

# ENERGY LOSS COMPUTATION AND ANALYSIS OF TUBULAR SLOTTED STAND-OFF-LAYER STRUCTURE USING MODAL STAIN ENERGY METHOD

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*Viscoelastic damping structures are widely used in many engineering designs. Their vibration-reducing characteristics are dependent on the viscoelastic material properties to a large extent. In order to acquire the constitutive model parameters of viscoelastic material, according to ISO 37:2005, the static tension experiments were carried out in order to obtain the stress and strain relationship of damping layer material under large deformation and small deformation, respectively. In addition, dynamic mechanical analysis tests were done to acquire the elastic modulus and material loss factor of damping layer material. At last, the damping layer material is applied to a tubular slotted stand-off layer structure(TSSS), the whole structural energy loss abilities are evaluated by the structural loss factor based on the modal strain energy method. Two structures with the axial and circumferential slots are analyzed, respectively. The results could provide the help for the various engineering practice of the damping treatments.*

**Keywords:** damping structure, loss factor, viscoelastic material, modal strain energy method, engineering designs

## 1. Introduction

In many engineering practice, vibration is often harmful. At present, laying a viscoelastic damping layer on the structure is one of the important methods for high-efficiency vibration reduction, which could be seen in Fig.1. From Fig.1, it is known that free damping structures(FLD) and constrained damping structures(CLD) are mainly two types of damping structures based on viscoelastic damping materials. The former uses the direct alternating stress and strain of viscoelastic damping materials to dissipate energy to achieve the effect of vibration and noise reduction; The other uses the shear deformation inside the damping layer to consume structural energy[1]. Based on the research of CLD,

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slotted constrained damping structures(SCLD) and partial constrained damping structures(PCLD) also attract people's attention[2]. Although the above mentioned three-layer constrained damping structures have been relatively mature, their performance still needs to be further improved.

With further research of damping structures, multi-layer structures such as 5-layers could also increase the structural energy dissipation[3-5]. However, five-layer structures are always expensive and their manufacturing process is also complex. Therefore some scholars put forward that the middle elastic layer of five-layer structures may be abolished. Thus, the four-layer damping structures are formed, which consist of two viscoelastic material layer[6-7].

In addition, the viscoelastic materials are important for the various damping structure in order to obtain good vibration isolating performance. Viscoelastic damping materials exhibit different physical states under different external conditions, such as glass, viscoelastic, rubber (hyperelastic) and viscous fluid state. There is no definite boundary between each state, and the constitutive models of materials are generally assumed to be different according to actual needs[8-9]. For example, the statistical and phenomenological models of incompressible rubber-like materials were reviewed by Boyce et al, and the constitutive equations of Linear viscoelastic solids were established by Holzapfel et al based on the four-parameter Kelvin-Maxwell model. Caruthers et al. derived the nonlinear thermo-viscoelastic constitutive equation of polymer by generalized strain measurement[10-12].

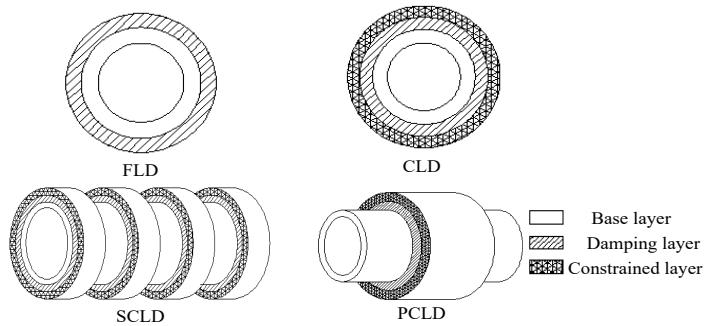


Fig. 1 Several damping structures

In this paper, viscoelastic material properties are obtained through the related test. After that, a kind of tubular slotted stand-off-layer structure(TSSS) is analyzed, which is shown in Fig.2. The TSSS adds the stand-off-layer between the base layer and viscoelastic layer, and the stand-off-layer is segmented along the

circumferential and axial direction, respectively. Based on the loss factor of viscoelastic materials, the structural loss factors are analyzed using modal strain energy method in order to grasp the whole damping performance of structures. The computation results will be very helpful for many engineering vibration-reducing designs.

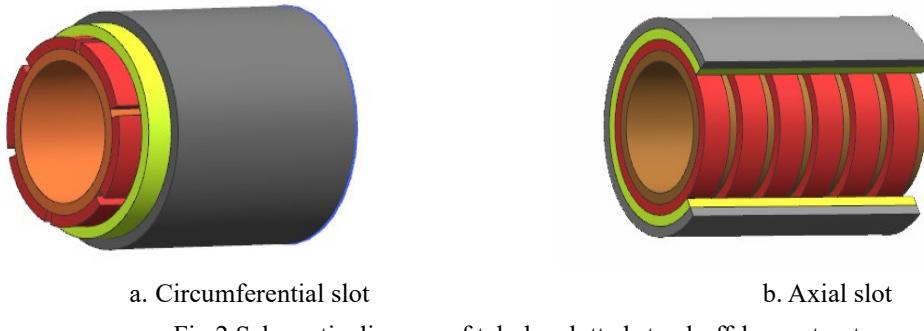


Fig.2 Schematic diagram of tubular slotted stand-off-layer structure

## 2. Experimental determination of viscoelastic material properties

### 2.1 Tension experiments

According to ISO 37:2005, static tension experiments of viscoelastic material used for damping layer were carried out on the M350-10kN-type precise elongation apparatus (see Fig.3a) (Testometric, Britain), for I-dumbbell specimen (see Fig.3b). The test temperature was chosen to be room temperature of 25°C, and the tension specimen of damping layer material needed to deposited at room temperature for at least 1 hour. Here, it should be noted that the test results are obtained when the specimens were subjected to two loading tensile tests.

The test results are given in Fig.4. In Fig.4, Fig.4(a) denotes the stress and strain of damping layer material under large deformation. In the meantime, Fig.4(b) denotes the stress and strain under small deformation. In addition, the two test curves in Fig. 4 show that when the stress is equal to zero, there is still a certain degree of residual deformation. It is found that this phenomenon may be related to two factors, one is that the lower fixture of the sample is loosened to some extent due to the influence of heavy load, the other is that the rubber itself is a macro-molecule material, and the strain lags behind the stress. Through the stress and strain curves, constitutive model parameters of viscoelastic material used for the damping layer could be known.

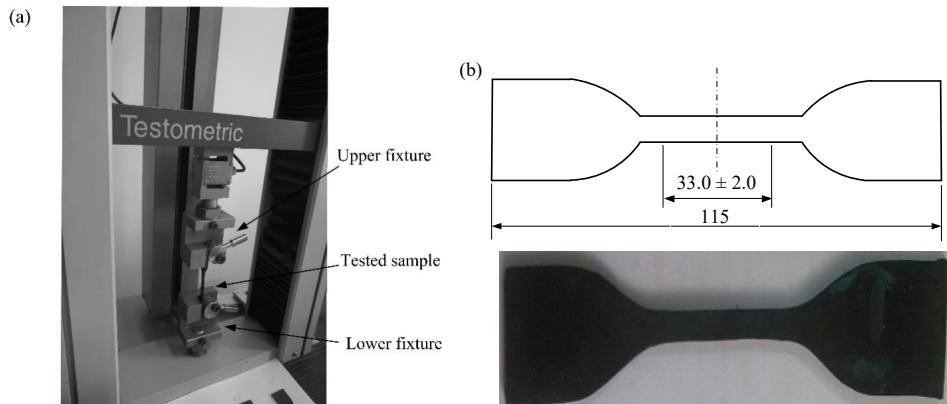


Fig. 3 Viscoelastic material test. (a) Elongation apparatus and (b) tension specimen

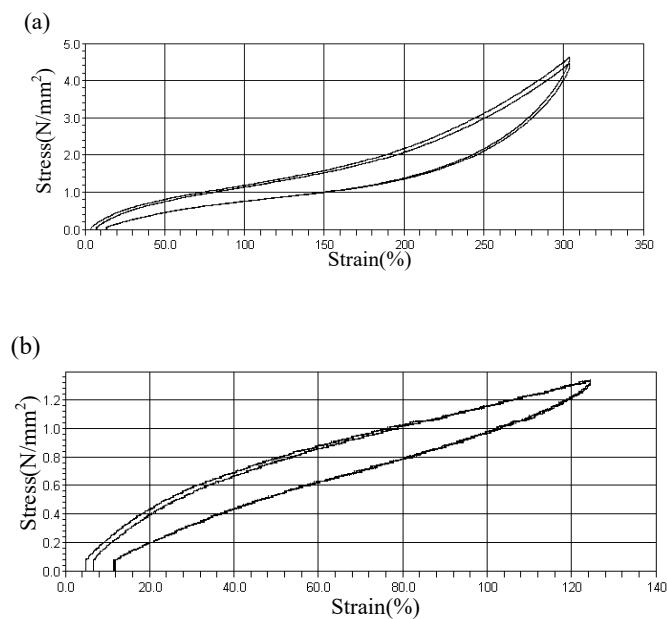


Fig. 4 Stress-strain curves of experiments. (a) large deformation and (b) small deformation

## 2.2 Tests by dynamic mechanical analysis (DMA)

DMA242C is used for measuring and characterizing viscoelastic material performances under dynamic deformation conditions with variable frequency and temperature. Its measure system is shown in Fig.5. The analyzer DMA242C has six measurement mode, including standard, creep, relaxation, stress scan, strain scan and TMA. In the aspect of deformation mode, besides providing standard deformation mode such as three-point bending, single cantilever bending, double cantilever bending, linear shearing, tension and compression, we also offer a

variety of advanced deformation modes, such as single cantilever plus free push rod mode, low guide mode, viscous liquid mode, etc. A three-point bending test mode is used in this paper. The specimen size is 50mm  $\times$  10 mm  $\times$  5 mm (length  $l$ , width  $b$  and height  $h$ ).

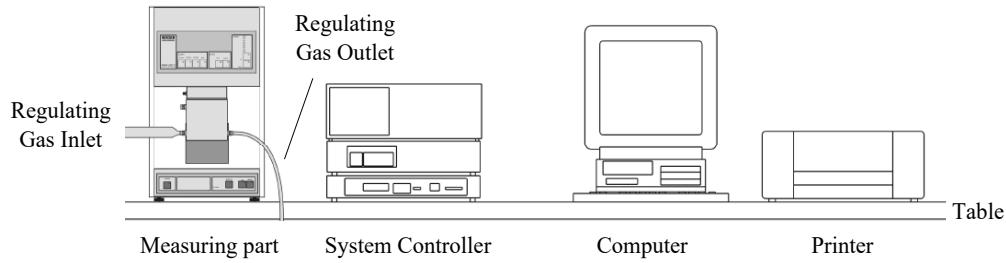


Fig. 5 DMA 242C measure system

The viscoelastic material loss factor of damping layer may be expressed as follows:

$$\beta = \frac{E''}{E'} = \frac{\frac{l}{4bh^3} \cdot \frac{F}{a''}}{\frac{l}{4bh^3} \cdot \frac{F}{a'}} = \frac{a'}{a''} \quad (1)$$

where  $E'$ 、 $E''$  are the real part and the imaginary part of the complex modulus of elasticity  $E^*$  respectively.  $a'$ 、 $a''$  are the real part and the imaginary part of the dynamic deformation  $a^*$  respectively.  $F$  is the load of exerting on the specimen.

In the meantime, the complex tensile Modulus  $E^*$  (unit, Pa) of viscoelastic materials can be obtained by:

$$E^* = \frac{l}{4bh^3} \cdot \frac{F}{a^*} \quad (2)$$

The obtained dynamic parameter curves of material for damping layer are given in Fig.6. Fig.7 gives relationship between real part and imaginary part of complex modulus  $E^*$ . This curve is also frequency dependent.

The above experimental results of viscoelastic material can provide the related parameters support for the following analysis of damping structures.

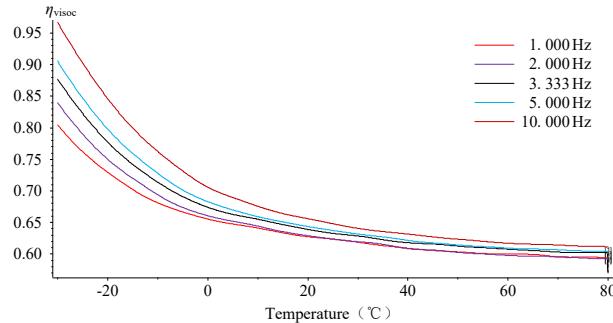


Fig. 6 Material loss factor varying with temperature and frequency

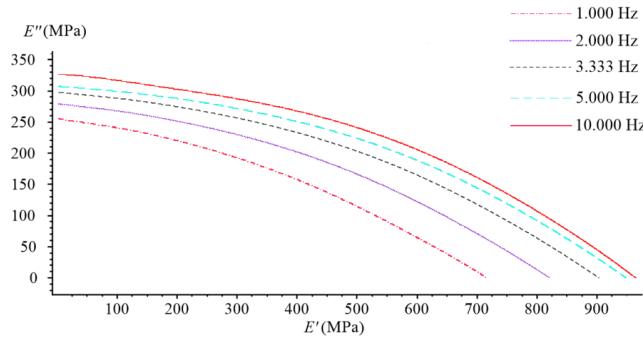


Fig. 7 Relationship between  $E'$  and  $E''$

### 3. Energy loss computation and analysis of TSSS

#### 3.1 Mathematical model of structural loss factor

In order to evaluate the external vibration energy dissipation characteristic of different damping structure, structural loss factor is adopted through modal strain energy method[13-14]. For a random damping structure, its structural loss factor is expressed as

$$\eta = \frac{D_0 + D_{visco}}{2\pi(W_0 + W_{visco})} \quad (3)$$

where  $D_0$  and  $D_{visco}$  represent the energies dissipated by the original structures and the damping treatment structures, respectively.  $W_0$  and  $W_{visco}$  represent the total deformation energy of the original structures and the damping structures, respectively.

Here,  $W_0$  and  $D_0$  of the original structures could be written as

$$W_0 = \frac{1}{2} \{\psi\}_k^T [K] \{\psi\}_k = \frac{1}{2} K_k \quad (4)$$

$$D_0 = \eta_0 \cdot 2\pi \cdot \frac{1}{2} K_k \quad (5)$$

where  $K_k$  represents the  $k$ -order modal stiffness of the original structure.  $\eta_0$  is the loss factor of the original structure.  $\{\psi\}_k$  denotes the eigenvector of real mode and it also denotes the displacement of structure.

At the same time, for the damping structure, its total deformation energy and dissipation energy could be express as follows:

$$W_{\text{visco}} = \frac{1}{2} \{x\}_k^T [K]_{\text{visco}} \{x\}_k \quad (6)$$

$$D_{\text{visco}} = \eta_{\text{visco}} \cdot 2\pi \cdot W_{\text{visco}} = \eta_{\text{visco}} \cdot \pi \{x\}_k^T [K]_{\text{visco}} \{x\}_k \quad (7)$$

where  $[K]_{\text{visco}}$  is the stiffness of the damping structure, and  $\eta_{\text{visco}}$  is material loss factor of damping layer,  $\{x\}_k$  denotes the displacement of damping treatment structure.

Finally, the  $k$ -order structural loss factor of the damping structure could be written as

$$\eta_k = \frac{\eta_0 K_k + \eta_{\text{visco}} \{x\}_k^T [K]_{\text{visco}} \{x\}_k}{K_k + \{x\}_k^T [K]_{\text{visco}} \{x\}_k} \quad (8)$$

### 3.2 Structural loss factor analysis of TSSS

The tubular slotted stand-off-layer structure(TSSS) has been given in Fig.2. Its radius is 60mm and its length is 460mm. The other geometric parameters and material properties of TSSS are given in Table1. In addition, the two ends of the base layer are fixed, the number of circumferential segments  $m = \{10,14,18,22,26,30,34,38\}$ , circumferential uniform interval  $\theta = \{3^\circ, 4^\circ, 5^\circ, 6^\circ, 7^\circ, 8^\circ\}$ ; number of axial slots  $n = \{15,20,25,30,35,40,45,50\}$ ; axial clearance  $l$  is located between 0 and 25mm.

Table.1

Layer thickness and material parameters of TSSS

Name	Thickness/m	Modulus of Elasticity/Pa	Poisson Ratio	Density (kg/m <sup>3</sup> )
Base-layer	0.005	$2.1 \times 10^{11}$	0.3	7800
Stand-off-layer	0.002	$2.9 \times 10^8$	0.35	1000

Damping-layer	0.005	-	0.49	1100
Constrained-layer	0.004	$2.1 \times 10^{11}$	0.3	7800

Fig.8 gives the first two-order structural loss factors varying with the number of slot parameters. Fig. 8a shows the curve of the first-order modal loss factor of TSSS with the circumferential slots. It could be known that when the circumferential slots is equal to 36 and circumferential uniform interval  $\theta$  is  $6^\circ$ , the modal loss factor is the biggest and it is 0.473 at this time. In addition, when the circumferential slots is equal to 18 and circumferential uniform interval  $\theta$  is  $7^\circ$ , the minimum loss factor of the structure is 0.443. Therefore, the modal loss factors at this situation are greatly influenced by the number of segments. Similarly, Fig. 8b is a second-order bending mode for TSSS with the circumferential slots. Its change trend is similar to that of Fig. 8a, but the change range is large. The maximum modal loss factor is 0.479, and the minimum value is 0.447.

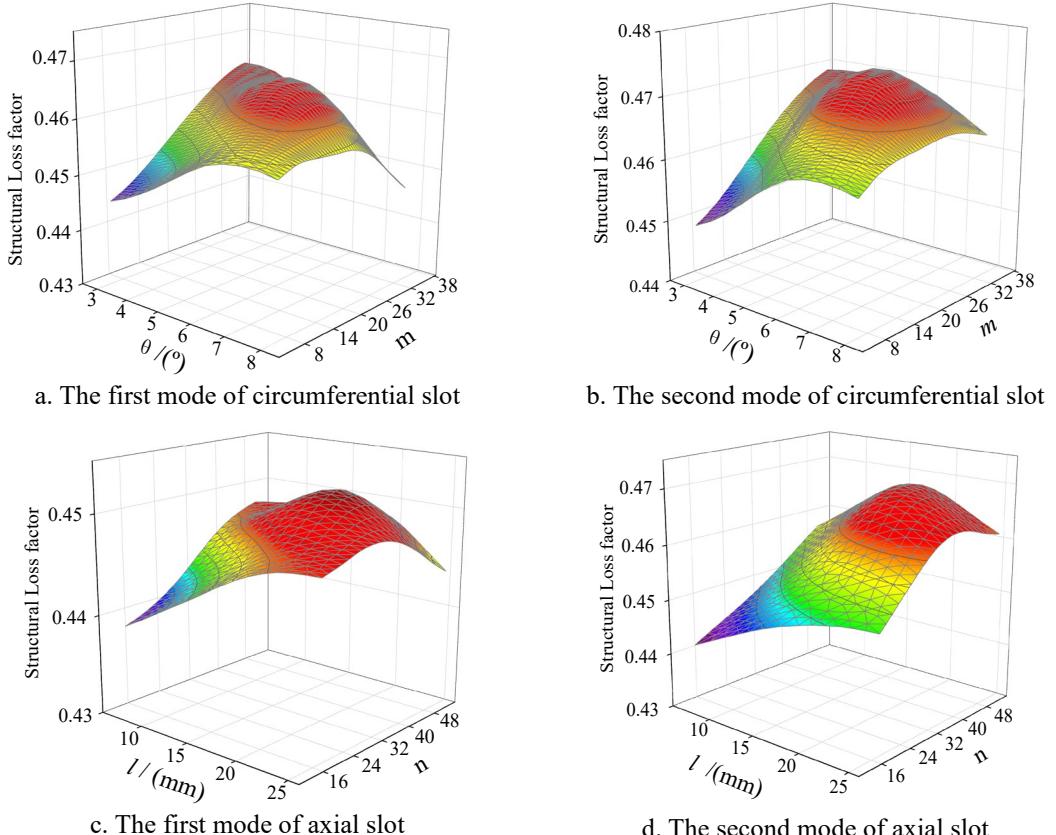


Fig.8 The first two-order loss factor varying with the number of slot parameter

Fig. 8c is the first-order bending mode for TSSS with slots in axial direction. The maximum value is 0.453, the change of structure loss factor is gentle, and the minimum value is 0.438. Fig. 8d shows the variation law of the structural loss factor of the second-order structure with axial slots, in which the maximum value of 0.468 appears at the position where the axial slot gap is  $l = 16$  mm and the number of segments is  $n = 48$ .

Through the above comparison and analysis, the axial and circumferential slots parameters have a great influence on the damping characteristic of the original structure. Both the second-order structural loss factor have a larger value. The damping effect of TSSS with the circumferential slots is better than that of the axial slots.

#### 4. Conclusions

In this paper, viscoelastic material is applied to tubular slotted stand-off-layer structure. Using modal strain energy modal, the structure loss factors are calculated. Through the above researches, the main results are given as follows:

1. The constitutive model parameters of viscoelastic material are acquired through the static tension experiments of viscoelastic material under large deformation and small deformation, respectively.
2. DMA242C is used for measuring and characterizing viscoelastic material performances under dynamic deformation conditions with variable frequency and temperature. Through tests, the relationship between real part and imaginary part of complex modulus  $E^*$  are obtained. More importantly, the material loss factor of damping layer is acquired.
3. When the number of slots in circumferential direction is 36 and the interval is 6 degrees, the structural loss factor of the first-order bending mode reaches the maximum, but more slots and the interval of slots will reduce the loss factor. The damping effect of TSSS with the circumferential slots is better than that of the axial slots.

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