behind it in the team is also a CAV.

$$\begin{cases} Gap_{i,j-1,n}(t_{change,AV}) < Gap_{i,j-1,n}^{safe}(t_{change,AV}) \\ p_{i,j-1,n}^{dir}(t_{change,AV}) = 0 \text{ and } d_{i,j-1,n}^{dir}(t_{change,AV}) < d_{i+1,m,1}^{dir}(t_{change,AV}) \\ p_{i+1,m,p}^{dir}(t_{change,AV}) = 1, p = 1, 2, \dots, k, k \geq 2 \\ Gap_{i,j,p+1}(t_b) > Gap_{i,j,p+1}^{safe}(t_b) \\ d_{i,j,p}^{dir}(t_b) > d_{i+1,m,1}^{dir}(t_b) \\ Gap_{i,j,p}(t_b) > Gap_{AV}^{safe}(t_b) \end{cases} \quad (4)$$

In addition, in order to reduce the computational complexity, in the design of the algorithm, the CAV lane change and vehicle speed guidance are divided into two stages, firstly, the CAV that meets the above lane change conditions is controlled for lane change, and then the vehicle speed guidance is carried out after the lane change is completed. In order to achieve the goal of minimizing the internal conflict of the intersection through the speed guidance of the head vehicle of the CAV, it is necessary to ensure that the head CAV vehicle (if any) entering the control section in the time window does not pass the stop line at the time when the CAV completes the lane change, and can be guided to the lowest speed  $v_{min}$ , therefore, the constraints are as follows:

$$l_{dir,i}^1(t_{change,AV}) - \int_{t_{change,AV}}^{t_{e,max}} v_{dir,i}^1(t) dt \geq \frac{v_{dir,i}^1(t_{e,max})^2 - v_{min}^2}{2a_c} \quad (5)$$

wherein,  $t_{e,\max}$  is the latest lane change end time required among all controlled lane change vehicles in this time window.

#### 4.2 Speed guidance constraints

Under the guidance speed, the rear car shall be evenly accelerated and decelerated until the guidance speed is reached, and in this process, it is necessary to meet the safety time distance from the front vehicle, that is, there are the following constraints:

$$\begin{cases} x_{dir,i}^k(t_a) + \int_{t_a}^T v_{dir,i}^k(t) dt \geq x_{dir,i}^{k+1}(t_a) + \int_{t_a}^t v_{dir,i}^{k+1}(t) dt + v_{dir,i}^{k+1}(t) T_{safe}, \quad \forall dir, k, i \\ t_a < T < t_{dir,i}^{k+1,e} \\ v_{\min} < v_{dir,i}^{k+1}(t) < v_{\max}, \quad \forall k, i \end{cases} \quad (6)$$

Wherein,  $t_a$  is the  $k$  time when the first vehicle starts to guide,  $x_{dir,i}^k(t_a)$  and is the position  $dir$  of the first  $i$  vehicle in the  $k$  direction lane at the time  $t_a$  (according to the lane where the vehicle is located,  $x_{dir,i}^k(t_a)$ ) it can be represented by the coordinates of the front corner point of the vehicle),  $v_{dir,i}^k(t)$  and is the  $dir$  direction lane  $i$  The  $k$  speed at the time  $t$  of the first car  $v_{\min}$  is the minimum guiding speed, and  $v_{\max}$  the maximum guiding speed  $t_{dir,i}^{k+1,e}$  is the moment when the  $dir$  first vehicle in the  $i$  direction lane  $k+1$  enters the intersection.

In the actual scene, if the guidance speed is very small, it can be converted into the time from the beginning of guidance to leaving the intersection, and then the time of leaving the intersection is obtained, and the vehicle can drive normally after conversion, but the speed and time of leaving from the stop line need to be consistent with the optimized result (the control mode is changed to the time control of entering the intersection at this time), therefore, if the vehicle needs to stop and wait before entering the intersection and is the first vehicle on the lane, its stop point should be some distance from the stop line, The distance is the distance from the acceleration of the vehicle speed from 0 to the guide speed, then at this time, the moment when the vehicle starts from the stop point and the stop point position are calculated as follows:

$$\begin{cases} \frac{V_{dir,i}^k - v_{dir,i}^k(t_a)}{a_c} + \frac{l_{dir,i}^k(t_a) - S_{dir,i}^k}{V_{dir,i}^k} = t_{dir,i}^k + \frac{V_{dir,i}^k}{a_c} \\ S_{dir,i}^k = \frac{(V_{dir,i}^k)^2 - (v_{dir,i}^k(t_a))^2}{2a_c} \\ l_{dir,i}^{k,s} = \frac{V_{dir,i}^{k,s}}{2a_c} \end{cases} \quad (7)$$

Wherein,  $l_{dir,i}^k(t_a)$  is the  $dir\ i$  distance from the stop line when the vehicle speed guidance on the direction lane begins, and  $k$  is the moment when  $t_{dir,i}^k$  the vehicle starts  $dir$  from the stop  $i$  point on the  $k$  direction lane, and  $l_{dir,i}^{k,s}$  is the  $dir$  direction lane  $i$  The  $k$  distance between the parking position of the vehicle and the parking line  $a_c$  is the comfortable acceleration of the vehicle,  $V_{dir,i}^k$  the guiding speed, and  $S_{dir,i}^k$  the distance traveled when  $dir$  the vehicle on the  $i$  direction lane  $k$  starts from the speed guidance to the guiding speed.

When the speed guidance is carried out on the head vehicle of the controllable fleet, the vehicle should be able to reach the guiding speed before the stop line at the intersection, therefore, the guiding speed also needs to satisfy the formula (12):

$$\frac{(V_{dir,i}^k)^2 - (v_{dir,i}^k(t_a))^2}{2a_c} < l_{dir,i}^k(t_a) \quad (8)$$

Before the vehicle reaches the stop line (enters the intersection), when there is speed guidance, it reaches the guiding speed and then drives at a constant speed. Due to the influence of the vehicle in front during acceleration guidance, the  $t_a$  following conditions should be met at a constant speed at a time when it is sufficient to  $a_c$  accelerate to the guidance speed, so the following conditions should be met respectively according to the attainability of acceleration guidance and deceleration guidance

(1) When decelerating and directing:

$$\begin{cases} l_{dir,i}^k(t_a) - \frac{(V_{dir,i}^k)^2 - (v_{dir,i}^k(t_a))^2}{2a_c} = V_{dir,i}^k(t_s - t_1) \\ t_1 = \frac{V_{dir,i}^k - v_{dir,i}^k(t_a)}{a_c} \end{cases} \quad (9)$$

Wherein,  $t_a$  is the time when the vehicle speed starts to guide, the  $t_1$  time when the vehicle reaches the guiding speed, and  $t_s$  the time when the vehicle arrives at the stop line.

(2) When speeding up booting:

$$\begin{cases} l_{dir,i}^k(t_{change,AV}) - \int_{t_{change,AV}}^{t_a} v_{dir,i}^k(t) dt - \frac{(V_{dir,i}^k)^2 - (v_{dir,i}^k(t_a))^2}{2a_c} = (t_s - t_1)V_{dir,i}^k \\ v_{dir,i}^k(t) = v_{dir,i}^k(t_{change,AV}) + \int_{t_{change,AV}}^{t_a} a_{dir,i}^k(t) dt, \quad t_{change,AV} \leq t \leq t_a \\ t_1 = \frac{V_{dir,i}^k - v_{dir,i}^k(t_a)}{a_c} \\ a_{dir,i}^{k,f}(t) = a_{\max} \left[ 1 - \left( \frac{v_{dir,i}^k(t)}{v_{\max}} \right) - \left( \frac{S^*(t)}{\Delta s(t)} \right)^2 \right] \\ a_{dir,i}^k(t) = a_{dir,i}^{k,f}(t), \quad a_{dir,i}^{k,f}(t) < 0, \quad t_{change,AV} \leq t \leq t_a \\ a_{dir,i}^k(t) = 0, \quad a_{dir,i}^{k,f}(t) \geq 0, \quad t_{change,AV} \leq t \leq t_a \end{cases} \quad (10)$$

Wherein,  $t_a$  is the time when the vehicle speed starts to guide, which  $t_a$  is obtained by the following formula, and the constraints also need to meet the formula (10), (11), (12):

$$\max f(t_a) = l_{dir,i}^k(t_a) - \frac{(V_{dir,i}^k)^2 - (v_{dir,i}^k(t_a))^2}{2a_c}, \quad t_{e,\max} \leq t_a \leq t_1 \leq t_s \quad (11)$$

The vehicle without speed guidance uses the following model to calculate the trajectory:

$$\begin{cases} l_{dir,i}^k(t_{change,AV}) = \int_{t_{change,AV}}^{t_s} v_{dir,i}^k(t) dt \\ v_{dir,i}^k(t) = v_{dir,i}^k(t_{change,AV}) + \int_{t_{change,AV}}^T a_{dir,i}^{k,f}(t) dt, \quad t_{change,AV} \leq T \leq t_s \\ a_{dir,i}^{k,f}(t) = a_{\max} \left[ 1 - \left( \frac{v_{dir,i}^k(t)}{v_{\max}} \right) - \left( \frac{S^*(t)}{\Delta s(t)} \right)^2 \right] \end{cases} \quad (12)$$

On the basis of the above constraints, based on the position and speed of the CAV head vehicle of each controllable fleet at the  $t_{e,\max}$  moment, the speed-time range of the CAV reaching the stop line can be calculated, and then the speed and time of the controllable fleet reaching the stop line can be used as the decision variables, and the optimization solution is carried out within the value range. Since the end position and lane change time of CAV lane change trajectory planning are not unique, when the end position and lane change time of CAV lane change trajectory planning, a new speed-time range will be formed, and then the speed-time space domain when it reaches the stop line will be formed.

Based on the above analysis, it can be seen that when the speed-time of the CAV head vehicle of each team is optimized, there may be more than one end position and lane change time of the corresponding lane change CAV trajectory, so on the basis of the optimization solution of the speed-time optimization of the CAV head car of each controllable team, reverse optimization is required, that is, the

optimal trajectory is found from the CAV lane change trajectory that can achieve the optimization result, that is, the optimal end position and lane change time.

### 4.3 Conflict constraints

To reduce the complexity of calculations, this paper focuses on conflict zones as the subject of analysis, addressing conflicts involving vehicles from different directions within these zones. To prevent the occurrence of "deadlock" at intersections, it is stipulated that vehicles are not allowed to stop within any conflict zones. A typical intersection conflict area is depicted in Fig. 3, which divides the intersection space into three sections based on the spatial position attributes of vehicles before crossing the intersection: the buffer zone, conflict zone, and downstream impact area. The buffer zone refers to the interval from the stop line to the boundary of the conflict zone. The conflict zone is the area where vehicles from different directions intersect, and the downstream impact area is the section between the downstream conflict zone and the target conflict zone. The design of the intersection can lead to variations in the extents of the buffer zones, conflict zones, and downstream impact zones, and specific calculations can establish a coordinate system for the intersection to analyze the ranges of these three areas in detail.

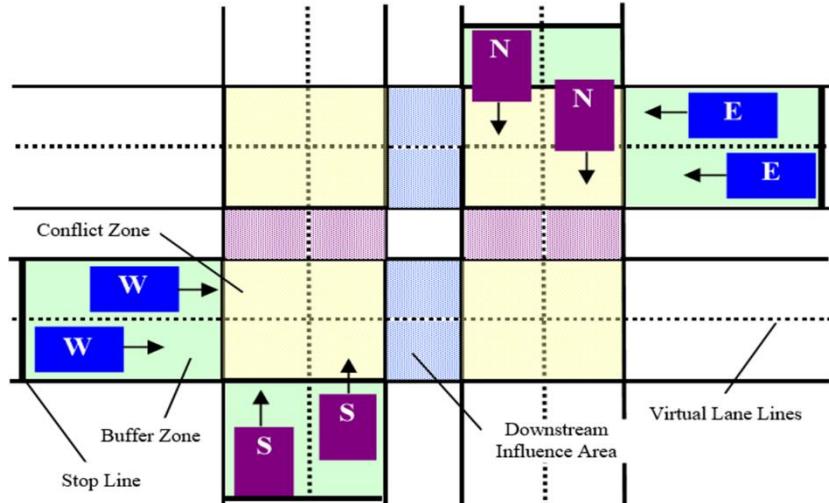


Fig.3 Schematic Diagram of Various Areas within the Intersection

As illustrated in Fig. 3, for platoon combinations on two lanes heading in the same direction, vehicles influence each other as they pass through the conflict area. Due to one platoon occupying a position within the conflict zone, vehicles from the other platoon in the same direction, although reaching the conflict area later, are still able to proceed.

#### 4.4 Objective function

On the basis of the above constraint analysis, the two-stage control objectives are designed as follows:

Decision variables: vehicle guidance speed  $V_{i,j,m}^{dir}$ , vehicle speed guidance start time  $t_{i,j,m}^{dir}$ .

Objective function 1: Through the reorganization of the controllable fleet and the speed guidance of the head CAV of the controllable fleet, the actual number of collision points generated by vehicles in all directions of the intersection at this time is minimized.

$$Z_1 = \min num_{C_r}(V_{i,j,m}^{dir}, t_{i,j,m}^{dir}) \quad (13)$$

On the basis of the optimal solution of objective function 1, the optimal solution of objective function 2 is obtained. The objective function 2 is the  $\Delta T$  shortest average delay time for vehicles entering the control area within the time window. The difference between the actual travel time of a single vehicle and the travel time at free flow speed, i.e.:

$$d_{dir_1,dir_2,i,j}^k = (t_{dir_1,i,e}^k - t_{dir_1,i,b}^k) - \frac{l_{dir_1,i} + l_{dir_1,dir_2,i,j}}{v} \quad (14)$$

Wherein,  $l_{dir_1,i}$  is the  $dir_1$  length of the control area of the  $i$  directional entrance lane  $l_{dir_1,dir_2,i,j}$ , is the distance from the  $dir_1$  directional entrance lane  $i$  to the  $dir_2$  directional exit lane  $j$  in the intersection,  $v$  represents the free flow speed, and  $t_{dir_1,i,b}^k$  represents the time window  $\Delta T$  The  $dir_1$  time when the first vehicle in the directional entrance lane enters the control area,  $i$  which indicates  $k$   $t_{dir_1,i,e}^k$  the moment when the  $\Delta T$  first vehicle in the  $dir_1$  directional entrance lane  $i$  leaves the intersection within the  $k$  time window.

Therefore, the  $\Delta T$  total vehicle delays within the time window are:

$$Delay = \sum_{dir_1=N,S,W,E} \sum_{dir_2=N,S,W,E} \sum_{i=1}^{I_{dir_1}} \sum_{j=1}^{J_{dir_2}} \sum_{k=1}^{K_{dir_1,dir_2,i,j}} d_{dir_1,dir_2,i,j}^k \quad (15)$$

where represents the number of vehicles  $K_{dir_1,dir_2,i,j}$  that entered the control section within  $\Delta T$  the time window  $dir_1$  and went from the direction lane  $i$  to the direction  $dir_2$  lane  $j$ .

The expression of the objective function 2 (the  $\Delta T$  average delay of vehicles entering the control section in the time window is the smallest) is further obtained as:

$$Z_2 = \min \left( \frac{\text{Delay}}{\sum_{\substack{dir_1=N,S,W,E \\ dir_2=N,S,W,E}} \sum_{i=1}^{I_{dir_1}} \sum_{j=1}^{J_{dir_2}} K_{dir_1, dir_2, i, j}} \right) \quad (16)$$

## 5. Conclusion

This paper addresses the issue of vehicle optimization control at intersections in a mixed human-autonomous driving traffic environment. Firstly, it introduces the concepts of controllable and uncontrollable vehicle platoons, considering the impact of multi-lane vehicles traveling in the same direction and conflict zones upstream and downstream on the internal conflict processes at the intersection. Conflict handling algorithms between different types of platoons are designed accordingly. Building upon this, the operational process of vehicles within the conflict zones of the intersection is considered, employing a scan line method to achieve decoupling of actual conflict points in a closed-loop manner. Furthermore, based on the speed-time space domain analysis of the leading vehicle in controllable platoons reaching the stop line, a two-stage optimization model for intersection vehicle scheduling under a mixed human-autonomous traffic flow environment is constructed. The objective function aims to minimize the number of actual conflict points and reduce the average vehicle delay. On the basis of model optimization and solution, further reverse optimization of lane-changing trajectories for autonomous vehicles is achieved. This paper constructs and solves a model for the optimization control problem of intersection vehicles in a human-machine hybrid traffic environment, which can minimize the average delay of vehicles. Future research directions can further verify the effectiveness of the proposed method through virtual simulation experiments.

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