

NYQUIST STABILITY FOR HYSTERESIS SWITCHING MODE CONTROLLERS

Dan OLARU¹

The stability analysis of the non-linear circuits is a challenging problem. Recently, new results were obtained in linear modeling using a discrete-time correspondence method. The presented paper extends this research for Nyquist stability analysis of a feedback controlled hysteresis switching buck circuit. Moreover, quantitative information was obtained about the control robustness, using a discrete differential evaluation of the Nyquist diagram characteristics. Also, numerical simulations are achieved, in order to validate the methods and analyze the circuit behavior.

Keywords: switching circuit, hysteresis, control loop, Nyquist stability

1. Introduction

In power control domain, the switching systems need reactive filters in order to obtain sufficient smooth output voltage. Thus, in a buck configuration with feedback, control loop stability problems arise. Classical stability analysis can't be applied because the non-linear switching characteristics. A recent research proves that the hysteresis operation allow a linear discrete-time approach using Z transform. This is done by an equivalence method described in reference [1]. Using a linear model for the driving comparator circuit, the authors succeed to obtain an analytical mathematical expression for the open loop gain of the control circuit. These important results enable to achieve a quantitative analysis of the circuit behavior, allowing useful comparisons for hysteresis, PWM, and phase delay circuit configurations.

This paper continues the study and is focused on the Nyquist stability analysis for the hysteresis configuration (Fig.1). Because the complex profile of the resulting Nyquist diagram, special differential methods are used. Mathcad numerical representation and simulation are used in order to confirm the methods validity and circuit operation.

¹ Lecturer, Dept. Mes. El. Ap.& Stat. Conv., University POLITEHNICA of Bucharest, Romania,
e-mail: dan.olaru@upb.ro

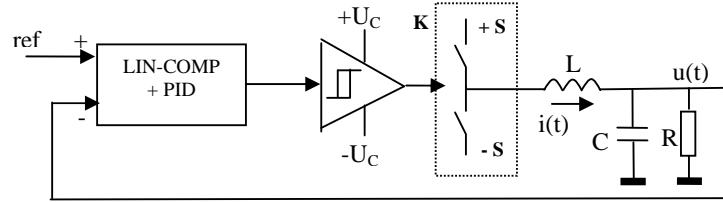


Fig.1. Principial circuit diagram

2. The Equivalence method

In the following the buck configuration of Fig.1 is considered.

From paper [1] two very important results are derived. First, the comparator circuit is modeled by a linear one. The hysteresis transition process is modeled by discrete-time representation using the Z transform. Second, an analytical expression is derived for circuit open-loop gain. A continuous-time domain expression is obtained by a mixed representation in s-Laplace domain and z-variable domain.

This equivalence principle is based on the following reasons. The transitions of the non-linear comparator arise when the input error signal exceeds the hysteresis thresholds. For a stationary operation, the feed-back controlled circuit acts as an auto-oscillate one. Due to the low-pass filter the output voltage swing near the constant reference value. At the filter input we have a discrete variation. The hysteresis levels control the duty cycle of the comparator output. This determines an equivalent (non-linear) gain of the comparator, taking into account the integration process achieved by the low-pass filter.

The main contribution of paper [1] is the fact than the Z-transform representation of the comparator model gives the same behavior, if we assume that the sampling frequency correspond with the nonlinear auto-oscillate frequency and the gain of the linear model equals the non-linear one. Thus, the sampling controlled operation in discrete-time domain models the non-linear transitions controlled by hysteresis in voltage value domain.

In reference [1] the following two important expressions are obtained:

a) The z-domain transfer function of the comparator linear model:

$$CTF(z) = \frac{K_z}{1 + K_z \cdot H_z(z)} \quad (1)$$

where: CTF(z) - the z-domain comparator model transfer function;

K_z - comparator equivalent gain;

$H_z(z)$ - low-pass filter transfer function (z-domain).

b) The (s,z)-domain mixed representation input-output (reference to output) transfer function:

$$Gro(s) = \exp(-s \cdot T_d) \cdot H_s(s) \cdot CTF(z) \quad (2)$$

where: T_d - the circuit loop delay;

$H_s(s)$ - low-pass filter transfer function (s-domain).

The mixed representation with both s and z variables is non-contradictory. If z is substituted by $\exp(st)$ this correspond to a physical experiment where for every frequency component applied to the input, only this component is measured at output (by a selective analyzer, for example).

3. Analysis method

In our case, the Z-transform of the LC filter is necessary. After some symbolical calculations, we have:

$$H_z(z) = \frac{1}{b \cdot LC} \cdot \frac{z \cdot \exp(-aT) \cdot \sin(bT)}{z^2 - 2\exp(-aT)\cos(bT) + \exp(-2aT)} \cdot T \quad (3)$$

where: T – the sampling period;

$$a = \frac{RC}{2LC} \quad \text{and} \quad b = \sqrt{\frac{1}{LC} - \left(\frac{RC}{2LC}\right)^2} \quad (4)$$

Nyquist diagram and criterion are valuable methods that may determine if a linear dynamical system is stable and, moreover, a so called "reserve of stability" may be determined, when robustness requests are implied.

The results from paper [1] enable to extend these methods to above described circuit, which is highly nonlinear. In this case, the instability situation is dependent on the phase delay, due to the LC low-pass filter.

But for the non-linear system model, the open loop transfer function has an infinity of poles that lead to complicated profiles of the Nyquist diagram. Thus, sometimes is difficult to observe the stable behavior and to appreciate the robustness of the system. For this purpose, here, a differential procedure is conceived.

Nyquist criterion take under consideration the encirclement of the (-1,0) coordinate point by the trajectory. In this situation, the vector radius with the

origin in (-1,0) will describe a 2π angle. Here, an incremental numerical procedure is used, in order to measure the vector radius angle variation. For short trajectory segments, we can define the movement tangent vector τ_k and radius vector R_k with coordinates (X_{k-1}, Y_{k-1}) and (X_k, Y_k) respectively. The angle increments (in radians) U_k may be approximated by the ratio of modules:

$$U_k = \frac{|\tau_k|}{|R_k|} \quad (5)$$

For each point, the sense of angle variation may be defined as the sign of vectorial product $\tau \times R$, considering a 3-dimensional coordinates extension. If an encirclement of the (-1,0) point appears, the angle variation must cumulate at least 2π . Thus, in a recursive representation: $\Sigma U_{k+1} = \Sigma U_k + \text{sign}(\tau_k \times R_k) \cdot U_k$.

The stability criterion taken under consideration becomes: $\Sigma U_N < 2\pi$. Thus the robustness evaluation may use as indicator the value of difference $\Delta = \Sigma U_N - 2\pi$.

Due to the non-linear behavior, the stability reserve hasn't a smooth dependence on the circuit phase delay, which in this case has a periodic variation. Instead, the robustness of the circuit may be evaluated with the Δ value corresponding to the Nyquist diagram profile.

Also, it is important to remember that a determinant factor for the whole behavior is the sampling period T . This is a specific parameter only for the discrete model approach. For the original non-linear circuit, which normally uses a PID regulator for stabilization, the correspondence must be done with the ripple auto-oscillating frequency.

4. Simulation results

For numerical simulation purposes, was considered a hysteresis “buck” circuit followed by a reactive filter with the following parameters: $R=1\Omega$, $L=1mH$; $C=1\mu F$. The filter is in fact a LC one. A low value of resistance is considered for a good mathematical conditioning and a realistic approach. The Fig.2 and Fig.3 show, respectively, the profiles of $H_z(z)$ and $\text{Gro}(s)$ modulus.

The Nyquist diagram is plotted for three representative cases in Fig. 4,6,8 respectively. The first is for a very stable case and the last for an unstable one. The middle case is poor stable situation. For each situation, at the left, the frequency dependence of the cumulative angle ΣU_k of the vectorial radius is plotted in Fig. 5,7,9 respectively. As shown, instability arises when 2π value is exceeded.

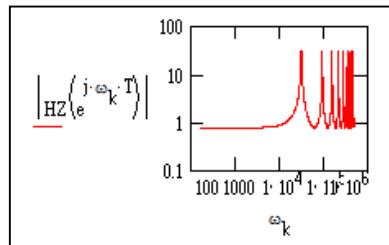


Fig.2. Filter Z transfer function

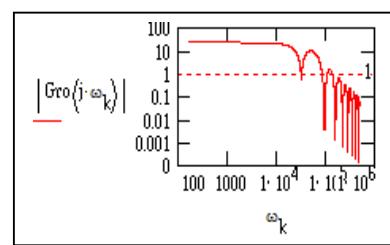


Fig.3. Filter Z transfer function

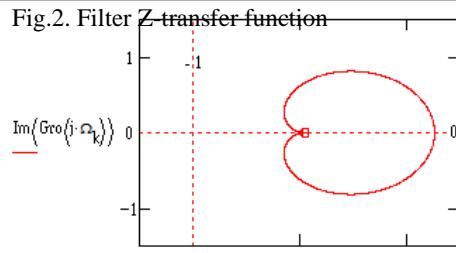


Fig.4. Stable Nyquist diagram

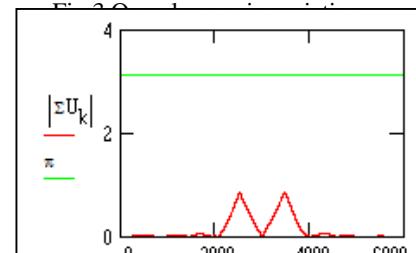
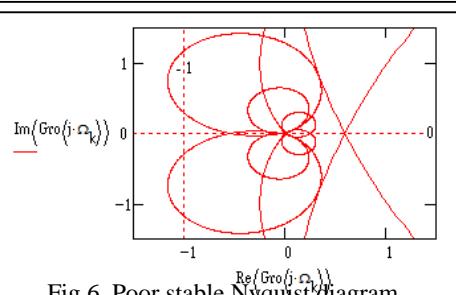
Fig.5. Stable cumulative ΣU angle

Fig.6. Poor stable Nyquist diagram

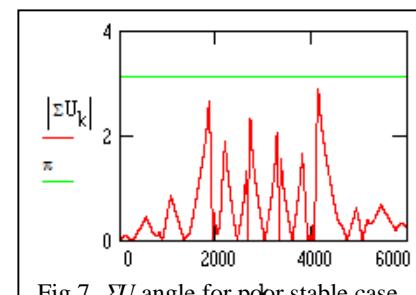
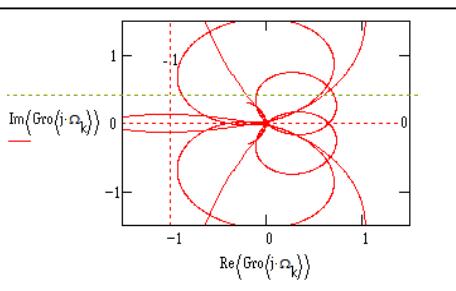
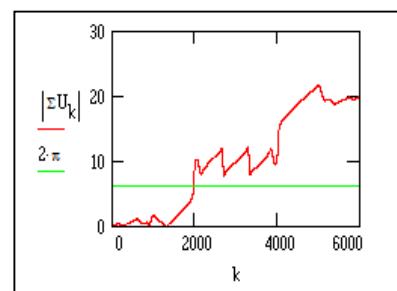
Fig.7. ΣU angle for poor stable case

Fig.8. Instable Nyquist diagram

Fig.9. Instable cumulative ΣU angle

5. Conclusions

The presented method enables the stability analysis of a non-linear switching circuit with hysteresis operation mode. Using the Z-transform linear model from [1], a Nyquist diagram representation was derived.

The main contribution of the paper is a new geometrical and algebraical method for robustness evaluation of a switching mode circuit. This is based on a discrete differential method and algebraic calculation giving a stability margin evaluation for non-linear circuits. Thus, Nyquist criterion may be applied even for complex trajectory profiles where visual observations on diagram give poor results. Moreover, using the cumulative angle representation as in Fig. 5,7,9 a potential instability situation may be presumed.

The numerical simulations are done for only few representative cases, but these procedures are useful in many different situations.

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