

## PULSATING FLOW THROUGH VERTICAL POROUS CHANNEL WITH VISCOUS DISSIPATION EFFECT

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*This study investigates the effect of viscous dissipation on the pulsatile flow through a vertical porous channel subjected to periodic heating at the heated walls. The flow governing equations are transformed into corresponding non-dimensional form. The dimensionless nonlinear coupled system of partial differential equations are then reduced to ordinary differential equations. Approximate solutions in the form of Adomian decomposition method (ADM) are obtained and the solutions are shown to be convergent. Important properties of the overall structure of the flow are presented and discussed including skin friction and Nusselt number.*

**Keywords:** ADM, pulsatile flow, porous channel, periodic heat input, skin friction, heat transfer

### 1. Introduction

Pulsatile flows has gained considerable attention over the past few decades due to its extensive applications in many industrial, engineering and physiological processes. The phenomena occurs in the cooling of electronic components, air-conditioning, thermal insulation, geothermal systems, flow of urine in the ureter, movement of spermatozoa's, blood flow in arterial network and human respiration to mention just a few. In recent times, a number of significant work have been done. For instance, in the purely oscillatory case Makinde and Mhone [1] investigated the radiative heat transfer to oscillatory flow of hydromagnetic heat absorbing fluid through a channel saturated with porous medium. This study was extended by Mehmood and Ali [2] by introducing effect of Navier slip at the interface of the porous wall and the porous walls. Bitla and Iyengar [3] studied the pulsating flow of an unsteady flow in which a periodic variation in flow velocity is superimposed on steady velocity.

Eldabe *et al* [4] presented pulsatile flow behavior of oscillatory hydromagnetic couple stress fluid without heat transfer. Since the result in [4], there have been a lot of appreciable work on the pulsatile fluid flow. For instance, Zuecco and Beg [5] applied the Network numerical simulation to the pulsatile case of the result in

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[4]. In addition, Adesanya and Ayeni [6] extended the pulsatile case examined in [5] to variable viscous case in which the channel is filled with porous medium. Similarly, Adesanya and Makinde [7] extended the result in [4] to the thermodynamical case in which the effect of radiative heat transfer to was considered in the pulsatile fluid flow.

In all the above studies, the effect of viscous dissipation was neglected. However, as explained by Jha and Ajibade [8], of all fluid properties viscosity is the most sensitive to temperature changes. This fluid property has a great influence on viscous dissipation because variation in fluid viscosity due to temperature may affect the flow characteristics as well as the efficient operation of industrial machinery where lubrication is important. Motivated by [8], the specific objective of this paper is to investigate the mixed convective pulsatile fluid flow through a heated porous channel with time-periodic boundary conditions which has not been accounted for in the previous models in the literature.

In the rest of the paper, section 2 gives the Mathematical formulation of the problem including model assumptions and non-dimensionalization. Section 3, presents the detailed method of solution via the rapidly convergent Adomian decomposition method. Similar method has been used for nonlinear problems [9] – [13]. Results are presented and discussed in section 4 while section 5 concludes the paper

## 2. Mathematical Analysis

We consider an unsteady flow of a viscous dissipating fluid flow through infinite vertical plates located at  $y = \pm h$  and subjected to steady-periodic heating. The  $x'$  - axis is taken along the infinite plate and  $y'$  - axis normal to it as shown in Fig. 1. The flow is acted upon by a pulsatile pressure gradient in the direction of the flow together with buoyancy forces. The fluid is assumed to be injected with a certain constant velocity  $v_0$  on one part of the plate and sucked off with the same velocity on the other plate. All fluid properties are assumed constant except the density which varies with temperature.

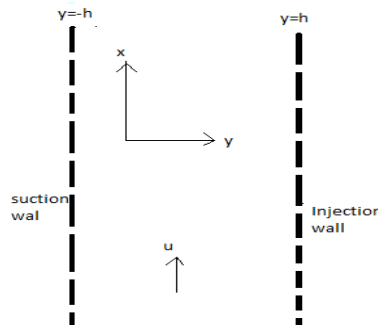


Fig. 1: Flow geometry

The equations governing the forced convective fluid flow can be written as [8]:

$$\left. \begin{aligned} \frac{\partial u'}{\partial t'} + v_0 \frac{\partial u'}{\partial y'} &= -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_0) \\ \frac{\partial T}{\partial t'} + v_0 \frac{\partial T}{\partial y'} &= \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \end{aligned} \right\} \quad (1)$$

together with appropriate initial-boundary conditions;

$$\begin{aligned} u'(t', y') &= 0, \quad T(y', t') = T_0 \quad t' \leq 0 \\ u'(t', y') &= 0, \quad T(y', t') = T_1 + (T_1 - T_0) \cos(\omega t') \quad y' = \pm h \quad t' > 0 \end{aligned} \quad (2)$$

additional terms in (1) is the pressure gradient and  $v_0$  is due suction/injection of fluid at porous walls. Introducing the following dimensionless parameters and quantities,

$$\begin{aligned} t = \omega t', \quad u = \frac{u'}{U}, \quad y = \frac{y'}{h}, \quad x = \frac{x'}{h}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad s = \frac{v_0 h}{\nu}, \quad p' = \frac{\mu U p}{h} \\ Gr = \frac{g\beta(T_1 - T_0)h^2}{\nu U}, \quad Ec = \frac{U^2}{C_p(T_1 - T_0)}, \quad Pr = \frac{\mu C_p}{k}, \quad St = \frac{\omega h^2}{\nu} \end{aligned} \quad (3)$$

we get, the following coupled partial differential equations

$$\left. \begin{aligned} St \frac{\partial u}{\partial t} + s \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Gr \theta \\ St \frac{\partial \theta}{\partial t} + s \frac{\partial \theta}{\partial y} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 \end{aligned} \right\} \quad (4)$$

subject to the boundary conditions

$$\begin{aligned} u(0, y) &= 0, \quad \theta(y, t) = 0 \quad t \leq 0 \\ u(t, y) &= 0, \quad \theta(y, t) = 1 + \cos(t) \quad y = \pm 1 \quad t > 0 \end{aligned} \quad (5)$$

where

$C_p$  – specific heat at constant pressure,  $\mu$  – dynamic fluid viscosity,  $\nu$  – kinematic viscosity,  $g$  – acceleration due to gravity,  $h$  – channel width,  $Pr$  – Prandtl number,  $(p, p')$  – dimensional and dimensionless fluid pressure,  $(t', t)$  – dimensional and dimensionless time,  $T_0$  – initial fluid temperature,  $T_1$  – fluid temperature,  $(T, \theta)$  – dimensional and dimensionless temperature of the fluid,  $u'$  – fluid velocity,  $u$  – dimensionless velocity of fluid,  $U$  – characteristic velocity,  $(x', x)$  – dimensional and dimensionless horizontal component of velocity,  $\beta$  – coefficient of thermal expansion,  $\kappa$  – thermal conductivity,  $\rho$  – density of the fluid,  $\omega$  – pulsating frequency,  $(v_0, s)$  is the dimensional and dimensionless suction/injection

parameter,  $Gr$  is the Grashof number,  $Ec$  is the Eckert number and  $St$  is the Strouhal number.

### 3. Method of Solution

To obtain the solution of the coupled nonlinear equations (4)-(5), we assume a perturbative solution in the form:

$$-\frac{\partial p}{\partial x} = \lambda_0 + \lambda_1 e^{it}, \quad u(t, y) = A(y) + B(y)e^{it}, \quad \theta(t, y) = F(y) + P(y)e^{it}, \quad (6)$$

and neglecting higher orders. Where  $A(y)$  – steady velocity profile,  $B(y)$  – unsteady velocity profile,  $F(y)$  – steady temperature profile,  $P(y)$  – unsteady temperature profile and  $\lambda_0, \lambda_1$  are positive constants.

with (6) in (4) – (5), we have

$$A''(y) - sA'(y) = -\lambda_0 - GrF(y); \quad A(\pm 1) = 0 \quad (7)$$

$$B''(y) - sB'(y) - iStB(y) = -\lambda_1 - GrP(y); \quad B(\pm 1) = 0 \quad (8)$$

$$F''(y) - sF'(y) = -Ec(A'(y))^2; \quad F(\pm 1) = 1 \quad (9)$$

$$P''(y) - sP'(y) - iStP(y) = -2EcA'(y)B'(y); \quad P(\pm 1) = 1. \quad (10)$$

Transforming (7) – (10) to integral equations, we get

$$A(y) = \int_{-1}^y \frac{dA(-1)}{dY} dY + \int_{-1}^y \int_{-1}^y (sA'(Y) - \lambda_0 - GrF(Y)) dY dY \quad (11)$$

$$B(y) = \int_{-1}^y s \frac{dB(-1)}{dY} dY + \int_{-1}^y \int_{-1}^y (sB'(Y) - \lambda_1 + iStB(Y) - GrP(Y)) dY dY \quad (12)$$

$$F(y) = 1 + \int_{-1}^y \frac{dF(-1)}{dY} dY + \int_{-1}^y \int_{-1}^y (sF'(Y) - Ec(A'(Y))^2) dY dY \quad (13)$$

$$P(y) = 1 + \int_{-1}^y \frac{dP(-1)}{dY} dY + \int_{-1}^y \int_{-1}^y (sP'(Y) + iStP(Y) - 2EcA'(Y)B'(Y)) dY dY. \quad (14)$$

By ADM [9] – [13], we assume a series solution in the form

$$A(y) = \sum_{n=0}^{\infty} A_n(y), \quad B(y) = \sum_{n=0}^{\infty} B_n(y) \quad (15)$$

$$F(y) = \sum_{n=0}^{\infty} F_n(y), \quad P(y) = \sum_{n=0}^{\infty} P_n(y) \quad (16)$$

using (15) – (16) in the Volterra integral equations (11) – (14) leads to an iterative scheme given by

$$\begin{aligned}
A_0 &= -\int_{-1}^y \int_{-1}^y \lambda_0 dY dY + \int_{-1}^y \frac{dA(-1)}{dY} dY, \\
B_0 &= -\int_{-1}^y \int_{-1}^y \lambda_1 dY dY + \int_{-1}^y \frac{dB(-1)}{dY} dY, \\
F_0 &= 1 + \int_{-1}^y \frac{dF(-1)}{dY} dY, \\
P_0 &= 1 + \int_{-1}^{y-1} \left( \frac{dP(-1)}{dY} dY \right)
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
A_{n+1} &= \int_{-1}^y \int_{-1}^y \left( s \frac{dA_n}{dY} - Gr F_n \right) dY dY \\
B_{n+1} &= \int_{-1}^y \int_{-1}^y \left( iSt B_n - Gr P_n + s \frac{dB_n}{dY} \right) dY dY \\
F_{n+1} &= \int_{-1}^y \int_{-1}^y \left( s Pr \frac{dF_n}{dY} - Ec Pr(Q_n) + \right) dY dY \\
P_{n+1} &= \int_{-1}^y \int_{-1}^y \left( iSt Pr P_n + s Pr \frac{dP_n}{dY} - 2 Pr Ec M_n \right) dY dY
\end{aligned} \tag{18}$$

where the nonlinear terms written as

$$M = \left( \frac{dA}{dY} \frac{dB}{dY} \right), \quad Q = \left( \frac{dA}{dY} \right)^2 \tag{19}$$

are decomposed into Adomian polynomials as follows

$$\begin{aligned}
Q_0 &= (A'_0(Y))^2 \\
Q_1 &= 2A'_0(Y)A'_1(Y) \\
Q_2 &= (A'_1(Y))^2 + 2A'_0(Y)A'_2(Y) \\
&\dots
\end{aligned} \tag{20}$$

and

$$\begin{aligned}
M_0 &= A'_0(Y)B'_0(Y) \\
M_1 &= A'_0(Y)B'_1(Y) + A'_1(Y)B'_0(Y) \\
M_2 &= A'_0(Y)B'_2(Y) + A'_1(Y)B'_1(Y) + A'_2(Y)B'_0(Y) \\
&\dots
\end{aligned} \tag{21}$$

equations (17)-(21) are then coded on a computer symbolic package- mathematica version 8 and mathematical expression for the unknown constants are obtained using the rest boundary conditions at  $y = 1$ .

Let us define  $m$  to be the truncation point, then the partial sums given by

$$A(Y) = \sum_{n=0}^m A_n(Y), \quad F(Y) = \sum_{n=0}^m F_n(Y), \quad B(Y) = \sum_{n=0}^m B_n(Y), \quad P(Y) = \sum_{n=0}^m P_n(Y) \quad (22)$$

are the approximate solutions of the coupled system of equations (7)-(10) which are substituted into the assumed solution (6). Due to large output of the symbolic series solutions only the graphical results will be presented.

Other parameter of interest includes the skin friction at the heated plates

$$Sf = \frac{\partial u}{\partial y} = \frac{\partial A}{\partial y} + \frac{\partial B}{\partial y} \cos(t), \quad (23)$$

and the rate of heat transfer

$$Nu = -\frac{\partial \theta}{\partial y} = -\frac{\partial F}{\partial y} - \frac{\partial P}{\partial y} \cos(t). \quad (24)$$

#### 4. Results and Discussion

Table 1:

Convergence result for  $Ec = 0, m = 5, t = 0.1, 1 = s = Gr = \lambda_0 = \lambda_1 = St$

$y$	$u_{EXACT}$	$u_{ADOMIAN}$	$ABS. Error$	$\theta_{EXACT}$	$\theta_{ADOMIAN}$	$ABS. Error$
-1	0	0	$8.40745 \times 10^{-18}$	1.995	1.995	$4.44089 \times 10^{-16}$
-0.75	0.585714	0.58624	$5.26129 \times 10^{-4}$	1.95208	1.95175	$3.34076 \times 10^{-4}$
-0.5	1.06577	1.06701	$1.245290 \times 10^{-3}$	1.90692	1.90613	$7.92176 \times 10^{-4}$
-0.25	1.42206	1.42431	$2.25663 \times 10^{-3}$	1.86734	1.8659	$1.43945 \times 10^{-3}$
0	1.62986	1.63349	$3.63445 \times 10^{-3}$	1.84116	1.83883	$2.33120 \times 10^{-3}$
0.25	1.65389	1.65917	$5.27784 \times 10^{-3}$	1.83585	1.83243	$3.42464 \times 10^{-3}$
0.5	1.44332	1.44992	$6.59776 \times 10^{-3}$	1.85775	1.85338	$4.36695 \times 10^{-3}$
0.75	0.925758	0.931744	$5.98569 \times 10^{-3}$	1.91087	1.9068	$4.07589 \times 10^{-3}$
1	0	0	$3.64385 \times 10^{-17}$	1.995	1.995	0

Table 1 shows the convergence of the solution in the special case when  $Ec = 0$ . The accuracy of the solution can be improved by computing more terms, that is, as  $n \rightarrow \infty$ . To illustrate the effects of fluid parameters of the flow, the following graphs are presented. Computations were done for  $\lambda_0 = \lambda_1 = 1$ . Figs. 2 represents the effect of variations in Eckert number on velocity profile. From the plot, it is observed that an increase in Eckert number has an increasing effect on the velocity profile. This is due to rise in the kinetic energy due to heat source. Figure 3 shows the effect of Grashof number on velocity profile. The result shows that an increase in Grashof number increases the velocity profile due to rise in buoyancy forces. Figure 4 depicts the plot of velocity profile for variations in Strouhal

number. The result also shows that an increase in the Strouhal number reduces the flow velocity. This is because as the pulsating frequency increases there is reduction in the maximum displacement and the flow tends toward a mean steady state. Similar phenomenon is seen in Figure 5 for the temperature distribution within the channel. Figure 6 shows that, as Eckert number increases, the fluid temperature also increases. This is true since additional heat is generated within the moving fluid as a result of rise in the kinetic energy within the channel. Figure 7 represents the effect of rise in Prandtl number on the fluid temperature. The results shows that an increase in Prandtl number means decrease in the thermal conductivity of the fluid. This eventually lowers the fluid temperature within the channel as observed in the plot. Figures 8 and 9 show the effect of Eckert number on the fluid flow and temperature distribution respectively. As observed, an increase in Eckert number enhances the flow and temperature distribution with increase in time. However, it is observed in Figures 10 and 11, that an increase in the pulsating frequency parameter lowers the fluid velocity and the temperature. Figure 12 shows the effect of Eckert number on the skin friction. It is observed that an increase in Eckert number enhances the skin friction at the suction wall while it weakens skin friction at the wall with injection. Finally in Figure 13, an increase in Eckert number is observed to enhance the rate of heat transfer from the walls to the fluid. However, the rate of heat transfer is observed to decrease along of the heated plates. This is because as the plate get hotter, the rate of heat transfer decreases.

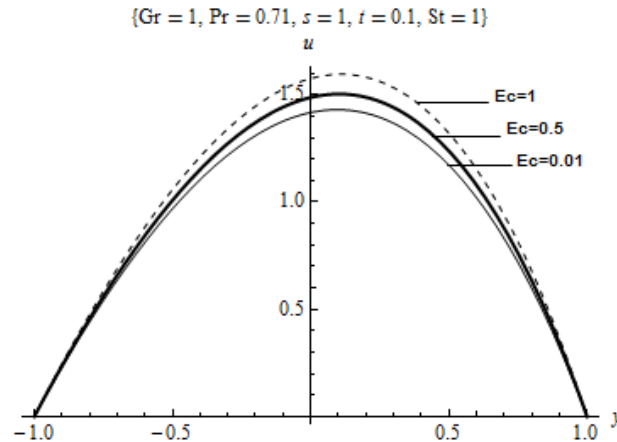


Fig. 2: velocity profile with different values of Eckert number

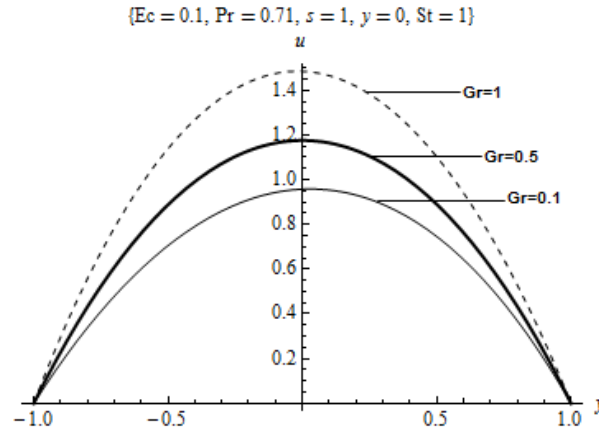


Fig. 3: velocity profile with different values of Grashof number

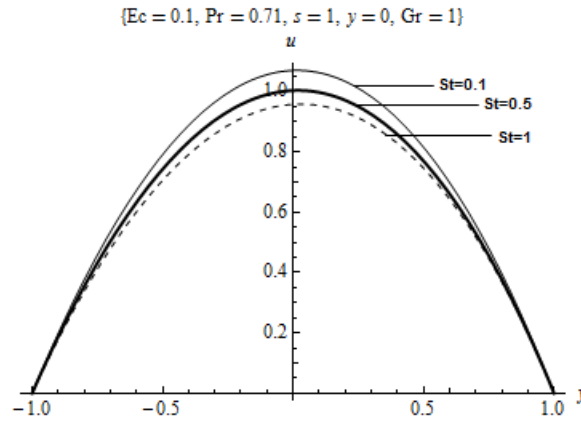


Fig. 4: velocity profile with different values of Strouhal number

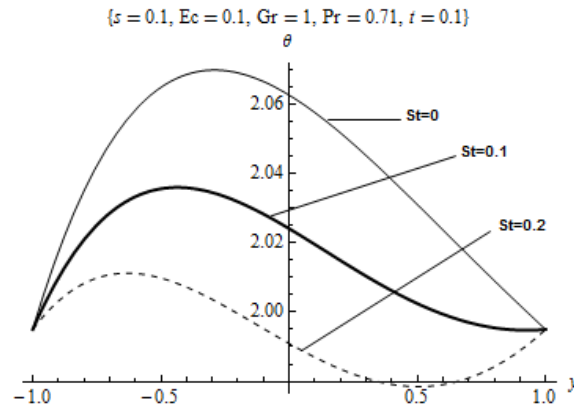


Fig. 5: Temperature profile with different values of Strouhal number



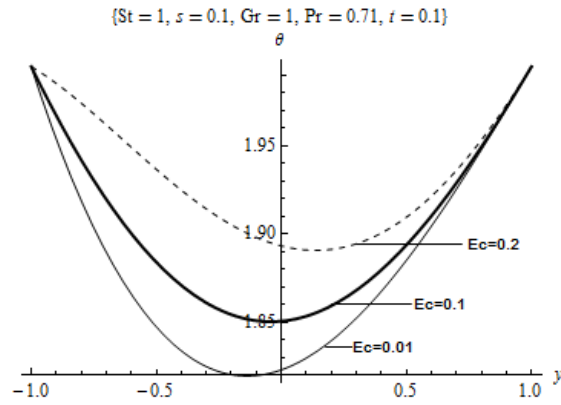


Fig. 6: Temperature profile with different values of Eckert number

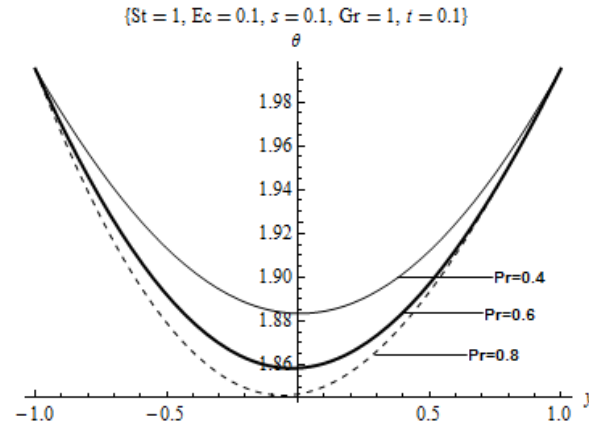


Fig. 7: Temperature profile with different values of Prandtl number

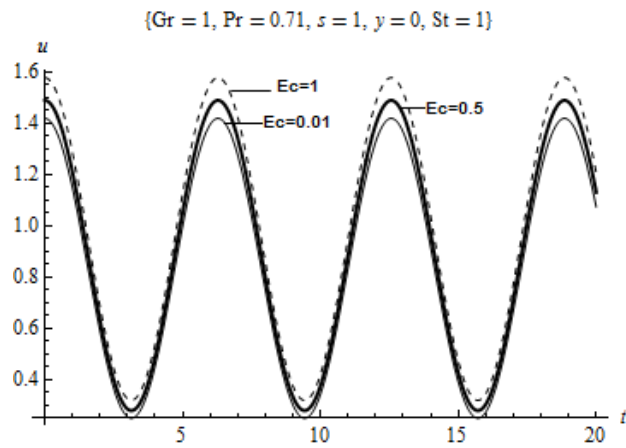


Fig. 8: pulsating velocity profile with different values of Eckert number

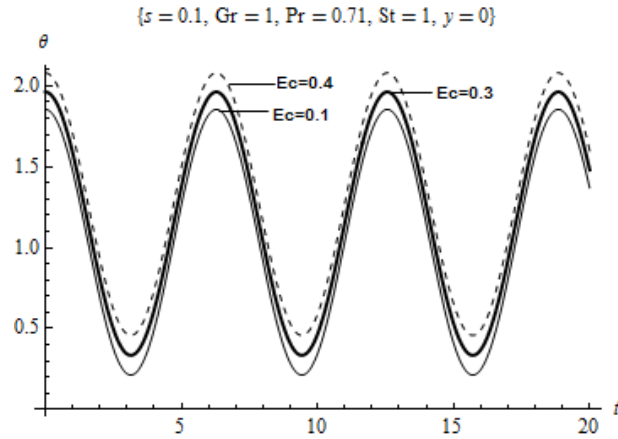


Fig. 9: temperature distribution with different values of Eckert number

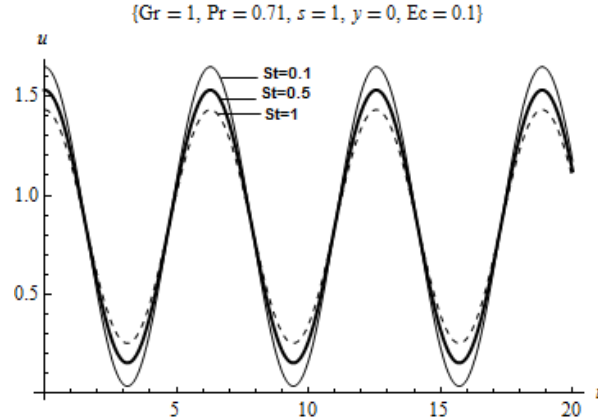


Fig. 10: pulsating velocity profile with different values of Strouhal number

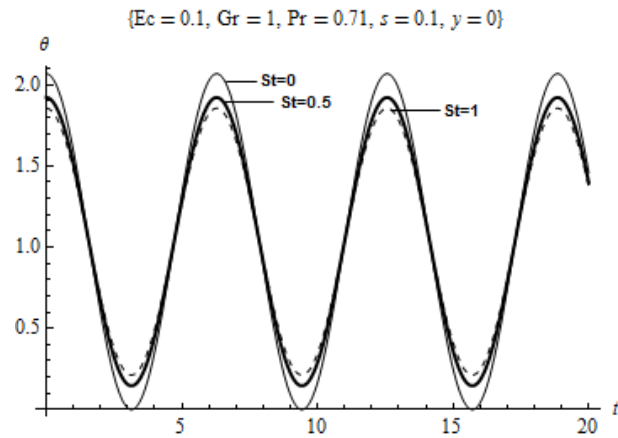


Fig. 11: temperature distribution with different values of Strouhal number

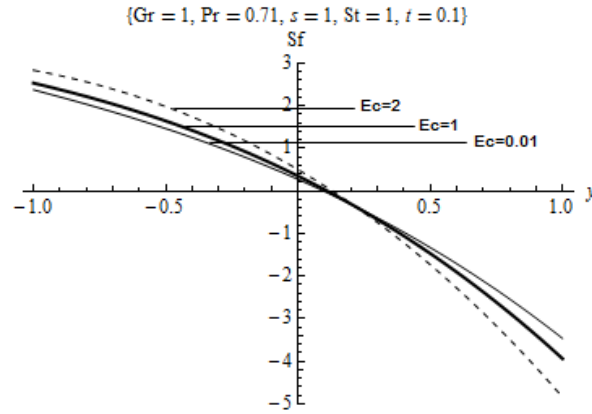


Fig. 12: wall skin friction with different values of Eckert number

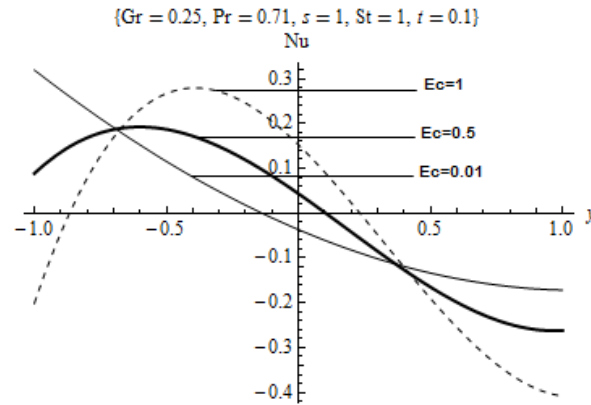


Fig. 13: heat transfer rate with different values of Eckert number

## 5. Conclusion

We have studied the mixed convective flow of pulsatile fluid through a vertical porous channel with viscous dissipation and time periodic boundary conditions. Based on oscillatory flow assumption, the coupled partial differential problems are reduced to couple nonlinear ordinary differential equations. The system of equations are uncoupled by using Adomian decomposition approach. In the present analysis, the contributions to knowledge are as follows: An increase in the Eckert number enhances the fluid temperature and velocity due to additional heat generated as a result of increase in the kinetic energy of the fluid particles. Moreover, as Eckert number increases, rate of heat transfer from the walls to the fluid increases while skin friction increases at the wall with suction on the other hand it got weakened at the injection wall.

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