

## OPTIMIZATION OF MAGNETIC COUPLED RESONATOR STRUCTURE USED IN WIRELES ELECTROMAGNETIC ENERGY TRANSFER

Maria – Lavinia IORDACHE (BOBARU)<sup>1</sup>, Mihai IORDACHE<sup>2</sup>

*This paper presents a procedure for wireless power transfer system (WPTS) to optimize the structure of the two magnetic coupled coils (inductance) used in the wireless electromagnetic energy transfer (WEET). The optimization of the two magnetic coupled coils is performed in function of their parameter values, for different structures of the two coils. Using the ANSYS Extractor Q3D Program we have been computed the C, L, R, and G matrices, for different configurations, structures and frequencies of the two coils. For each structure, we keep the same relative position between the two coils, the same turn number, the same geometrical dimension of the conductors and the same conductor materials. Finally, comparing the obtained results we can select the optimal solution.*

**Keywords:** electromagnetic field, mutual inductance, frequency, resonator, wireless power transfer, ANSYS Q3D Extractor, optimization

### 1. Introduction

The parameter identification can be defined as: “the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent” (Zadeh 1962). To simplify the analog linear system (circuit) parameter identification process, system (circuit) parameters can be estimated from manufacturer data by the transfer function, [1-7].

For magnetic coupled resonators, used in electromagnetic wireless power transfer systems, the corresponding parameters (R, L, C and G) should be identified using some performing electromagnetic field simulation software. One of these programs widely used for electromagnetic system parameter estimation is ANSYS Q3D Extractor 2D or 3D software. This program has special sub-routines for generating R, L, C and G matrices, for different frequencies, for any magnetic coupled resonators system [13-18].

For the size  $> \lambda/10$ , when the radiation is important, the Q3D Extractor used the full-wave solvers to generate the S, Y, and Z parameters and fields.

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<sup>1</sup> PhD Eng., Electrical Engineering Departament, University POLITEHNICA of Bucharest, Romania, e-mail: laviniabobaru@gmail.com

<sup>2</sup> Prof., Electrical Engineering Departament, University POLITEHNICA of Bucharest, Romania, e-mail: iordache@elth.pub.ro

ANSYS Q3D Extractor is also capable of generating S-parameters exported using the Touchstone format, which represent the n-port network parameter data of a device or passive interconnect network. Such frequency dependency can be included in simulations through dynamic links with ANSYS Designer SI and ANSYS Simplorer products, [3].

Finally, comparing the obtained results we can select the optimal solution. In Section 4 the identification parameters of the two magnetic coupled coils is performed in function of their parameter values, for different structures of the two coils. Using the program ANSYS Extractor Q3D to compute the R, L, C and G matrices for different configurations, structures, frequencies and distances, between magnetic coupled resonators. For each configuration, it is kept the same relative position between the two magnetic coupled resonators (coils), the same turn number, the same geometrical dimension of the conductors and the same conductor materials. Finally, by comparing the obtained results, we can select the optimal solution.

## **2. Description of ANSYS Extractor Q3D simulator**

ANSYS Q3D Extractor program calculates the parasitic parameters of frequency-dependent resistance, inductance, capacitance and conductance (RLCG) for electronic products:

- The capacitance matrix  $C$ , the inductance (self-inductances and mutual inductances) matrix  $L$ , and resistance  $R$ , by a dc *analysis*;
- The inductance (self-inductances and mutual inductances) matrix  $L$ , and resistance  $R$ , by *ac high frequency analysis*.

Apply extracted data in studying crosstalk, ground bounce, interconnect delays and ringing, as well as to understand performance of high-speed electronic designs — such as multi-layer printed circuit boards, advanced electronic packages and 3-D on-chip passive components. Review the per bump R, L, G, & C parasitics of a package or die along with per cell R, L, G, & C of a Touch Panel/Screen.

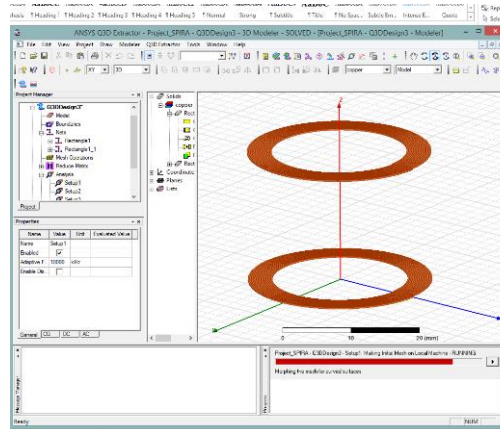


Fig. 1. Presentation software interface

To compute all elements in a matrix for an  $n$  conductor transmission line, the 2D (3D) Extractor performs a sequence of field simulations, then computes the energy stored in the simulated field. In each field simulation, 1 volt (for capacitance and conductance matrices) or 1 amp (for inductance and resistance matrices) is applied to a single conductor and 0 volts or 0 amps are applied to all other conductors. Therefore, for an  $n$ -conductor system,  $n$  field simulations are performed.

The  $C$ ,  $G$  matrices that are generated by the software from the field simulator outputs are in a Maxwell matrix format. Since a standard SPICE component only has two terminals, the software derives a SPICE matrix format from the Maxwell capacitance and conductance elements. This is only necessary for the capacitance and conductance matrices, because both formats yield the same results for the inductance and resistance matrices.

For field values computation, the user should choose the appropriate method (combination of Finite Element Method –FEM and moments method –MM), which depends on the computed element quantity. For example, MM is used to compute capacitance and inductance problems and for DC conductors problems, FEM is used.

For a transmission line, Ansoft Extractor Q3D (Q2D) gives you the option of extracting any combination of four parameters: capacitance  $C$ , inductance  $L$ , resistance  $R$ , and conductance  $G$ . The generating process of the  $C$ ,  $L$ ,  $R$ , and  $G$  follows, in general, two aims:

1. for a lossless transmission line, both capacitance and inductance matrices are required to generate a SPICE equivalent circuit;
2. for a loss transmission line, one of the following combinations is required to generate a SPICE equivalent circuit:

- resistance, capacitance and inductance;
- conductance, capacitance and inductance.

The electromagnetic field simulations are based on the Maxwell's equations:

### 3. The algorithm of the parameter extraction

All currents in ac analysis are surface currents. To take this into account, the system computes the surface magnetic field  $\bar{H}$ , such that, [1] - [7]:

$$\bar{H} = \frac{1}{\mu_0} \nabla \times \bar{A}, \quad (1)$$

where  $\bar{A}$  is the magnetic vector potential.

At high frequencies, the magnetic field is tangential to the surface of a good conductor. After solving for the magnetic field, the system computes the surface current density,  $\bar{J}_S$ :

$$\bar{J}_S = \bar{n} \times \bar{H}, \quad (2)$$

and inductance is found using the relationship

$$L_{kj} = \iiint_V \bar{A}_k \cdot \bar{J}_{Sj} dV \quad (3)$$

For a matrix with entries  $L_{ij} = L_{ji}$ ,  $i, j = 1, \dots, n$ , magnetic coupling coefficient for the row  $i$  and column  $j$  is given by the relation:

$$k_{ij} = \frac{|L_{ij}|}{\sqrt{L_i L_j}}. \quad (4)$$

Between each pair of conductors exists a service capacitance denoted by  $C_{ij} = C_{ji}$ ,  $i, j = 1, \dots, n$ ,  $i \neq j$ , and each conductor has a capacitance between its own conductor and ground, denoted by  $C_{i0}$ ,  $i=1, \dots, n$  (see Fig. 2 for three conductors).

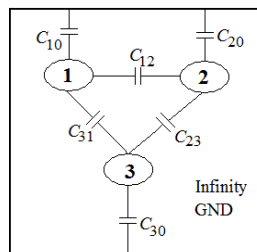


Fig.2. Capacitance structure of the three conductors.

The total amount of electric charge of the capacitor  $C_i$ , according to the Maxwell equations, has the following expression:

$$Q_i = C_{i0}V_i + C_{i1}(V_i - V_1) + \dots + C_{in}(V_i - V_n). \quad (5)$$

For the three conductors (Fig. 2) the relation (9) becomes

$$Q_1 = C_{10}V_1 + C_{12}(V_1 - V_2) + C_{13}(V_1 - V_3) \quad (6)$$

$$Q_2 = C_{20}V_2 + C_{21}(V_2 - V_1) + C_{23}(V_2 - V_3).$$

$$Q_3 = C_{30}V_3 + C_{31}(V_3 - V_1) + C_{32}(V_3 - V_2)$$

If the physical length of the package is comparable to wavelength  $\lambda/10$  of the corresponding frequency, the model is valid.

Ac solvers compute high frequency asymptotes of inductance and resistance. While external inductance values do not grow with frequency, the resistance values are unbounded and increase with the square root of frequency. The default operating frequency is 100 MHz. The skin depth used by the software is given by the following relationship (for reference, remember that the skin depth in copper ( $s = 5.8 \times 10^7$  S/m) at 100 MHz is 6.6 mm):

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_o \mu_r}}. \quad (7)$$

where:

- $\omega$  is the angular frequency, which is equal to  $2\pi f$  ( $f$  is the frequency in hertz)
- $\sigma$  is the conductor's conductivity in S/m
- $\mu_r$  is the conductor's relative permeability. For most metals,  $\mu_r = 1$ .
- $\mu_o$  is the permeability of free space, which equals to  $4\pi \times 10^{-7}$  Wb/Am (or  $4\pi \times 10^{-7}$  H/m).

Thus, given a conductor with thickness  $d$ , you can calculate the lower bound of the ac region by calculating the smallest frequency that produces a skin depth  $d$  much smaller (in general, three times smaller) than this thickness. Using the formula for  $d$  and the definition of  $\omega = 2\pi f$ , you have the frequency limit:

$$f \geq \frac{9}{\pi \sigma d^2 \mu_o \mu_r}. \quad (8)$$

Q3D Extractor calculates this number and includes it in the Spice sub-circuit in .cir format, but we do not report it. Even though we do not report this number does not mean that it is zero, however, it is very small and omitting this term does not necessarily introduce an error. The results matrix, permit the generation for new matrix for any selected parameters how have solutions in the field, without need to calculate new solutions. The non-net conductors are objects

connected to ground. Since a model can contain many nets, electromagnetic interaction between them is described by a matrix, calculated as follows:

- 1) In each source terminal (excitation), the field solution is used to calculate the capacity, inductance or resistance for a conductor to grown voltage. Each excitation produced a column of the matrix;
- 2) The resistance matrix and inductance matrix have one row for each source terminal. However, capacity matrix contains one row for each net;
- 3) After finding the field solution and complete calculation of the respective matrix, the simulator carries out an error analysis in each network loop element.

In the next step of adjustment, the elements with the greatest error are divided into smaller elements, producing a more accurate solution in these areas [5], [6], [7].

#### **4. Optimization of the magnetic coupled resonators structure: using Ansys Q3D Extractor Program**

In general, for electrical vehicle battery charging by WPTS, are used the pancake magnetic coupled coils. We use the program ANSYS Extractor Q3D to optimize the configurations. To each configuration, we keep the same turn number, the same geometrical dimension of the conductors and the same conductor materials, but we vary the frequency. After we finish to design the structure, we compute the C, L, R, and G matrices for different frequencies, between magnetic coupled resonators. Finally, analyzing and comparing the obtained results we can select the optimal solution. For each structure of the two magnetic coupled resonators (coils), we want to choose the configuration of the two coils, which has the most grated value of the mutual inductance. We performed the simulation of the two coils with Ansoft Q3D Extractor for frequency 50, 5000 and 10000 kHz. From all analyzed results we have selected only the best four representative configurations (see Figs. 3, 4, 5 and 6), as it follows:

1. The two spiral parallel coils. The coil parameters are: the initial radius of a turn  $r = 10$  mm, the pitch  $p = 0.21$  mm, the wire section  $0.2 \times 0.2$  mm<sup>2</sup>, and the distance between the coils  $z=20$  mm, and the turn number  $n = 15$  (Figs. 3, a, and b).

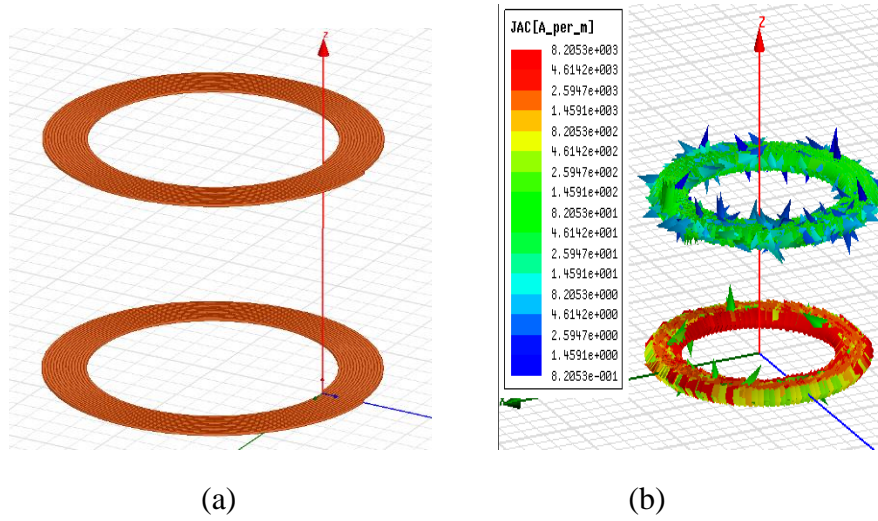


Fig.3. a) Structure of the two spiral parallel coils; b) Surface current density.

The two spiral parallel coil parameters, determined by Ansoft Q3D Extractor simulation, are presented in Table 1

*Table 1*

**Results obtained by the Ansoft Q3D Extractor simulation for the two spiral parallel coils**

Frequency [kHz]	50	5000	10000
Resistance matrix [ohm]	$\begin{pmatrix} 0.1635 & 0.0004 \\ 0.0009 & 0.1790 \end{pmatrix}$	$\begin{pmatrix} 2.1451 & 0.0394 \\ 0.0394 & 2.3414 \end{pmatrix}$	$\begin{pmatrix} 7.7438 & 0.1892 \\ 0.1892 & 7.6332 \end{pmatrix}$
Capacitance matrix [pF]	$\begin{pmatrix} 0.9601 & -0.2977 \\ -0.2977 & 0.9602 \end{pmatrix}$	$\begin{pmatrix} 0.9601 & -0.2977 \\ -0.2977 & 0.9602 \end{pmatrix}$	$\begin{pmatrix} 0.9601 & -0.2977 \\ -0.2977 & 0.9602 \end{pmatrix}$
Inductance matrix [ $\mu$ H]	$\begin{pmatrix} 8.8699 & 0.4971 \\ 0.4971 & 8.8821 \end{pmatrix}$	$\begin{pmatrix} 8.5028 & 0.4950 \\ 0.4950 & 8.5032 \end{pmatrix}$	$\begin{pmatrix} 8.7034 & 0.4927 \\ 0.4927 & 8.7154 \end{pmatrix}$

- The two truncated con coils with the parallel bases, at a distance of 20 mm (distance between the highest point and lowest point). The coil parameters are: the initial radius of a turn  $r = 10$  mm, the distance between the centres of the two consecutive turns on the OY axis 0.21 mm, the wire section  $0.2 \times 0.2$  mm<sup>2</sup>, the distance between the coils  $z=20$  mm, and the turn number  $n = 15$  (Figs. 4, a, and b).

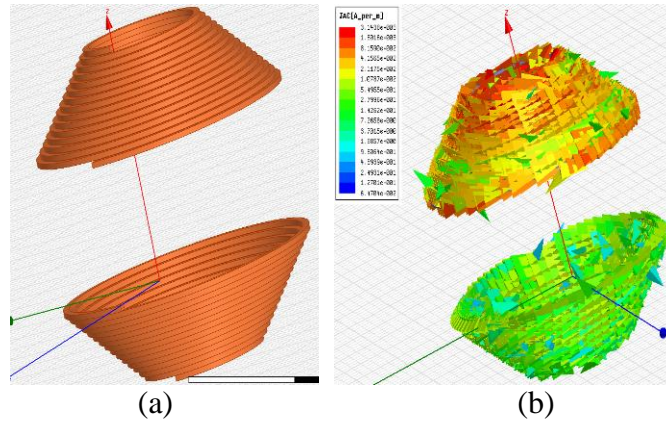


Fig. 4. a) Truncated con coils with the parallel bases; b) Surface current density c) Mesh surface

Simulating the truncated con coils with the parallel bases by the Ansoft Q3D Extractor Program we obtain the coil parameters presented in Table 2.

Table 2

Results obtained by the Ansoft Q3D Extractor simulation for the truncated con coils with the parallel bases

Frequency [kHz]	50	5000	10000
Resistance matrix [ohm]	$\begin{pmatrix} 0.1826 & 0.0012 \\ 0.0012 & 0.1807 \end{pmatrix}$	$\begin{pmatrix} 2.4503 & 0.0772 \\ 0.0772 & 2.5167 \end{pmatrix}$	$\begin{pmatrix} 4.1709 & 0.1374 \\ 0.1374 & 4.0615 \end{pmatrix}$
Capacitance matrix [pF]	$\begin{pmatrix} 1.3023 & -0.5986 \\ -0.5986 & 1.3031 \end{pmatrix}$	$\begin{pmatrix} 1.3023 & -0.5986 \\ -0.5986 & 1.3031 \end{pmatrix}$	$\begin{pmatrix} 1.3023 & -0.5986 \\ -0.5986 & 1.3031 \end{pmatrix}$
Inductance matrix [ $\mu$ H]	$\begin{pmatrix} 13.348 & -1.9729 \\ -1.9729 & 13.339 \end{pmatrix}$	$\begin{pmatrix} 13.0880 & -1.9689 \\ -1.9689 & 13.173 \end{pmatrix}$	$\begin{pmatrix} 13.4930 & -1.9727 \\ -1.9727 & 13.5310 \end{pmatrix}$

- The two truncated con coils with the nonparallel bases, one of them is rotated with an angle of  $45^\circ$ . The coil parameters are: the initial radius of a turn  $r = 10$  mm, the distance between the centres of the two consecutive turns 0.21 mm, the distance between the centres of the two consecutive turns on the OZ axis 0.21 mm, the wire section  $0.2 \times 0.2$  mm<sup>2</sup>, the distance between the coils  $z=20$  mm, and the turn number  $n = 15$  (Figs. 5. A, and b).

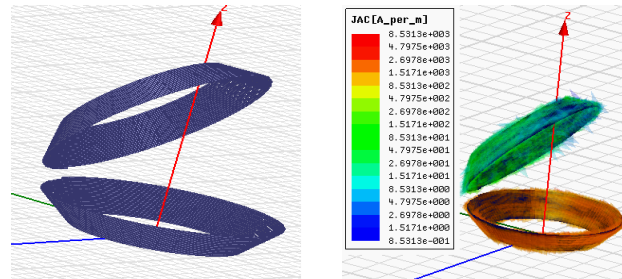


Fig. 5. a) Truncated con coils with nonparallel bases, one of them are rotated with an angle of  $45^\circ$ ; b) Surface current density



Simulating the truncated con coils with the nonparallel bases, one of them is rotated with an angle of 450, by the Ansoft Q3D Extractor Program we obtain the R, L, G, and C matrices presented in Table 3.

Table 3

**Results obtained by the Ansoft Q3D Extractor simulation for the truncated con coils with the nonparallel bases, one of them is rotated with an angle of 450.**

Frequency [kHz]	50	5000	10000
Resistance matrix [ohm]	$\begin{pmatrix} 0.1846 & 0.0015 \\ 0.0015 & 0.1787 \end{pmatrix}$	$\begin{pmatrix} 2.3284 & 0.0196 \\ 0.0196 & 2.3955 \end{pmatrix}$	$\begin{pmatrix} 4.5941 & 0.2569 \\ 0.2569 & 4.9967 \end{pmatrix}$
Capacitance matrix [pF]	$\begin{pmatrix} 1.3743 & -0.6593 \\ -0.6593 & 1.3667 \end{pmatrix}$	$\begin{pmatrix} 1.3743 & -0.6593 \\ -0.6593 & 1.3667 \end{pmatrix}$	$\begin{pmatrix} 1.3743 & -0.6593 \\ -0.6593 & 1.3667 \end{pmatrix}$
Inductance matrix [ $\mu$ H]	$\begin{pmatrix} 13.3680 & \mathbf{1.8555} \\ \mathbf{1.8555} & 13.3730 \end{pmatrix}$	$\begin{pmatrix} 13.2090 & \mathbf{1.8560} \\ \mathbf{1.8560} & 13.0710 \end{pmatrix}$	$\begin{pmatrix} 13.5420 & \mathbf{1.8485} \\ \mathbf{1.8485} & 13.4200 \end{pmatrix}$

- The two truncated con coils with the parallel bases, the distance on the OZ  $z = 20$  mm and the distance on the OY is  $y = 15$  mm. The coil parameters are: the initial radius of a turn  $r = 10$  mm, the distance between the centers of the two consecutive turns 0.21 mm, the distance between the centers of the two consecutive turns on the OZ axis 0.21 mm, the wire section  $0.2 \times 0.2$  mm<sup>2</sup>, and the turn number  $n = 15$  (Figs. 6, a, and b).

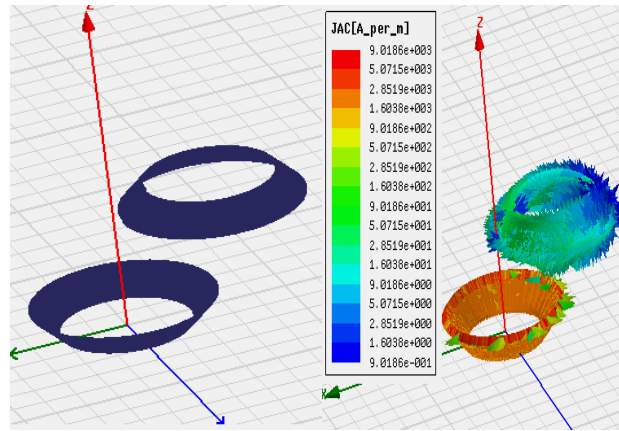


Fig. 6. a) Truncated con coils with the parallel bases, the distance on the OZ  $z = 20$  mm and the distance on the OY is  $y = 15$  mm; b) Surface current density

Simulating the truncated con coils with the parallel bases, the distance on the OZ  $z = 20$  mm and the distance on the OY is  $y = 15$  mm, by the Ansoft Q3D Extractor Program we obtain the coil parameters presented in Table 4.

From the four configurations of the two magnetic coupled resonators we can see that the best configuration is the second one (Fig. 4, a) where the mutual inductance has the greatest value  $M = 1.9729$   $\mu$ H (see Table 2).

Table 4

**Results obtained by the Ansoft Q3D Extractor simulation for the truncated con coils with the parallel bases, the distance on the 0Z z = 20 mm and the distance on the 0Y is y = 15 mm.**

Frequency [kHz]	50	5000	10000
Resistance matrix [ohm]	$\begin{pmatrix} 183.3 & 0.9199 \\ 0.9199 & 181 \end{pmatrix}$	$\begin{pmatrix} 2720 & 32.731 \\ 32.731 & 2144.7 \end{pmatrix}$	$\begin{pmatrix} 4655.2 & 132.27 \\ 132.27 & 4715.4 \end{pmatrix}$
Capacitance matrix [pF]	$\begin{pmatrix} 1.1973 & -0.4423 \\ -0.5523 & 1.975 \end{pmatrix}$	$\begin{pmatrix} 1.1973 & -0.4423 \\ -0.5523 & 1.975 \end{pmatrix}$	$\begin{pmatrix} 1.1973 & -0.4423 \\ -0.5523 & 1.975 \end{pmatrix}$
Inductance matrix [ $\mu$ H]	$\begin{pmatrix} 13.366 & 0.6389 \\ 0.6389 & 13.347 \end{pmatrix}$	$\begin{pmatrix} 13.183 & 0.6387 \\ 0.6387 & 13.185 \end{pmatrix}$	$\begin{pmatrix} 13.4930 & 0.6357 \\ 0.6357 & 13.489 \end{pmatrix}$

The values of the mutual inductance obtained by measurements ( $M_m$ ), those computed by integration in MATLAB ( $M_p$  – relation (8)), and the ones obtained by ANSOFT Q3D EXTRACTOR simulation ( $M_{Q3D}$ ), for the four cases presented above, are exposed in Table 5.

Table 5

**Experimental results and their comparison with the computed ones**

Case	Case 1	Case 2	Case 3	Case 4
$M_{Q3D}$ [ $\mu$ H]	<b>0.4971</b>	<b>1.9729</b>	<b>1.8555</b>	<b>0.6389</b>
$M_p$ [ $\mu$ H]	<b>0.4985</b>	<b>1.9862</b>	<b>1.876</b>	<b>0.6445</b>
$M_m$ [ $\mu$ H]	<b>0.5004</b>	<b>1.9875</b>	<b>1.8844</b>	<b>0.6482</b>

From Table 5 we can see that the values of the mutual inductance, determined by the three procedures, are much closed.

The ANSOFT Q3D EXTRACTOR simulations are performed at three frequencies: 0.05 MHz, 5 MHz, and 10 MHz, however the values of the self-inductances, the capacitances, and the mutual inductance remain unchanged, but the resistance values are higher due to the skin effect and to the proximity effect, as it is shown in Tables 1 - 4.

## 5. Conclusions

The efficiency of the wireless transfer power deeply depends on the resonator parameters (L - self-inductance, M - mutual inductance, C – parasitic capacitance, and R - Ohmic resistances) of the two magnetically-coupled coils, at several frequencies working at several angles coil assembly. Consequently, parameter identification is a very important objective in the automatic design system. In this paper the optimization of the two magnetic coupled coils is performed in function of their parameter values, for different structures of the two

coils, using the program ANSYS Extractor Q3D to compute the C, L, R, and G matrices for different configurations, structures and frequencies of the two magnetic coupled resonators. For each configuration, it was kept the same relative position between the two magnetic coupled resonators (coils), the same turn number, the same geometrical dimension of the conductors and the same conductor materials. Finally, comparing the obtained results we can select the optimal solution. The optimal configuration has a maximum value of the mutual inductance - the second configuration in Fig. 4, a. A great importance in obtaining more accurate solutions, using Q3D Extractor software, it has the mesh. Although the program has adaptive function, the original network can lead to convergence problems of the solution or increasing the computational effort. The advantage of the mesh in Ansoft Q3D Extractor program is that the discretized coil is only in the area of interest, it is not necessary to divide the air in the vicinity of the coil or use conditions indefinitely. The mutual inductance  $M_{21}$  depends only on the geometrical properties of the two coils, such as the number of turns and the radii of the two coils. The mutual inductance is also influenced, by the angle of the one of the two coils, where the field lines pass by the receiving coil. The mutual inductance is an important parameter in the determining of the performances of the wireless power transfer systems (the load power and the efficiency). The resulting RLC matrices allow you to generate new matrices for any selected parameters that have field solutions, without having to compute new field solutions. In addition, any model that has been successfully solved can have its matrix results exported to an equivalent circuit for further signal integrity analysis. After the field solutions and matrix calculations are complete, the simulator performs an error analysis in each element in the mesh. Upon the next adaptive pass, the elements with the highest error are broken down (refined) into smaller elements — producing a more accurate solution in these areas. Because the final circuit parameters are computed properly, using the mesh solution we generated net current field, this solution, increasing the accuracy of circuit parameters results. WPT efficiency is determined by the coil: shape, location and inclination.

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