

COMPARATIVE STUDY OF SUSPENSION OPTIMIZATION CRITERION FUNCTIONS

Ciprian TABACU¹, Dan Lucian CÂMPAN², Cristian EREMIA³, Mihai TERTIȘCO⁴

This paper presents a new optimization criterion for the suspension's behavior of vehicles under a profile of random rough road. A Monte-Carlo method is proposed for searching minimum criterion function. The proposed criterion function is defined by the ratio between the change of random output signal and the change of random road profile. The results obtained through simulation for the proposed criterion were compared with the results obtained for the criterion used by many researchers. This criterion depends only on the suspension's random acceleration without taking into account the characteristics of the road. Simulations have demonstrated a high efficiency of the new proposed criterion in assessing the comfort of optimized suspension.

Keywords: vehicle suspension, simulation, optimization, road simulation, Matlab, Simulink, random road- suspension system , Monte-Carlo algorithm

1. Introduction

For the vehicle's optimal comfort, a number of disruptive factors are needed to be diminished, suspension vibrations being one of these. For this purpose are being used various assessment criteria for the suspension's performances presented in Table 1. Most literature works use a criterion based on a full body motion acceleration of the sprung mass.

Given a suspended mass constant $m = \text{const}$, the values of acceleration $a(t)$ also expresses the destructive forces of inertia $F = ma(t)$ acting the body mass m as being suspended. Our work is entirely based on the acceleration criterion. If for vehicle suspension optimization is proposed, all full criteria, but based on displacement amplitude oscillating vertical sprung mass compared to the amplitude of the oscillating stochastic perturbation applied to the system input- road suspension, suspended mass. To test the simulation of the two criterion functions mentioned above are used in the case study as simple a model type

¹ PhD, University POLITEHNICA of Bucharest, Romania, e-mail: tabacu_ciprian@yahoo.com

² PhD, University POLITEHNICA of Bucharest, Romania, e-mail: ludan840808@yahoo.com

³ Eng., University POLITEHNICA of Bucharest, Romania, e-mail: ceremial2003@yahoo.com

⁴ Prof. Department of Automatics and Systems Engineering, University POLITEHNICA of Bucharest, Romania, e-mail: tertisco.mihai@gmail.com

quarter-car to accommodate detailed presentation of the testing procedure and the results of the comparative study of the two criteria optimization.

These indicators are determined by theoretical analysis of the mathematical model of the suspension, insuring the comfort of the vehicles when driving on roads involves reducing the discomfort due to some disturbance factors of the physiological and the mental state of the driver and the passengers such as: mechanical vibration, climatic conditions of the vehicle enclosure, excessive noise, solar radiation, etc. The comfort of the road vehicles as well as their degree of maneuverability and stability are among the most frequently used in a vehicle evaluation from the road point of view – suspension system. A large research effort was made to find the factors that affect the ride comfort [6]. For example, it was investigated the human behavior in the random vibrations conditions, and was demonstrated that people are very sensitive to very low frequencies below 1 Hz. This is the explanation for which were nominated a series of performance indicators of the suspension system for road vehicles presented in table 1. The majority of the performance indicators characterizes the suspension system through some point values regarding the system's response to the sinusoidal signal (or a sum of sinusoidal signals of different frequencies and amplitudes) that shape the road irregularities by deterministic signals applied to the input of the system. These theoretical analyses derive some expressions for the calculation of these indicators in terms of certain parameters pulsation of the mathematical model as: the sine wave frequency shaping the road, the natural the system, the maximum amplitude of response etc. The values of these indicators can be used to compare different model structures for quarter-car suspension type half car type with one, two, three or four degrees of freedom etc.. But these indicators are not convenient to use for testing the road - suspension system in real road conditions when the suspension must filter signals $u(t)$ of the white noise type of arbitrary dispersion, induced by the road at the system entry. Some deficiencies are eliminated by using an experimental criterion of the last position of table 1.

Table 1.

Indicators and benchmarks of the vehicle suspension system

Indicator	Calculation relation	Where:	
Indicator 1: Transmissibility by acceleration	$\eta = \left \frac{\max(\ddot{x})}{\max(\omega_n^2 y)} \right $	\ddot{x} - vertical acceleration; ω_n – natural pulsation;	[1] [2]
Indicator 2: Transmissibility by acceleration	$\eta = \frac{\omega^2 \sqrt{\omega_n^4 + (2\zeta\omega_n\omega)^2}}{\omega_n^2 \sqrt{(\omega_n^2 - \omega)^2 + 4\zeta^2\omega_n^2\omega^2}}$	ω - sinus signal pulsation that models the road in case 2DOF1/4M	[3]

Indicator 3: Transmissibility by movement	$\lambda = \frac{\max(x_r)}{\max(y)}$	<i>x - suspended mass movement; y - wheel movement;</i>	[4]
The AMP criteria: The square medium deviation (AMP)	$J = \frac{1}{T} \int_0^T [y(t) - y_0]^2 dt$	<i>y - suspended mass position; y₀ - position in pause; T - registration duration;</i>	[5] [16]
Root mean square: RMS of f(t)	$f_{RMS} = \lim_{T \rightarrow \infty} \sqrt{\frac{1}{T} \int_0^T [f(t)]^2 dt}$	<i>f(T) - suspended mass acceleration; T - registration duration;</i>	[14]

Fig. 1 shows the block diagram of the *dynamic road-suspension system (DRSS)*, where, $u(t)$ shapes the perturbations caused by the road while walking and $y(t)$ expresses the vertical oscillations of the vehicle suspended mass while moving with the velocity v of the vehicle.

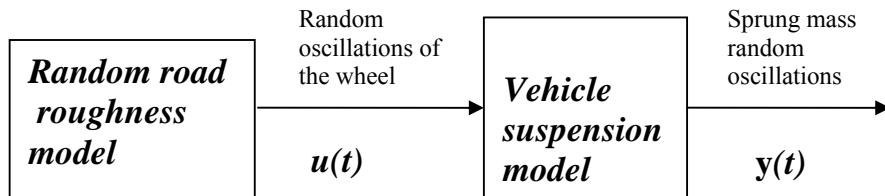


Fig. 1. Block diagram of a dynamic road-suspension system (DRSS)

The suspension design is based on mathematical models obtained considering the suspension as a linear system with concentrated parameters. These considerations make that the computer models describe just about the actual behavior of a vehicle's suspension system, as some of the suspension and body components (springs, dampers, tires, axles, etc.) have actually a slightly nonlinear behavior with distributed parameters [2]. For example, the suspended body weight is considered concentrated into a point while in reality its mass is distributed at the vehicle's gauge level. The method proposed in this paper is to organize the experimental correction of the suspension system parameters [3].

In the simplest case where only two parameters are considered: the stiffness k and the damping coefficient c of a quarter car, then the testing is done by changing the parameters around the nominal values c_0 and k_0 . The permissible variations of the parameters being of no more than 10% of the nominal value [4].

In section 2 are present some aspects regarding the modeling of the random road and the suspension in quarter-car version [7] and formulation of the problem system on road vehicles simulation and experimental optimization. In Section 3 is present the formulation of problems regarding the (RR-S) system quality characterizing through an integral criteria that reflects not only the parameters of the suspension, but also those of the road. Section 4 presents the results of tests by simulation in MATLAB-SIMULINK of the *road-suspension system* (RR-S). Section 5 contains some conclusions reached from testing in simulation of the new optimization criterion and statistical method proposed in this paper for optimizing a DRSS.

2. RR-S system simplified model

The structure simplified by neglecting the tire action of the (RR-S) system, in the $\frac{1}{4}$ car option with a semi-active damper is shown in Fig. 2. DRSS structure contains two distinct parts: the road and the suspension itself. These have different functions in *road-suspension system* and are modeled separately in order to simulate on the computer.

The road generates random disturbances $u(t)$ applied to the DRSS input. The road roughness is the most important disturbance for the driver and for the vehicle structure itself [8]. Traditionally, the road profile was modeled using some random processes [9,10].

The simplified structure of the suspension contains three main elements to be modeled. These three main elements are: the suspended mass m , the bow with the parameter k and the damper characterized by the coefficient c , by viscous friction, amended by an electromagnetic control valve to change the viscous friction by strangling the oil passage section [11].

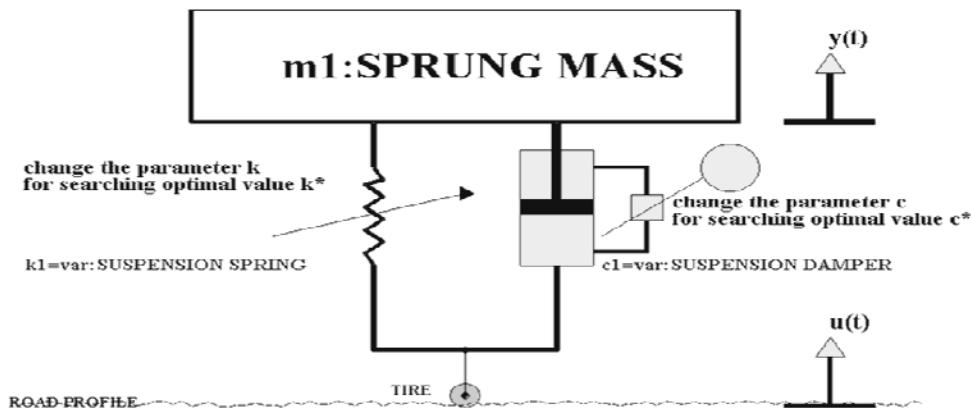


Fig. 2. Simplified structural model

From a theoretical perspective, the physical model is the damping behavior for a wide range of operating regimes. For determining these, many simplifying assumptions are made, such as the case of the suspension model type 1/4M from Fig. 3:

- The linear behavior of the dependence between the force and the displacement in case of flexible elements ($F_e = k(y-u) = kx$);
- The linear dependence between the viscous friction force and the damper piston movement speed and the speed of the oil passing through the damper piston passing hole ($F_{fv} = c \frac{dx}{dt}$);
- The suspension mass m considered constant and focused on a point (center of gravity) and the force of inertia calculated as the product between the mass and the acceleration ($m \frac{d^2 y}{dt^2}$);
- For simplicity we neglect the elastic force of the tires.

The dynamic behavior of the system 1/4 M:

$$m \frac{d^2 y}{dt^2} + c \frac{d(y-u)}{dt} + k(y-u) = 0 \quad (1)$$

respectively

$$y(t) = \frac{1}{m} [k \int \int (u - y) dt^2 - c \int (y - u) dt] \quad (2)$$

The road model is a critical component of the vehicle's simulation. If the complete simulation is to accurately represent reality the road model must accurately represent the terrain. In addition to modeling the large scale features of a particular road course the road model must also be capable of representing the small scale features such as bumps and other road surface irregularities. On the other hand, it is undesirable for the road model to be so detailed that massive amounts of data are required to generate a complete road course. More road surface profiles were measured, and several road models were discussed in the literature of specialty. In the vibration context, the road roughness is typically represented as a stationary stochastic process [12].

3. The estimation J and RMS criteria of the RR-S system performance optimization

In what concerns the integral criteria of suspension optimization in the last line of table 1, it presents the drawback that it doesn't take into account the characteristics of the road in an explicit coverage form in the performance criteria of the RR-S system. This is the *first problem* which is the subject of research

developed in our work is to find a performance indicator for RR-S system that depends on suspension settings (spring and damper) as well as random variations in intensity of road profile. The proposed criterion J is expressed by the ratio(3) of the average square for output signal $y(t)$ on the average square for input signal $u(t)$ and to compare calculated and RMS with equation (4):

$$J = -\frac{\int_0^T [y(t) - M(y)]^2 dt}{\int_0^T [u(t) - M(u)]^2 dt} \quad (3) \quad ; \quad RMS = \left[\frac{1}{T} \int_0^T a^2(t) dt \right]^{1/2} \quad (4)$$

Where $y(t)$ -sprung mass position; $u(t)$ -road profile; $a(t)$ -sprung mass acceleration; M -average operator

4. The simulation in MATLAB-SIMULINK of the optimization method

Fig. 3 presents the functional links between MATLAB and SIMULINK in the simulation optimization of RR-S System. In this interaction process, MATLAB send to Simulink data of suspension parameters m , k , c , and then Simulink send to Matlab $u(t)$, $a(t)$ and $y(t)$. Performance indicators J_1 and J_2 depend nonlinear on k and c parameters of RR-S system, (k , c). Thus it follows that the minimum performance index is obtained for the optimal values of the parameters $k = k^*$ and $c = c^*$. The designing and testing of a method for determining the k^* and c^* by simulating DRSS is the **second problem** analyzed in our work.

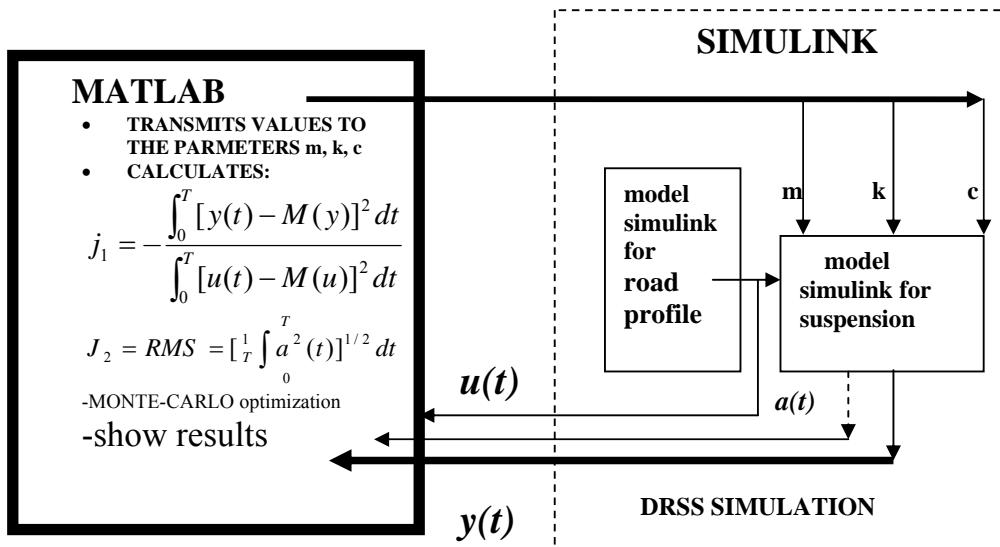


Fig. 3. The functional links between MATLAB and SIMULINK

Exchange of data, MATLAB → SIMULINK and SIMULINK → MATLAB, runs at each point ($k(i), c(i)$) as strings of random numbers generated by MATLAB to search MONTE_CARLO method (Fig. 4) optimal values c^* and k^* for the two optimality criteria J_1 and J_2 .

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STEP 1:
  MATLAB initialize,
  m=1000 , k1=k2=0 , c1=c2=0 and M1=M2=1(memory)

STEP 2:
  • MATLAB random value for parameters  $k(i)$  and  $c(i), i=1,2,...,50$ 
    FOR  $i=1:50$ 
  • MATLAB transmit random value of parameters  $k(i)$  and  $c(i)$  to SIMULINK;
  • MATLAB resives :  $u(t)$  MATLAB,  $y_i(t)$  and  $a_i(t)$  from SIMULINK;
  • MATLAB compute  $J_1(y_i)$  and  $J_2(a_i)$  with (3) and (4) relations
  • IF  $J_1(y_i) < M1$ 
    THEN  $M1 = J_1(y_i)$ ;  $k1=k(i)$ ;  $c1=c(i)$ 
    ELSE IF  $J_2(a_i) < M2$ ;
    THEN  $M2 = J_2(a_i)$ ;  $k2=k2(i)$ ;  $c2=c2(i)$ 
    END (if)
  END (for)
k*1 = k1 ; k*2=k2 ; c*1=c1; c*2=c2 J1* = M1 and J2* = M2

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Fig. 4. The MONTE-CARLO algorithm for optime values k^* , c^*
search of suspension model parameters

Matlab sends to Simulink the numerical of $m=1000$ $Kg=const$ and variable values of the parameters, $0 < k < 1000$ N/m and $0 < c < 500$ from 50 points according to the RR-S system model entry a random signal with uniform distribution and the other position to connect to the RR-S system input another Simulink block that generates a random signal of white noise type with limited band.

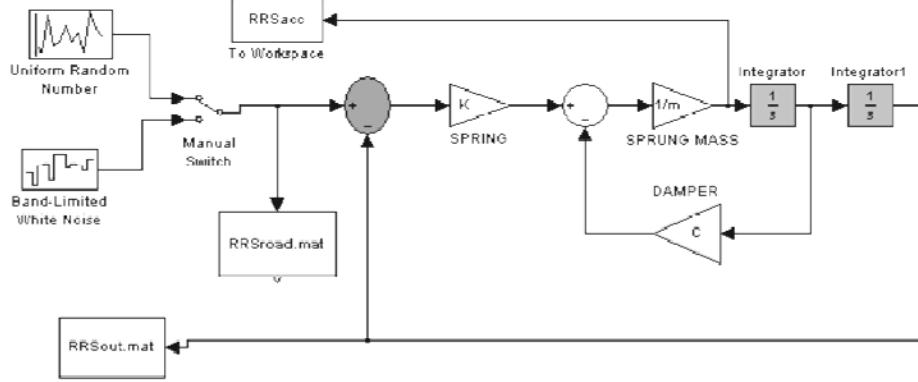


Fig. 5. The DRRS Simulink-model

Block called RRSout.mat in Fig. 5 send to Matlab $y(t)$ obtained from model output in response to $u(t)$ and $k(i)$ values and $c(i)$ received Simulink blocks.

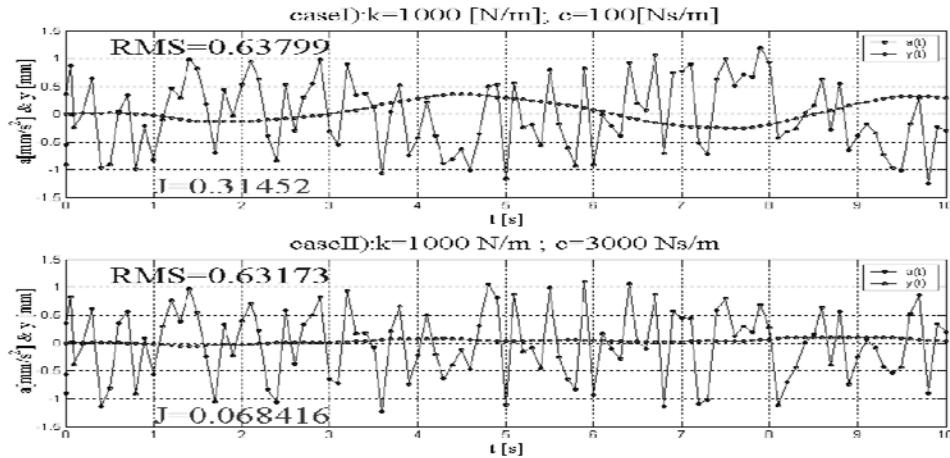


Fig. 6. Representing the worst outcome (CASE) and the optimal result (case II) MONTE-CARLO determined from 50 simulations for various combinations of random values for the parameters k and c .

The data transmission regarding $u(t), y(t)$ and $a(t)$ from Simulink to Matlab is made by the block called RRSroad.mat in Fig. 5. Note also that in the case of the random signal type white noise (the manual switch in Fig. 5, in "down" position), the results were similar, which also proves that the optimization criterion function (4) proposed in this paper is an indicator corresponding to

the suspension evaluation in road conditions with random behavior. RRSacc send to Matlab sprung mass acceleration $a(t)$ this $a(t)$ and $y(t)$ are represented in Fig.6. In this Fig. are represented worst outcome (case I), and the optimal result (case II) determined by Monte-Carlo simulations of 50 different combinations of random values in terms of the parameters (k, c). In the optimal suspension corresponds to case II value of criterion is $J(k=1000, c=3000)=0,068$ and for the worst case I was obtained $J=0.314$, a value of almost 5 times higher as the optimum value.

5. Conclusions

The proposed method is inspired from the optimal granting techniques of the automatic control systems parameters and was adapted for adjustment to some modifiable parameters of the suspension system of the vehicles. The criterion function adopted for optimization is the square standard deviation of the response signal of the suspended mass position (from a constant value). Experimental testing of vehicle suspension is justified by the fact that in vehicle design using approximate mathematical models which take into account the distributed nature of the sprung mass and nonlinear static and dynamic behavior of such components: spring, damper, etc. The method is inspired by techniques providing optimal automatic control systems and has been adapted for adjustment of modifiable parameters of the suspension system of the vehicle. Criterion function is adopted to optimize signal response ratio dispersion sprung mass position and dispersion of random signal road roughness induced RR-S system. Analysis of simulation results in Fig. 6 provides the following conclusions:

- RMS comparison for pairs (I, II) shows that an increase of 3 ... 10 times the damping coefficient c does not result in an appropriate increase in RMS values as [RMS (I, $c = 100$) = RMS (III, $c = 300$) = 0.29 for $k = 500$] & [RMS (II, $c = 100$) = RMS (IV, $c = 3000$) = 0.63 for $k = 1000$];
- comparison of the proposed criterion value J for pairs (I, III) and (II, IV) shows that an increase of 3 ... 10 times the damping coefficient c leads to an increase in appropriate J values as $J (I) = 0.1$ and $J (III) = 0.29$ and $J (S) = RMS (IV) = 0.63$;

The contributions of this paper are their main objectives:

- 1) An algorithm to optimize vehicle suspension either by simulation or bench test or in actual road conditions;
- 2) Introduction of performance indicators (3) to assess the dynamic behavior of the RR-S system;

3) Optimization problem formulation of the dynamic behavior of vehicle suspension model and proposed a Monte-Carlo method to solve this problem from simulation.

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