

## A MODEL FOR PREDICTION OF STRESS STATE IN SOIL BELOW AGRICULTURAL TYRES USING THE FINITE ELEMENT METHOD

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*Această lucrare prezintă un model pentru estimarea stării de tensiune din solul agricol sub acțiunea pneurilor agricole, după direcția longitudinală și după direcția laterală, diferită una de alta, utilizând metoda elementelor finite.*

*În prezent, una dintre cele mai avansate metodologii pentru modelarea fenomenului de propagare a tensiunilor în solul agricol este metoda elementelor finite, care este o metodă numerică ce permite obținerea soluțiilor aproximative ale ecuațiilor diferențiale ce descriu această distribuție. În această lucrare solul agricol a fost idealizat ca un material elasto-plastic conform criteriului Drucker-Prager.*

*This paper presents a model for prediction the stress state in agricultural soil below agricultural tyres in the driving direction and perpendicular to the driving direction which are different from one another, using the finite element method.*

*In present, one of the most advanced methodologies for modelling the phenomenon of stresses propagation in agricultural soil is the finite element method, which is a numerical method for obtaining approximate solutions of ordinary and partial differential equations of this distribution. In this paper, the soil has been idealised as an elastoplastic material by Drucker-Prager yield criterion.*

**Keywords:** Finite element method, agricultural soil, tyre, stress state

### 1. Introduction

The passage of wheels over agricultural soils, which is usually of short duration in the case of most vehicles, results in soil artificial compaction [5,6]. The compaction phenomenon of agricultural soil can be defined as an increase in its dry density and the closer packing of solid particles or reduction in porosity [E. McKyes, 1985] [12], which can result from natural causes, including rainfall impact, soaking, and internal water tension [1,2,5]. The most important factors, which have a significant influence in the process of artificial compaction of agricultural soil, are: the type of soil, moisture content of the soil, intensity of

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external load, area of contact surface between the soil and the tyre or track, shape of contact surface, and the number of passes [2].

Because the agricultural soil is not a homogeneous, isotropic, and ideal elastic material, the mathematical modelling of stress propagation phenomenon is very difficult.

Many mathematical models of stress propagation in the soil under different traction devices are based on the Boussinesq equations, which describes the stress distribution under a point load (Fig. 1) acting on a homogeneous, isotropic, semi-infinite, and ideal elastic medium [3,10].

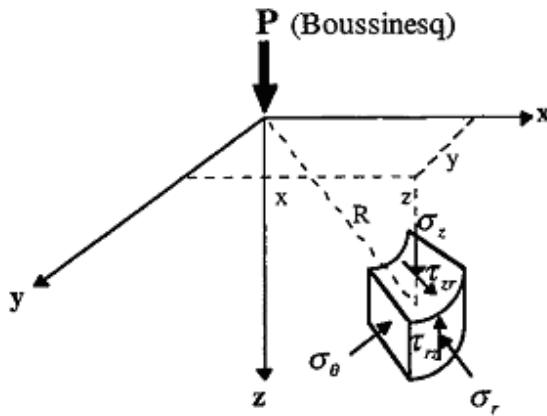


Fig. 1. Stress state produced by a concentrated vertical load [9]

Frohlich developed equations to account for stress concentration around the point of application of a concentrated load for the problem of the half-space medium subjected to a vertical load  $P$  [Kolen 1983] [8].

Many models of dynamic soil behaviour use elastic properties of soil, and when the soil is represented by a linearly elastic, homogenous, isotropic, weightless material, the elastic properties required to fully account for the behaviour of the material are Young's modulus ( $E$ ), shear modulus ( $G$ ), and Poisson's ratio ( $\nu$ ).

The main objective of this paper is to find a mathematical model for prediction of stress state in agricultural soil below tractors tyres using the most advanced mathematical tools.

The Finite Element Method (FEM) is proving to be very promising to modelling this propagation phenomenon. For agricultural soil, the relationships between stresses and strains are measured on soil samples in the laboratory or directly in the field. The stress-strain relationships are given by constitutive equations [4].

## 2. Analytical Model for Stress Propagation in Agricultural Soil

The stress levels under a point load like in figure 1 are given in cylindrical coordinates as follows [9]:

$$\sigma_z = \frac{3 \cdot P \cdot z^3}{2 \cdot \pi \cdot R^5} \quad (1)$$

$$\sigma_r = \frac{P \cdot z^3}{2 \cdot \pi} \cdot \left[ \frac{3 \cdot z \cdot r^2}{R^5} - \frac{1 - 2 \cdot \nu}{R \cdot (R + z)} \right] \quad (2)$$

$$\sigma_\theta = \frac{P \cdot (1 - 2 \cdot \nu)}{2 \cdot \pi} \cdot \left[ \frac{1}{R \cdot (R + z)} - \frac{z}{R^3} \right] \quad (3)$$

$$\tau_{rz} = \frac{3 \cdot P \cdot r \cdot z^2}{2 \cdot \pi \cdot R^5} \quad (4)$$

where  $P$  – is the point load,  $\nu$  – Poisson's ratio,  $\sigma_{z,r,\theta}$  – normal stress components, and  $\tau_{rz}$  – shear stress component.

Figure 2 shows the stress state in soil, of an infinitely cubic soil element, which can be written in a matrix, termed the matrix of the stress tensors [8]. Stresses acting on a soil element can be described by mechanical invariants, which are independent of the choice of reference axes. The invariants yield [7]:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_x + \sigma_y + \sigma_z \quad (5)$$

$$I_2 = \sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2 = \sigma_1 \sigma_2 + \sigma_1 \sigma_3 + \sigma_2 \sigma_3 \quad (6)$$

$$I_3 = \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{xz} \tau_{yz} - \sigma_x \tau_{yz}^2 - \sigma_y \tau_{xz}^2 - \sigma_z \tau_{xy}^2 = \sigma_1 \sigma_2 \sigma_3 \quad (7)$$

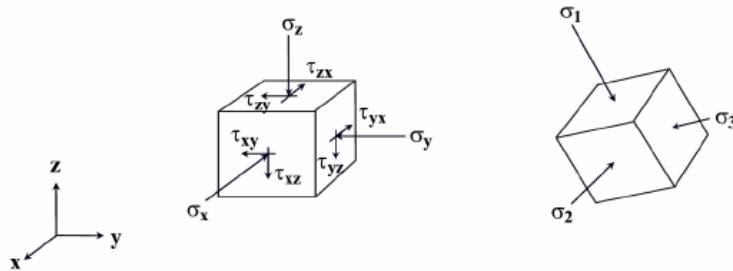


Fig. 2. Stress tensor components [8]

It is useful to define the stress measures that are invariant. Such stress is the octahedral normal stress and the octahedral shear stress:

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}I_1 \quad (8)$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} = \sqrt{\frac{2}{9} \cdot (I_1^2 - 3I_2)} \quad (9)$$

The critical state soil mechanics terminology uses the mean normal stress  $p$  and the deviator stress  $q$ . Whereas  $p = \sigma_{oct}$  Eq. (8),  $q$  is given as [7]:

$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} = \sqrt{(I_1^2 - 3I_2)} \quad (10)$$

The incremental methods are used to deal with material and geometrically non-linear problems. The basis of the incremental procedure is the subdivision of the load into many small increments. Each increment is treated in a piecemeal linear fashion with the stiffness matrix evaluated at the start of the increment. The tangent stiffness,  $E_t$  (Fig. 3) for each element is calculated from the stress-strain curves according to the current stress level of that element. It is worth noting that normally a strain increment,  $d\epsilon$ , is defined as the ratio of an incremental length to the original length.

In a FEM calculation when the coordinates are continually updated, the strain increment  $d\epsilon$ , has the mean of a ratio between an incremental length and the current length.

The relationship between  $\epsilon$  and  $\epsilon$  has the form [4]:

$$\epsilon = 1 - e^{-\epsilon} \quad (11)$$

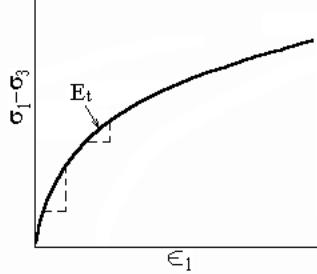


Fig. 3. Stress-strain curve for agricultural soil

According to the relationship between  $\epsilon$  and  $\epsilon$  the following revised stress-strain and tangent stiffness formulae were derived and used in the calculation [4]:

$$\sigma_1 - \sigma_3 = \frac{1 - e^{-\epsilon_1}}{a + b \cdot (1 - e^{-\epsilon_1})} \quad (12)$$

$$E_t = \frac{1}{a} \cdot [1 - b \cdot (\sigma_1 - \sigma_3)] \cdot [1 - (b + a) \cdot (\sigma_1 - \sigma_3)] \quad (13)$$

For saturated soil under an un-drained condition, the volume change is generally considered to be negligible. But for FEM calculation purposes, it is common to assume a constant Poisson's ratio slightly less than 0,5 [4].

In terms of the concept of the incremental method, for a soil with nonlinear properties when increments are very small, Hooke's law in which the Young's modulus,  $E_t$ , and Poisson's ratio,  $\nu_t$ , are variables (depending on current stress and strain values) is valid [4]. On the basis of this, for a plane strain problem, a formula for the volume modulus,  $K_t$ , can be derived:

$$K_t = \frac{d(\sigma_x + \sigma_y)}{d(\varepsilon_x + \varepsilon_y)} = \frac{E_t}{(1 - \nu_t - 2 \cdot \nu_t^2)} \quad (14)$$

where:  $\varepsilon_x$ ,  $\varepsilon_y$  are strains in  $x$  and  $y$  directions;  $\sigma_x$ ,  $\sigma_y$  are stresses in  $x$  and  $y$  directions.

If  $\nu_t$  is constant, as  $E_t$  decreases (soil failure),  $K_t$  also decreases. This means that soil volume changes can be large. Assuming  $K_t$  is constant, and the initial values of  $E_t$  and  $\nu_t$  are  $E_0$  and  $\nu_0$ , respectively, then the Poisson's ratio formula can be derived as in Eqn (15) in which a maximum  $\nu_t$  and a minimum  $E_t$  may be specified to avoid the calculation problem:

$$\nu_t = 0,25 \cdot \left( \sqrt{9 - \frac{8 \cdot E_t}{E_0} \cdot (1 - \nu_0 - 2 \cdot \nu_0^2)} - 1 \right) \quad (15)$$

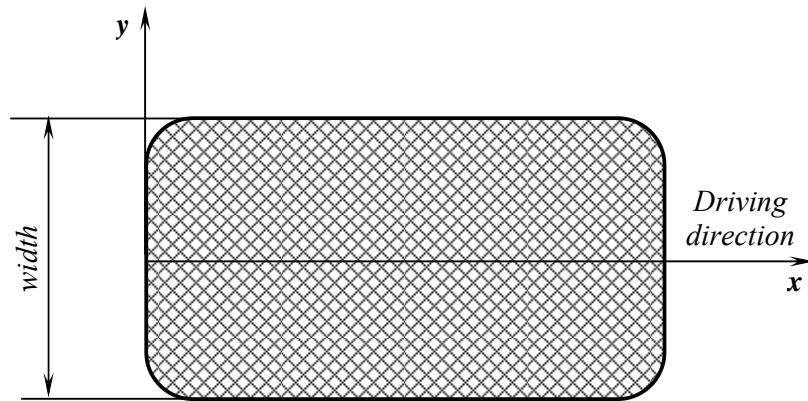


Fig. 4. Shape of contact surface between the soil and the tyre

Figure 4 shows the theoretical shape of contact area between the soil and agricultural tyres. The pressure distribution along to the width of tyre is described by a decay function [7]:

$$p(y) = C \cdot \left[ \frac{w(x)}{2} - y \right] \cdot e^{-\delta \cdot \left[ \frac{w(x)}{2} - y \right]}; 0 \leq y \leq \frac{w(x)}{2} \quad (16)$$

and the pressure distribution in the driving direction is described by a power-law function:

$$p(x) = p_{x=0,y} \cdot \left\{ 1 - \left[ \frac{x}{l(y)} \right]^\alpha \right\}; 0 \leq x \leq \frac{l(y)}{2} \quad (17)$$

where  $C$ ,  $\delta$  and  $\alpha$  are parameters,  $w(x)$  is the width of contact between the tyre and soil,  $p_{x=0,y}$  is the pressure under the tyre centre and  $l(y)$  is the length of contact between the tyre and soil.

Equation (16) is able to describe different cases of pressure distribution, e.g. maximum pressure under the tyre centre or pressure under the tyre edge. The parameters  $C$ ,  $\delta$  and  $\alpha$  are calculated from wheel load, tyre inflation pressure, recommended tyre inflation pressure at given wheel load, tyre width and overall diameter of the unloaded tyre. All these parameters are easy to measure or readily available from e.g. tyre catalogues.

### 3. Materials and Methods

The Drucker-Prager plasticity model can be used to simulate the behaviour of agricultural soil. The yield criterion can be defined as:

$$F = 3 \cdot \alpha \cdot \sigma_m + \bar{\sigma} - k = 0 \quad (18)$$

where  $\alpha$  and  $k$  are material constants which are assumed unchanged during the analysis,  $\sigma_m$  is the mean stress and  $\bar{\sigma}$  is the effective stress,  $\alpha$  and  $k$  are functions of two material parameters  $\phi$  and  $c$  obtained from experiments where  $\phi$  is the angle of internal friction and  $c$  is the material cohesion strength.

In using this material model, the following considerations should be noted: strains are assumed to be small; problems with large displacements can be handled provided that the small strains assumption is still valid; the use of NR (Newton-Raphson) iterative method is recommended; material parameters  $\phi$  and  $c$  must be bounded in the following ranges:  $90 \geq \phi \geq 0$  and  $c \geq 0$ .

The required input parameters for the constitutive model of the agricultural soil of wet clay type are [4,5,8]:

- Cohesion of soil ( $c$ ): 18.12 kPa
- Internal friction angle of soil ( $\phi$ ): 30°
- Soil density ( $\gamma_w$ ): 1270 kg/m<sup>3</sup>
- Poisson's ratio  $\nu_s$ : 0.329
- Young's modulus  $E$ : 3000 kPa

Figure 5 shows the distribution of vertical load in the contact area beneath agricultural tyres for three considerations: the real distribution with measured values (left), a model with uniform load distribution (centre), and a better model with un-uniform load distribution (right).

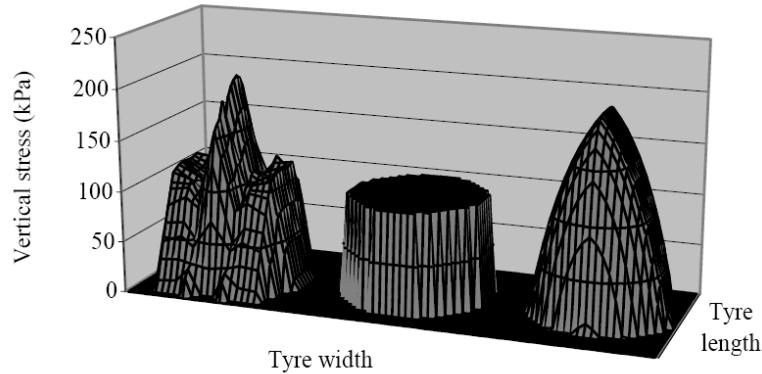


Fig. 5. Distribution of vertical load in the contact area [7]

In this paper, it was considered a soil volume with the depth of 1 meter, the width of 3 meter and length of 4 meter (Fig. 6) under the act of wheels of U-650 tractor, for two different cases of load distribution (Fig. 7) (first -uniform load distribution, and second - decay function). The structural nonlinear analysis was made on the ideal model, which was considered the soil as if it were homogeneous and isotropic material. It was used the COSMOS/M 2.95 Programme for FEM modelling [11].

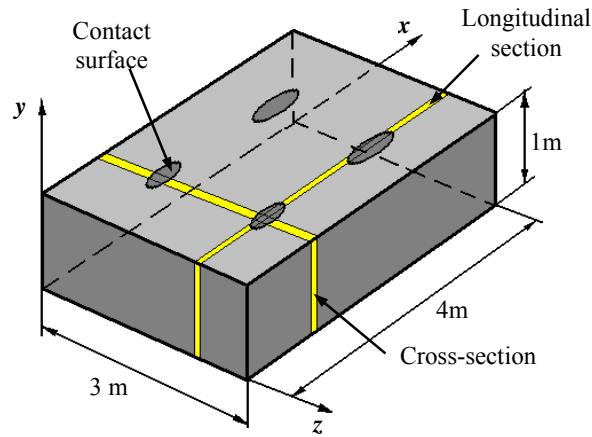


Fig. 6. Analyzed soil volume

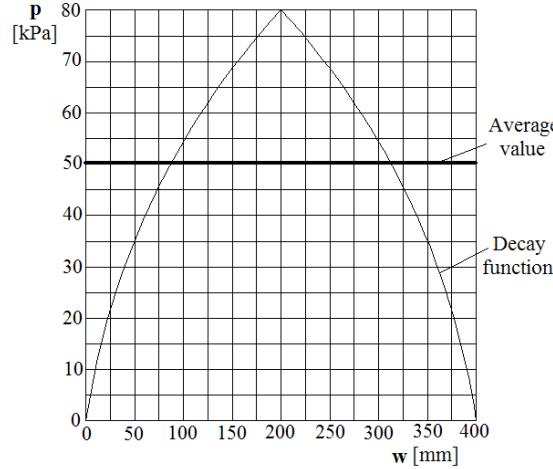


Fig. 7. Distribution of vertical load in the contact area along to the width of tyre for U-650 tractor

#### 4. Results and discussion

Figure 8 shows the results of FEM analysis in cross-section for a “1/2 symmetrical model” which consists in equivalent stresses distribution in agricultural soil under the action of a uniform load in the case of back wheel of U-650 tractor.

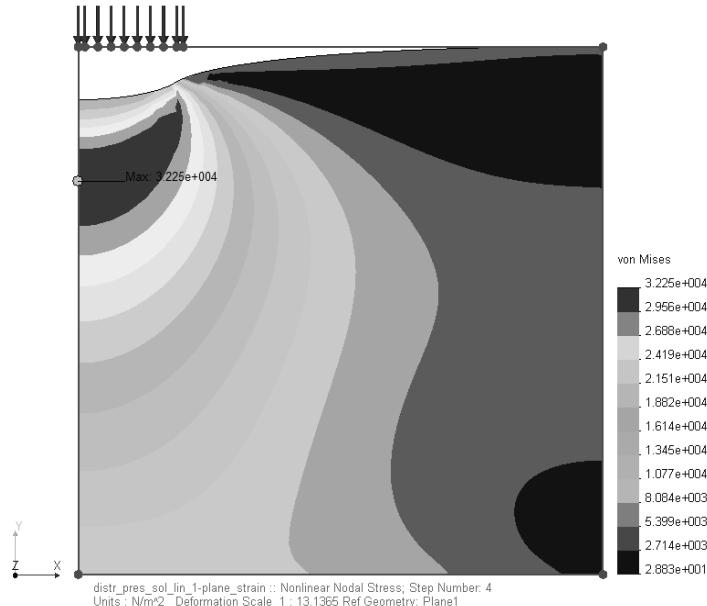


Fig. 8. Distribution of equivalent stresses for uniform load (Units: Pa)

Figure 9 shows the distribution of equivalent stresses in agricultural soil in cross-section for the same “1/2 symmetrical model” under the action of an un-uniform load (Decay function) in the case of back wheel of U-650 tractor.

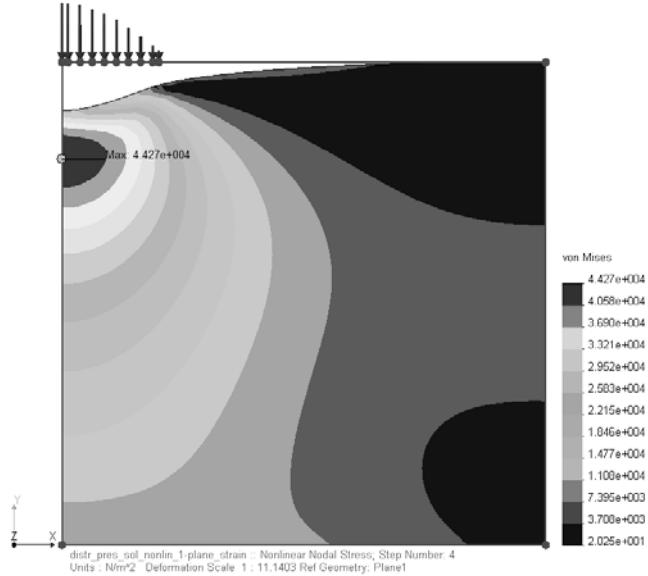


Fig. 9. Distribution of equivalent stresses for un-linear load (Units: Pa)

Figure 10 shows the graphical variation of equivalent stresses along the vertical-axial direction for the two cases of loading.

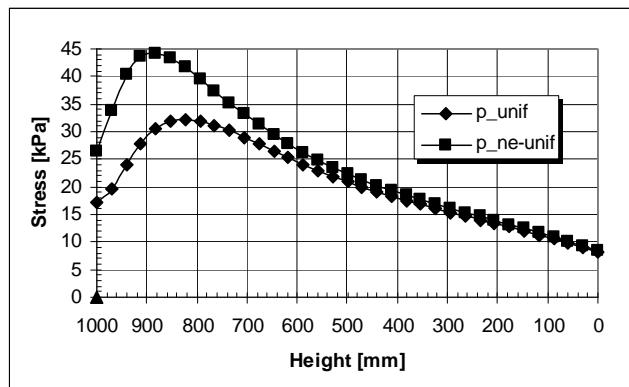


Fig. 10. Graphical variation of stresses along the vertical-axial direction

As we can see from the figures 8, 9, and 10, the distribution of equivalent stresses in soil volume are strongly influenced with the loading distribution in the contact area.

#### 4. Conclusions

Today, there is the true development's possibility of the pseudo-analytical procedures to modelling of stress propagation in agricultural soil, based on the work of Boussinesq, Fröhlich and Söhne, using the numerical calculus procedures, respectively the finite element method [1,2,4,7,10,11].

The Finite Element Method is in present the most advanced mathematical tool which can be used for the study of agricultural soil artificial compaction process. For mathematical modelling the soil is considered as a homogeneous and isotropic material, and the Drucker-Prager plasticity model can be used to simulate the behaviour of agricultural soil.

As it can be noticed from the figures 8, 9, and 10 the distribution of equivalent stresses in soil volume are strongly influenced with the loading distribution in the contact area.

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