

## MINIMIZATION OF THE WORKPIECE POSITIONING ERRORS ON THE CNC MILLING MACHINE

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*In the precision production, certain conditions must be observed. One of these conditions is minimized the defects of workpiece positioning.*

*This paper presents a new method for minimizing defects in the workpiece positioning for machining on the CNC milling machine. This method is based on the combination between the small displacements torsors and the EULER angles. For modeling, an experimental study was carried out to quantify the deviations. At the end, we presented a help module to choose the ideal position. An example was presented to validate these results.*

**Key words:** positioning errors, modeling, Euler angles.

### 1. Introduction

During the production of a mechanical part, it is impossible to avoid manufacturing errors. That is why each functional rate has an interval of tolerance, which depends on the role of this part in the mechanism. It must be respected. When this interval is smaller, making a product with the required accuracy become more difficult. In order to obtain a series of identical parts, it is impossible to make them with the same dimensions. Sometimes, it is very difficult to make a series of the parts without rejecting a number of the parts.

There are several sources of errors, related to the machine, to the raw part, to the tool used, to the external environment etc. Among these sources, there are the workpiece positioning errors (WPE).

Putting the part in position is an important operation in order to have the desired accuracy; the goal is position in the best possible way a workpiece in order to perform the series of actions planned in the phase. The workpiece position must be isostatic to minimize disparities. The more isostatic it is, the more accurate it will be. If the piece is placed hyper statically, there will be much more chance of disparity, and therefore, defects between each piece in the series will be found.

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In the context of manufacturing tolerance problems. A computational model of tolerance unidirectional based on machining dispersions has developed [1], in addition, mathematical model using the concept of small displacement Torsor (SDT) has been given [2]. Moreover, mathematical approach has presented in order to optimize the effect of the design variables on the manufacturing tolerances [3].

In the aim of minimize the workpiece location errors, mathematical model has been developed considering the local elastic deformations [4]. In addition, a formulation of the relationship between the local point displacements and the workpiece displacements, taking a count both locator position error and locator deformation has been developed [5]. Moreover, a general formulation has been proposed to give the relationship between the WPE and its source errors [6].

In order to integrate the design aided manufacturing in the manufacturing tolerance problems, the approaches of three-dimensional (3D) machining simulation were presented [7]. By combining the Taguchi method and the Monte-Carlo statistical method, a study on fixture locating layout was carried out with the robust design approach, in order to increase the quality of final machining workpiece [8]. A modeling and analyzing of the effect of the clamping force, and the geometric errors of the workpiece and the locator, on the tolerance manufacturing, using the concept of SDT [9]. In the aim of optimizing the manufacturing tolerances, a three dimensions mathematical model based on the concept of SDT had been carried out [10], and an analytic model of 3-2-1 fixturing system had been presented basing on the SDT hypothesis and the Langrangien formulation [11]. In addition, a modelling and the compensation of the effect of the WPE on the machining surface in numerically controlled (CNC) machine milling were presented [12].

In the context of modeling and analyzing the WPE. An analysis approach with general fixture layouts and parameterized tolerances had been given [13]. A study of the influence of systematic dispersion on manufacturing tolerances was presented [14]. Moreover, a mathematical model to calculate the distribution of the Registration fulcrums was developed [15]. In addition. An experimental study and a modeling of machining errors on the NC machine tool had been carried out [16]. In other works, a mathematical model was developed and validated with a computer aided design and an experimental study in order to investigate the robustness of the locating layout [17].

## **2. WPE modelling**

The movement of the workpiece was discretized in pure translation and pure rotation by creating three coordinate systems. The first is a fixed coordinate system (CS0), related to the workpiece theoretical position. The second is a mobile coordinate system (CS1), related to the workpiece actual position. In addition, the

third is an intermedium coordinate system (CS2), related to the workpiece intermedium position (Fig. 1). The pure translation is between the CS0 and the CS2, and the pure rotation is between the CS2 and the CS1. The Euler angles were used in the rotation movement.

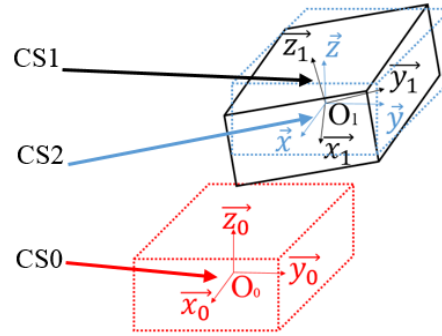


Fig. 1. Coordinate systems

There are several Euler angles chains and for each chain the names (or the order) of the angles vary according to the authors. In our case, the Yaw (rotation of  $\alpha$  about Z-axis), pitch (rotation of  $\beta$  about the Y-axis), and roll (rotation of  $\gamma$  about the X-axis) rotations were used (Fig. 2).

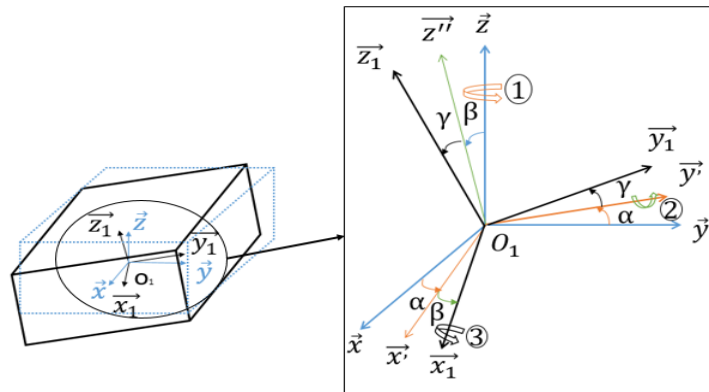


Fig. 2. Euler angles.

The rotation matrix is given by:

$$R_z(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_y(\beta) = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix}, R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix}$$

The yaw, pitch, and roll rotations can be used to place a 3D body in any orientation; a single rotation matrix can be formed by multiplying the yaw, pitch, and roll rotation matrices to obtain:

$$R(\alpha, \beta, \gamma) = R_z(\alpha)R_y(\beta)R_x(\gamma) \quad (1)$$

The  $\overrightarrow{P_0P_1}|_0$  vector coordinates represent the WPE, either  $P_1$  a point belongs to the piece in its current position, and  $P_0$  is the same point but belongs to the piece in its theoretical position, i.e. if  $(X_0, Y_0, Z_0)^T$  is the position vector of  $P_0$  in the CS0 and  $(X_1, Y_1, Z_1)^T$  is the position vector of  $P_1$  in the CS1.

The calculation of the  $\overrightarrow{P_0P_1}|_0$  coordinates is done by the projection of all the points on the CS0 (Fig. 3).

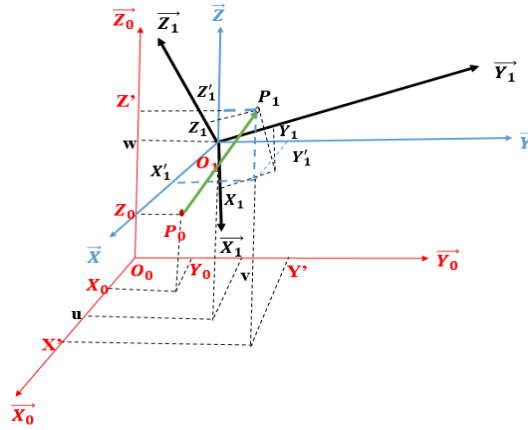


Fig. 3. WPE vector coordinates illustration.

According to the Fig. 3, we have the equations (2) and (3).

$$(X_0, Y_0, Z_0)^T = (X_1, Y_1, Z_1)^T \quad (2)$$

$$\overrightarrow{P_0P_1}|_0 = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} X' - X_0 \\ Y' - Y_0 \\ Z' - Z_0 \end{pmatrix} = \begin{pmatrix} X' - u \\ Y' - v \\ Z' - w \end{pmatrix} + \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \quad (3)$$

According to (2), (3), we have equation (4)

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{pmatrix} \cdot \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \quad (4)$$

In order to know which the optimum isostatic position it is, the average value of the  $u$ ,  $v$ ,  $w$ ,  $a_x$ ,  $b_x$ ,  $c_x$ ,  $a_y$ ,  $b_y$ ,  $c_y$ ,  $a_z$ ,  $b_z$  and  $c_z$  were calculated using the equation (4), and the measures taken from the series of test illustrating in the next section, after that, a comparison between the results was done.

### 3. Experimental WPE Study and Optimization

A series of 100 tests were conducted on prismatic standard holds of the same sizes, (L=100 mm, H=35 mm, P=9 mm). Varying the distances between the supports for each isostatic position as shows the Table 1, and measuring the errors of the in position on the points as show the Fig. 4.

Based on these tests, we measured the value of  $E_z$  for the points 1, 2 and 3, the value of  $E_y$  for the points 4 and 5 and the value of  $E_x$  for the point 6.

First notice that  $a_x$ ,  $b_x$ ,  $c_x$ ,  $a_y$ ,  $b_y$ ,  $c_y$ ,  $a_z$ ,  $b_z$  and  $c_z$  depend on  $\alpha$ ,  $\beta$  and  $\gamma$ , so we have six unknowns, which are  $u$ ,  $v$ ,  $w$ ,  $\alpha$ ,  $\beta$  and  $\gamma$ . Therefore, we need to solve a system of equations with six unknowns, to obtain this system we replaced on the equation (4) the coordinates of the points 1, 2 and 3 with their  $E_z$ , the coordinates of the Points 4 and 5 with their  $E_y$  and the and the coordinates of the point 6 with his  $E_x$ .

Each position gave a value of the average but we chose the optimal position, which give the minimum of the average, then these averages are used to obtain the final formula of the WPE.

Table 1

Informations of the isostatic positions.

position	x	y
1	L/3	H/4
2	L/4	H/4
3	L/4	3H/4
4	L/8	H/4
5	L/6	H/4
6	L/6	H/6

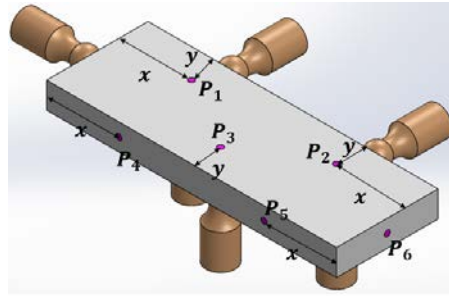
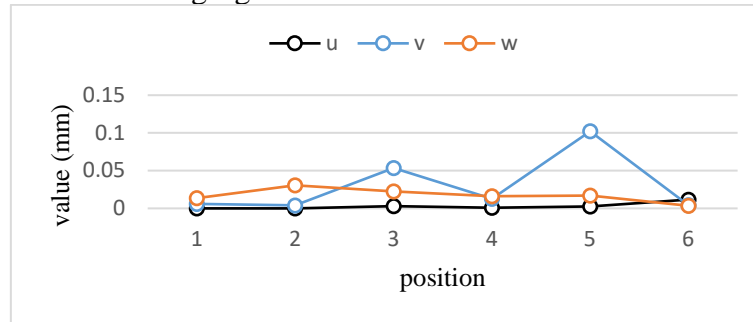
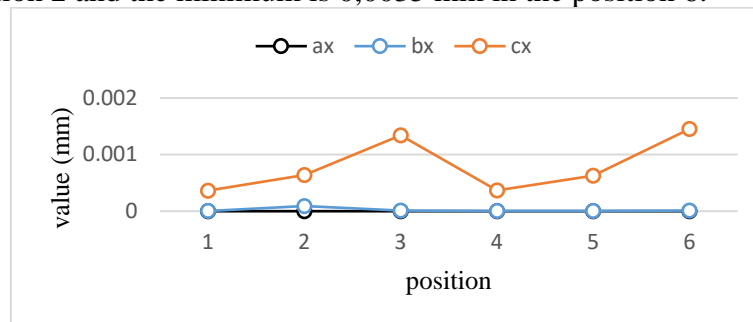


Fig. 4. Supports distribution

Using the results of the measures of the experimental study, the average value of  $u$ ,  $v$ ,  $w$ ,  $a_x$ ,  $b_x$ ,  $c_x$ ,  $a_y$ ,  $b_y$ ,  $c_y$ ,  $a_z$ ,  $b_z$  and  $c_z$  was calculated and illustrated on the following figures:

Fig. 5. The variation of the value of  $u$ ,  $v$  and  $w$  according to the position.

The graph in the Fig. 5 shows the evolution of the average value of  $u$ ,  $v$  and  $w$ , the maximum value of  $u$  is 0,0116 mm in the position 6 and the minimum is 0 in the position 4. The maximum value of  $v$  is 0,1021 mm in the position 5 and the minimum is 0.004 mm in the position 2. The maximum value of  $w$  is 0,0305 mm in the position 2 and the minimum is 0,0035 mm in the position 6.

Fig. 6. The variation of the value of  $a_x$ ,  $b_x$  and  $c_x$  according to the position.

According the graph of the Fig. 6, the average value of  $a_x$  is 0 on all the position. The minimums of  $b_x$  and  $c_x$  are 3,01E-06 mm and 0,0003 mm respectively in the position 4. And the maximums are 9,09E-05 mm and 0,0014 mm respectively, in the position 2 and 3 respectively.

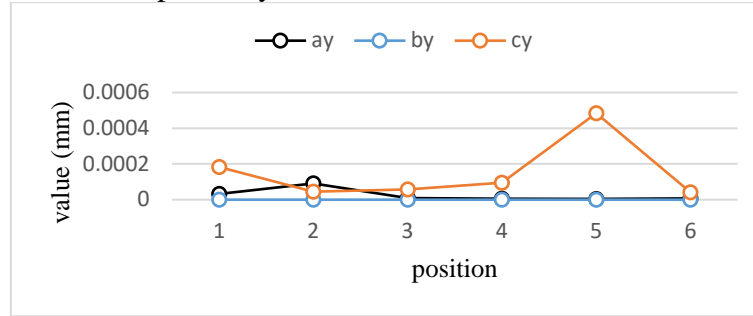


Fig. 7. The variation of the value of  $a_y$ ,  $b_y$  and  $c_y$  according to the position.

The graph of the Fig. 7 represent the variation of the average value of  $a_y$ ,  $b_y$  and  $c_y$ , that the maximum of  $a_y$  and  $c_y$  is 0.0001 mm and 0,0005 mm respectively, and the minimum is 0 mm and 4,1E-05 mm respectively. The maximums was found on the positions 2 and 5 respectively, moreover the minimums on the positions 4 and 6 respectively. In addition, the average of  $b_y$  is 0 mm in all the positions.

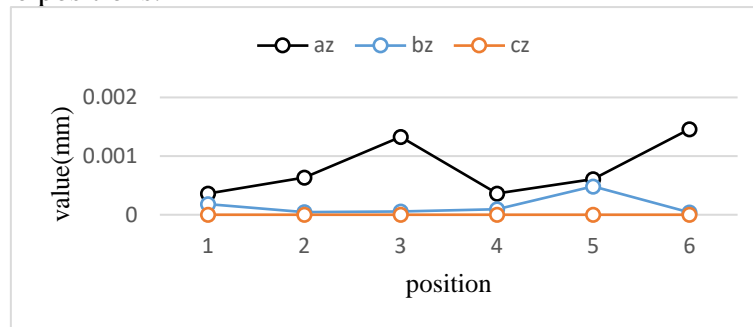


Fig. 8. The variation of the value of  $a_z$ ,  $b_z$  and  $c_z$  according to the position.

the Fig. 8 illustrates the evolution of the average of  $a_z$ ,  $b_z$  and  $c_z$ , the average of  $c_z$  is 0 mm for all the positions, and the maximum and the minimum of  $a_z$  which are 0,0003 mm and 0,0014 mm respectively, and are given by the positions 6 and 4 respectively. Moreover those for  $b_z$  are 4,02 E-05 mm and 0,0005 mm respectively, and are given by the positions 5 and 2 respectively.

According to the graphs of the Fig.5, 6, 7 and 8, the optimum position is the position 4 which is relative to  $x = L/8$ , and  $y = H/4$ . Therefore, according to this result, the optimum formulates of the WPE are:

$$Ex = 0.0009 + (3.01E - 6).Y_0 + 0.0003.Z_0 \quad (5)$$

$$Ey = 0.0125 + (5.33E - 06).X_0 + (9.43E - 05).Z_0 \quad (6)$$

$$Ez = 0.0162 + 0.0003.X_0 + (9.43E - 05).Y_0 \quad (7)$$

#### 4. The WPE(x) formulation

In this section, the equation 5,6 and 7 was more developed, fixing the value of  $y$  and varying the value of  $x$  in order to obtain a general formulation allow to calculate the WPE in the case of  $y = H/4$  and for any value of  $x$ . To develop this formulate, we used the experimental study results, and the interpolation. We obtained the equations below:

$$u(x) = (-8.64E - 06)x^3 + 0.0006x^2 - 0.0135x + 0.095 \quad (8)$$

$$v(x) = (-2.42E - 05)x^3 + 0.0016x^2 - 0.0335x + 0.256 \quad (9)$$

$$w(x) = (-1.37E - 06)x^3 + (8.73E - 05)x^2 - 0.001x + 0.0162 \quad (10)$$

$$b_x(x) = (1.61E - 05)x^2 - 0.0003x + 0.0019 \quad (11)$$

$$c_x(x) = -(8.68E - 05)x^2 + 0.0002x - 0.0009 \quad (12)$$

$$a_y(x) = (1.60E - 05)x^2 - 0.0019x + 0.0019 \quad (13)$$

$$c_y(x) = -(1.3E - 05)x^2 + 0.0003x - 0.0005 \quad (14)$$

$$a_z(x) = -(5.75E - 06)x^2 + 0.0001x - 0.0004 \quad (15)$$

$$b_z(x) = -(1.3E - 05)x^2 + 0.0003x - 0.0018 \quad (16)$$

Finally, using the equation 8 to 16, the general formulate of the WPE is:

$$Ex = (-8.64E - 06)x^3 + (0.0006 - (1.61E - 05).Y_0 - (8.68E - 06).Z_0)x^2 + (-0.0135 - 0.0003.Y_0 - 0.0002.Z_0).x + (0.095 + 0.0019.Y_0 - 0.0009.Z_0) \quad (17)$$

$$Ey = (-2.42E - 05)x^3 + (0.0016 - (1.60E - 05).X_0 - (1.30E - 05).Z_0)x^2 + (-0.0335 - 0.0019.X_0 - 0.0005.Z_0).x + (0.256 + 0.0019.X_0 - 0.0005.Z_0) \quad (18)$$

$$Ez = (-1.37E - 06)x^3 + ((8.73E - 05) - (5.57E - 06).X_0 - (1.3E - 05).Y_0)x^2 + (-0.001 + 0.0001.X_0 + 0.0003.Y_0).x + (0.0162 - 0.0004.X_0 - 0.0018.Y_0) \quad (19)$$

## 5. Module for the WPE calculation

Under the base of equations 5,6,7,17,18 and 19, a tool has been developed to automate the calculation of Mt and the choice of the optimal workpiece positioning. An example was used (Fig.9) to validate the results.

These results have been grouped in Fig.10.

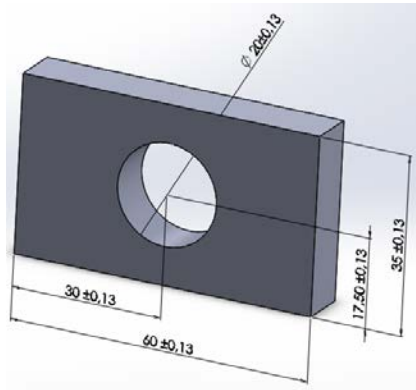


Fig.9. Example for validation

Please enter the value of each rate and click on Calculate:

L= 60 H= 30 P= 10 X= 30 Y= 17,5

Manufacturing ratings:

Mrx= 30 Mry= 17,5 Mrz= 10

the Workpiece positioning errors:

Ex= 0,0039326 Ey= 0,0136029 Ez= 0,02658025

the normals distribution:

Plan supports:			Linear supports:		Single support:	
x1= 7,5	x2= 32,5	x3= 30	x4= 7,5	x5= 32,5	x6= 0	
y1= 26,25	y2= 26,25	y3= 8,75	y4= 30	y5= 30	y6= 17,5	
z1= 0	z2= 0	z3= 0	z4= 5	z5= 5	z6= 5	

Exit Clear Calculate

Fig.10. Results for the example

## 6. Case study

On machined prismatic part (L= 100 mm, H= 50 mm, P= 20 mm), a hole and a groove was created as illustrated on the Fig.11, the objective is to calculate the WPE( $\Delta E$ ) for each manufacturing rate (Mr), the HMr and GMr represent the Mr for the hole and groove respectively, and the supports distribution.

The results were grouped on the table 2.

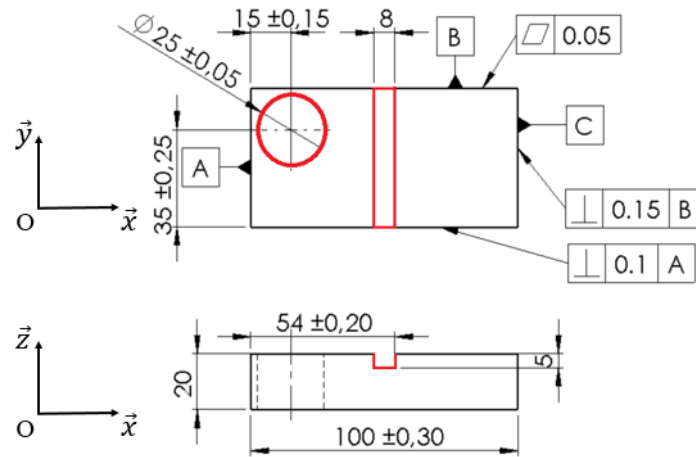


Fig.11. example for the case study

Table 2

The case study results.

The Mr and the WPE (mm)			Supports distribution (mm)						
Mr		$\Delta E$	coordinates	Plan			linear		single
HMrx	15	0,007	X	12.5	87.5	50	12.5	87.5	0
HMry	35	0,014	Y	37.5	37.5	12.5	50	50	25
GMr	54	0,006	Z	0	0	0	0	0	10

According to these results, the WPE can be predict and compensate to obtain the desired accuracy on a series of the identical parts; and to simplify the calculation, a module was given based on our mathematical model.

## 7. Conclusions

This work led to position a part in an optimal way to obtain more precision and minimize the random dispersions; in addition, from mathematical modeling based on the experimental tests and by using the angles of Euler a reliable

mathematical model allows calculate the WPE in a precise way, was developed. To obtain these objectives the work was devised on four steps, on the first; a general mathematical model was developed in order to describe the deviation of the workpiece, containing the translations and the rotations, using the Euler angles matrices. On the second step; to use this model in order to calculate the WPE on the prismatic workpiece, the results of an experimental study were used to calculate the translation and the Euler angles for six isostatic positions. After comparing between the results, an optimum position was founded and the value of the translation and Euler angles of this position were inject on the mathematical model, on the third step; this model was more developed using the same experimental study results. To obtain this object, an parameter was fixed and another was varied, this new model help to calculate the WPE on the cases that the optimum position can't be used, and final step, an interface was programmed using visual studio 2017 to simplify the calculation of WPE and the optimum supports distribution.

On further works, the WPE formulate will be more developed, injecting other factors in order to calculate the WPE under constraint of clamping.

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