HYDROMAGNETIC THERMAL BOUNDARY LAYER OF NANOFLUIDS OVER A CONVECTIVELY HEATED FLAT PLATE WITH VISCOUS DISSIPATION AND OHMIC HEATING

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This paper examines the effect of the complex interaction between the electrical conductivity of the conventional base fluid and that of the nanoparticles under the influence of magnetic field in a boundary layer flow with heat transfer over a convectively heated flat surface. Three types of water based nanofluids containing metallic or non-metallic nanoparticles such as copper (Cu), Alumina (Al_2O_3) and Titania (TiO_2) are investigated. Using a similarity analysis of the model transport equations followed by their numerical computations, the results for the nanofluids velocity, temperature, skin friction and Nusselt number are obtained. The effects of various thermophysical parameters on the boundary layer flow characteristics are displayed graphically and discussed quantitatively. It is observed that the presence of nanoparticles greatly enhance the magnetic susceptibility of nanofluids as compared to the convectional base fluid.

Keywords: Boundary layer flow; Magnetic field; Nanofluids; Convectively heated plate

1. Introduction

Studies related to hydromagnetic boundary layer flows of nanofluids have a wide range of industrial, engineering and biological applications. These include; magnetics drug targeting, MHD blood flow meters, production of magneto-rheostatic (MR) materials known as smart fluids, boundary layer control in aerodynamic and crystal growth [1]. In recent years, we find several applications in polymer industries, cooling of metallurgical materials, cooling of microchips in computers and other electronics which use microfluidic applications, cooling of automobile engine, wire drawing, glass-fibre production. The nanofluid containing magnetic nanoparticles also acts as a super-paramagnetic fluid, which, in an alternating electromagnetic field, absorbs energy and produces a controllable hyperthermia. Choi [2] pioneered the study of nanofluids. The term nanofluid describes a solid liquid mixture which consists of base liquid with low volume fraction of high conductivity solid nanoparticles. The particles are usually of nanometer-size (10–50nm) and are made by a high-energy-pulsed process from a conductive material. They include particles of metals such aluminium, copper,

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gold, iron and or their oxides like titanium. Moreover, it is well known that the heat transfer properties of the conventional base fluids such as water; mineral oil and ethylene glycol are very poor compared to that of most solids. A comprehensive survey of convective transport in nanofluids was presented by Buongiorno [3]. He reported that a satisfactory explanation for the abnormal increase in the thermal conductivity and viscosity of nanofluids was yet to be found. Li and Xuan [4] experimentally investigated the various transport properties of nanofluids. Kuznetsov and Nield [5] examined the influence of nanoparticles on the natural convection boundary-layer flow past a vertical plate. Thereafter, several researchers [6-9] have investigated convention flows of nanofluids under various physical conditions. Ahmad et al. [10] extended the well known Blasius and Sakiadis boundary layer flow problems to include the nanofluids. Makinde and Aziz [11] presented a similarity solution for boundary layer flows of nanofluids over a convectively heated stretching sheet. The effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate were reported by Motsumi and Makinde [12]. Recently, the influence of magnetic field on the boundary layer flow of electrically conducting nanofluids was investigated in some studies [13, 14]. However, in their theoretical analysis, the complex interaction of the nanoparticles electrical conductivities with that of conventional base fluids was ignored. In reality, the electrical conductivities of nanoparticles are not equal to that of conventional base fluid and therefore cannot be ignored in order to obtain a realistic solution to the problem.

In this study, our objective is to investigate the combined effects of magnetic field, viscous dissipation and Ohmic heating on the boundary layer flow of nanofluids over a convectively heated flat plate. The complex interaction of the electrical conductivities of metallic or non-metallic nanoparticles such as copper (Cu), Alumina (Al_2O_3) and Titania (TiO_2) with that of conventional base fluid (water) is taken into consideration. Using an appropriate similarity transformation, the well-known governing partial differential equations are reduced to ordinary differential equations. The resulting problems are solved numerically using the Runge–Kutta–Fehlberg method with the shooting technique. The effects of various thermophysical parameters on velocity, temperature, skin friction and Nusselt number are discussed in detail. A comparative study between the previously published results and the present results in a limiting sense reveals excellent agreement between them. The organization of the paper is as follows: Problem is formulated and solved in section 2. Numerical results and discussion are given in Section 3. The conclusions have been summarized in Section 4.

2. Mathematical Formulation

We consider a steady boundary layer flow of an electrically conducting nanofluid past a semi-infinite convectively heated flat plate in the presence of a uniform transverse magnetic field of strength B_0 applied parallel to the y-axis (Fig. 1). It is assumed that the induced magnetic field and the external electric field are negligible. The nanofluid on the upper surface of the plate is made up of water as base fluid with copper (Cu), Alumina (Al_2O_3) or Titania (TiO_2) as the nanoparticles. The lower side of the plate is convectively heated by a hot conventional fluid of temperature T_f such that the nanofluid $T < T_f$.

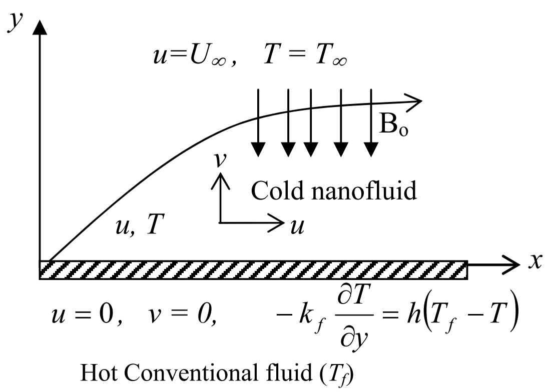

Fig 1. Schematic diagram of the problem

Under the usual boundary layer approximations, the flow is governed by the following equations [10-14];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B_0^2 (u - U_\infty)}{\rho_{nf}}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf} B_0^2}{(\rho c_p)_{nf}}(u - U_\infty)^2, \tag{3}$$

where (u, v) are the velocity components of the nanofluid in the (x, y) directions respectively, U_∞ is the free stream velocity, T is the temperature of the nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid, σ_{nf} is the electrical conductivity of the nanofluid, and $(\rho c_p)_{nf}$ is the heat capacitance of the nanofluid which are given by [8-10]

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s,$$
$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}, \tag{4}$$
$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s, \sigma_{nf} = (1-\varphi)\sigma_f + \varphi\sigma_s.$$

The thermal conductivity of the nanofluid is represented by k_{nf}, φ is the solid volume fraction parameter of the nanofluid, ρ_f is the reference density of the fluid fraction, ρ_s is the reference density of the solid fraction, σ_f is the electrical conductivity of the fluid fraction, σ_s is the electrical conductivity of the solid fraction, μ_f is the viscosity of the fluid fraction, k_f is the thermal conductivity of the fluid fraction, c_p is the specific heat at constant pressure and k_s is the thermal conductivity of the solid volume fraction. The boundary conditions at the plate surface and far into the cold nanofluid may be written as [11]

$$u(x,0) = 0, \; v(x,0)=0, \; -k_f \frac{\partial T}{\partial y}(x,0) = h_f[T_f - T(x,0)],$$
$$u(x,\infty) = U_\infty, \; T(x,\infty) = T_\infty, \tag{5}$$

where is the h_f plate heat transfer coefficient. The stream function ψ, satisfies the continuity Eq. (1) automatically with

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

A similarity solution of Eqs. (1)–(5) is obtained by defining an independent variable η and a dependent variable f in terms of the stream function ψ as [10-14]

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon_f x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}, \quad \psi = \sqrt{\upsilon_f x U_\infty} f(\eta), \; u = U_\infty f'(\eta),$$
$$v = \frac{1}{2}\sqrt{\frac{U_\infty \upsilon_f}{x}}(\eta f' - f), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \tag{7}$$

where a prime symbol denotes differentiation with respect to η and Re_x is the local Reynolds number ($=U_\infty x/\upsilon_f$). After introducing Eq. (6) into Eqs. (1) - (5), we obtain

$$f''' + \frac{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)}{2} ff''$$
$$- Ha(1-\phi)^{2.5}(1-\phi+\phi\sigma_s/\sigma_f)(f'-1) = 0, \tag{8}$$

$$\theta'' + \frac{k_f \Pr[1-\phi+\phi(\rho c_p)_s/(\rho c_p)_f]}{2k_{nf}} f\theta' + \frac{k_f Br}{k_{nf}(1-\phi)^{2.5}}\left(f''\right)^2 + \frac{HaBrk_f}{k_{nf}}(1-\phi+\phi\sigma_s/\sigma_f)(f'-1)^2 = 0, \tag{9}$$

$$f(0)=0, f'(0)=0,\ \theta'(0)=Bi[\theta(0)-1], \tag{10}$$

$$f'(\infty)=1,\ \ \theta(\infty)=0, \tag{11}$$

where

$$Bi = \frac{h_f}{k_f}\sqrt{\frac{\upsilon_f x}{U_0}},\ \ Ha = \frac{\sigma_f B_o{}^2 x}{\rho_f U_\infty},\ \ Br = \frac{\mu_f U_o{}^2}{k_f\left(T_f - T_\infty\right)},\ \Pr = \frac{\upsilon_\mathrm{f}}{\alpha_\mathrm{f}}$$

represents Biot number, magnetic field parameter, Brinkmann number and Prandtl number respectively. When $\varphi = 0$, we obtain the conventional fluid scenario. Bi and Ha in Eqs. (8)-(10) are functions of x and in order to have a similarity solution, all the parameters must be constant. Following [11] we therefore assume that

$$h_f = ax^{-\frac{1}{2}},\ \sigma_f = bx^{-1}, \tag{12}$$

where a and b are constants. The quantities of practical interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho_f U_0^2},\ \ Nu = \frac{xq_w}{k_f(T_f - T_\infty)}, \tag{13}$$

where τ_w is the skin friction and q_w is the heat flux from the plate which are given by

$$\tau_w = \mu_{nf}\left.\frac{\partial u}{\partial y}\right|_{y=0},\ \ q_w = -k_{nf}\left.\frac{\partial T}{\partial y}\right|_{y=0}, \tag{14}$$

Substituting Eqs. (14) into (13), we obtain

$$\mathrm{Re}_x^{\frac{1}{2}} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(0),\quad \mathrm{Re}_x^{-\frac{1}{2}} Nu = -\frac{k_{nf}}{k_f}\theta'(0). \tag{15}$$

The above set of Eqs. (8)–(9) subject to the boundary conditions (10)-(11) were solved numerically by the Runge-Kutta-Fehlberg method with shooting technique [15]. Both velocity and temperature profiles were obtained and utilized to compute the skin-friction coefficient and the local Nusselt number in Eq. (15).

3. Results and Discussion

Numerical evaluation of the model equations is performed for three types of water based Newtonian and electrically conducting nanofluids containing metallic or non-metallic nanoparticles such as copper (Cu), Alumina (Al_2O_3) and Titania (TiO_2). The solid volume fraction φ of the nanoparticles is investigated in the range of $0 \le \varphi \le 0.2$ and the Prandtl number of the based fluid (water) is kept constant at 6.2. The thermophysical properties of water and the nanoparticles Cu, Al_2O_3 and TiO_2 are shown in table 1.

Table 1

Thermophysical Properties of Water and Nanoparticles [4, 7-9]

Materials	ρ(kg/m^3)	c_p (J/kgK)	k (W/mK)	σ (S/m)
Pure water	997.1	4179	0.613	5.5×10^{-6}
Copper (Cu)	8933	385	401	58×10^{6}
Alumina (Al_2O_3)	3970	765	40	35×10^{6}
Titania (TiO_2)	4250	686.2	8.9538	2.6×10^{6}

In order to validate the accuracy of our numerical procedure, the special case of infinite Biot number in the absence of viscous dissipation and magnetic field effects is considered as shown in table (2), our results agreed perfectly with the one reported by Ahmad et al. [10].

Table 2

Values for $Re^{1/2}C_f$ showing the comparison with Ahmad et al. [10] results for *Ha=0, Br=0, Bi* = ∞.

φ	Cu-water [10]	Al_2O_3–water [10]	TiO_2 –water [10]	Cu –water (Present)	Al_2O_3–water (Present)	TiO_2 –water (Present)
0	0.3321	0.3321	0.3321	0.3321	0.3321	0.3321
0.002	0.3355	0.3339	0.3340	0.3355	0.3339	0.3340
0.004	0.3390	0.3357	0.3359	0.3390	0.3357	0.3359
0.008	0.3459	0.3394	0.3398	0.3459	0.3394	0.3398
0.01	0.3494	0.3412	0.3417	0.3494	0.3412	0.3417
0.012	0.3528	0.3431	0.3436	0.3528	0.3431	0.3436
0.014	0.3563	0.3449	0.3456	0.3563	0.3449	0.3456
0.016	0.3597	0.3468	0.3476	0.3597	0.3468	0.3476
0.018	0.3632	0.3487	0.3495	0.3632	0.3487	0.3495
0.02	0.3667	0.3506	0.3515	0.3667	0.3506	0.3515
0.1	0.5076	0.4316	0.4362	0.5076	0.4316	0.4362
0.2	0.7066	0.5545	0.5642	0.7066	0.5545	0.5642

3.1. Effects of parameters variation on the velocity profiles

The nanofluids velocity profiles are shown in figs. 2-4. Generally, the flow over a stationary convectively heated plate surface is driven by the combined action of magnetic field, Newtonian heating and free stream velocity. The nanofluid's velocity is zero at the plate surface and increases gradually until it attains the free stream value far away from the plate, satisfying the prescribed boundary conditions. It is interesting to note that of TiO_2-water nanofluid produced a thicker momentum boundary layer thickness than Al_2O_3–water and Cu-water nanofluids under the influence of magnetic field as illustrated in fig. 2. In fig. 3, it is observed that the momentum boundary layer thickness decreases with increasing magnetic field intensity. The application of a magnetic field normal to the flow direction has the tendency to slow down the movement of the nanofluids because it gives rise to a resistive force called the Lorentz force which acts opposite to the flow direction. Similar trend is observed in fig. 4 with increasing nanoparticles volume fraction. As the φ increases, the momentum boundary layer thickness decreases.

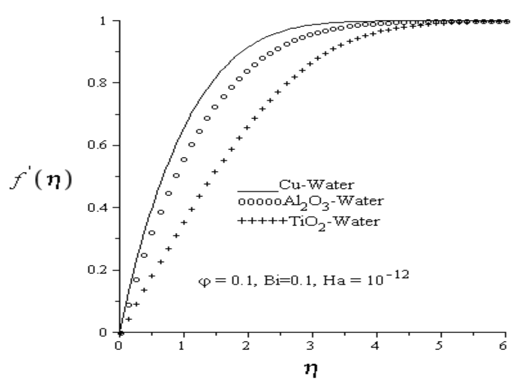

Fig. 2: Velocity profiles for different nanofluids

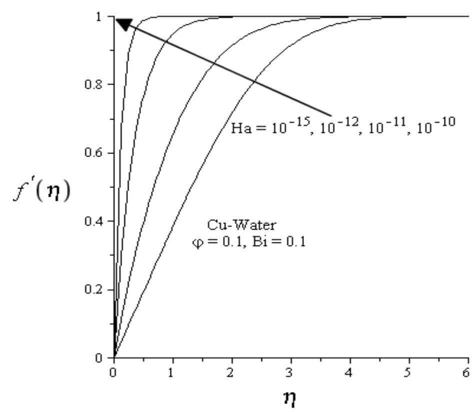

Fig. 3: Velocity profiles with increasing magnetic field intensity.

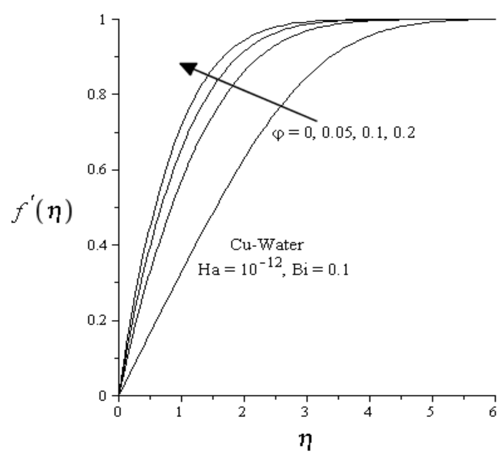

Fig. 4: Velocity profiles with increasing nanoparticles volume fraction

3.2. Effects of parameters variation on the temperature profiles

Effects of various thermophysical parameters on the nanofluid's temperature profiles are displayed in figs. 5-9. The temperature is highest at the plate surface due to Newtonian heating and decreases to zero far away from the plate satisfying the free stream conditions. It is noted that the plate surface temperature is highest with Cu-water nanofluid followed by Al_2O_3-water nanofluid while TiO_2-water nanofluid produced the lowest plate surface temperature as shown in fig. 5. Meanwhile, the thermal boundary layer thickness increases with an increase in the magnetic field intensity as illustrated in fig. 6, consequently, the plate surface temperature increases as well. This can be attributed to the influence of Ohmic heating due to magnet field in the flow system. Fig. 7 depicts the effect of increasing the nanoparticles solid fraction for Cu-water on the temperature profiles. As expected, the thermal boundary layer thickness increases with increasing values of nanoparticle volume fraction (φ), leading to an increase in the plate surface temperature and thermal boundary layer thickness. Moreover, as the values of Biot number (Bi) increasing, the rate of convective heat transfer from the hot conventional fluid below the plate surface to the nanofluid above the plate surface increases, leading to an increase in the thermal boundary layer thickness and the plate surface temperature as shown in fig. 8. Similar trend is observed with increasing values of Brinkmann number (Br) as shown in fig. 9. This can be attributed to the additional heating in the flow system due to viscous dissipation.

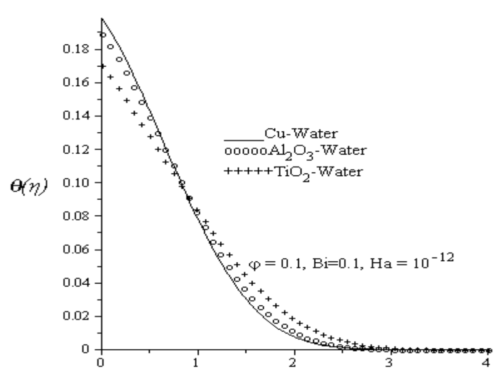

Fig. 5: Temperature profiles for different nanofluids

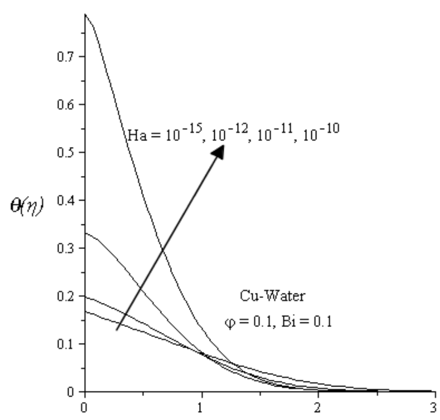

Fig.6: Temperature profiles with increasing magnetic field intensity.

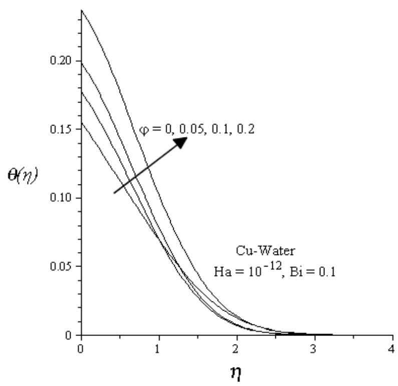

Fig. 7: Temperature profiles with increasing nanoparticles volume fraction

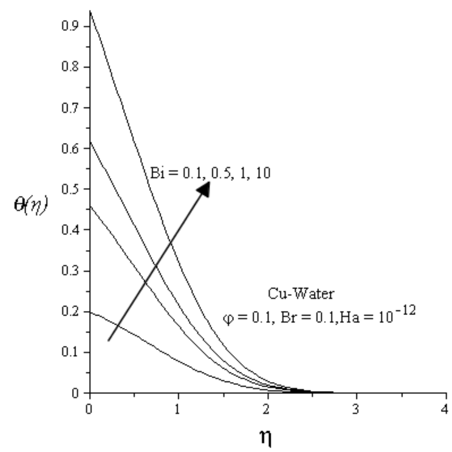

Fig. 8: Temperature profiles with increasing Biot number

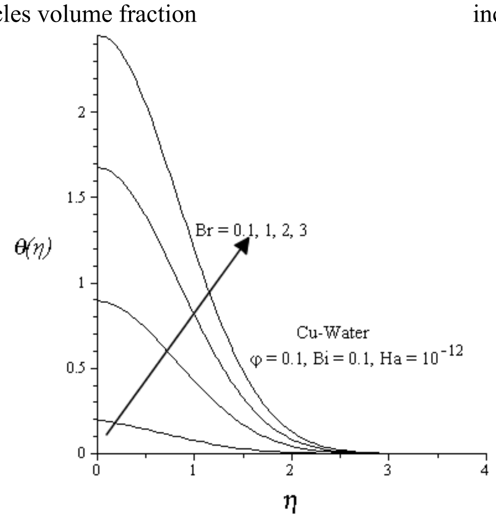

Fig. 9: Temperature profiles with increasing Brinkmann number

3.3 Effects of parameters variation on the skin friction and Nusselt number

Figs. 10–14 illustrate the effects of parameter variation on the skin friction and the Nusselt number. As seen in fig. 10, the skin friction grows with an increase in the nanoparticle volume fraction. Cu-Water nanofluid shows faster growth than Al_2O_3-water while TiO_2-water nanofluid produced the lowest skin friction. Moreover, an increase in the magnetic field intensity (Ha), leads to a further increase in the skin friction as shown in fig. 11 with Cu-Water as the working nanofluid. In fig. 12, it is observed that the Nusselt number increases with an increase in the nanoparticle volume fraction, with Cu-water showing a plate surface higher heat transfer rate than Al_2O_3-water while TiO_2-water nanofluid produced the lowest heat transfer rate at the plate surface. An increase in magnetic field intensity decreases the heat transfer rate at the plate surface as shown in fig. 13. As Biot number (Bi) increases, the heat transfer rate at the plate surface increases (see fig. 14). Meanwhile, an increase in Brinkmann number (Br) due to viscous heating decreases the heat transfer rate at the plate surface.

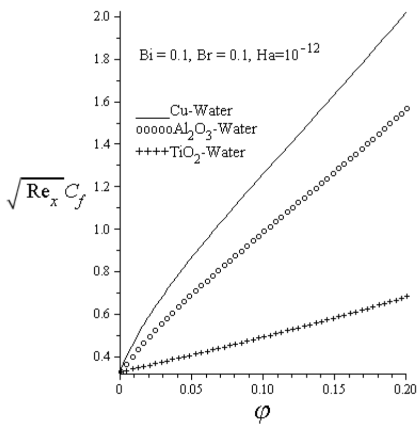

Fig. 10: Skin friction profiles for different nanofluids

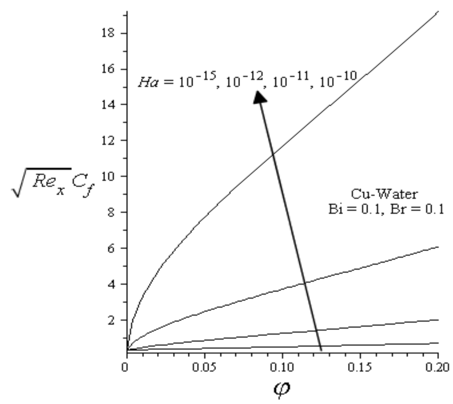

Fig. 11: Skin friction with increasing magnetic field intensity

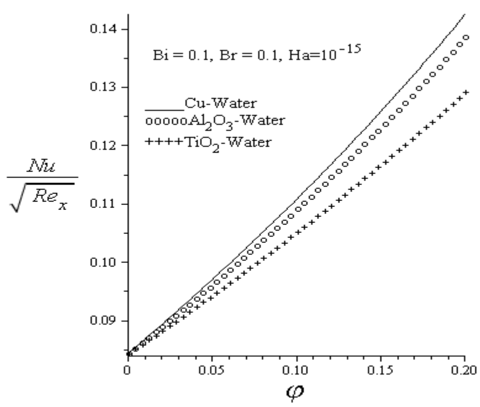

Fig. 12: Reduced Nusselt number for different nanofluids

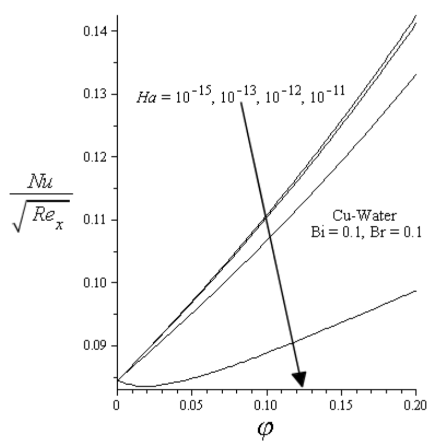

Fig. 13: Nusselt number with increasing magnetic field intensity

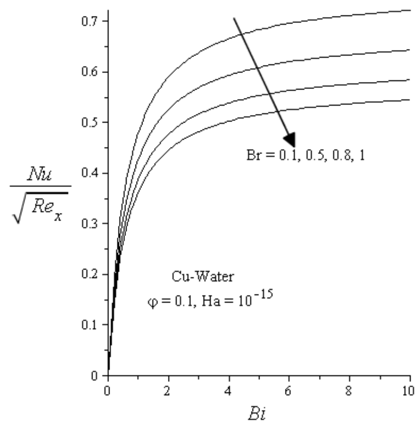

Fig. 14: Nusselt number with increasing Biot number and Brinkmann number

4. Conclusions

The combined effects of magnetic field, viscous dissipation and Ohmic heating on the thermal boundary layer of water-based nanofluids containing Cu, Al_2O_3 and TiO_2 as nanoparticles are investigated theoretically, taking into consideration the complex interaction between the electrical conductivity of the base fluid and that of the nanoparticles. The governing nonlinear partial differential equations were transformed into ordinary differential equations using the similarity approach and solved numerically using the Runge–Kutta–Fehlberg method coupled with the shooting technique. Generally, our results revealed that the susceptibility of nanofluids to the influence of magnetic field is extremely high compared to conventional base fluid due to the complex interaction of electrical conductivity of nanoparticles with that of base fluid. Some other results are as follows;

- TiO_2 – water shows a thicker momentum boundary layer than Al_2O_3 – water while Cu-water momentum boundary layer is the thinnest. As Ha and φ increase, the momentum boundary layer thickness decreases.
- TiO_2 – water produced highest plate surface temperature followed by Al_2O_3 –water while Cu-water produced the lowest surface temperature. As Ha, φ, Bi, Br increase, the thermal boundary layer thickness increases.
- Cu-water produced the highest skin friction followed by Al_2O_3 –water while TiO_2 – water produced lowest skin friction. As φ and Ha increase, the skin friction increases.
- Cu-water produced the highest Nusselt number followed by Al_2O_3 –water while TiO_2 – water produced lowest Nusselt number. The heat transfer rate at the plate surface decreases with increasing Brinknann number values

(Br) and magnetic field intensity (Ha), whereas it increases withincreasing values of Biot number (Bi) and nanoparticle volume fraction(φ).

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