

ATANASSOV'S INTUITIONISTIC FUZZY INTERIOR IDEALS OF Γ -SEMIGROUPS

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The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. The purpose of this paper is to introduce a generalization of ideas presented in [20]. Indeed, we study the concept of intuitionistic fuzzy interior ideals of a Γ -semigroup, by using Atanassov's idea and investigate some of their important properties. Specially, we obtain a characterization of a simple Γ -semigroup in terms of intuitionistic fuzzy interior ideals.

Keywords: Γ -semigroup, intuitionistic fuzzy set, intuitionistic fuzzy interior ideal, intuitionistic fuzzy characteristic interior ideal, simple Γ -semigroup.

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1. Introduction

Uncertainty is an attribute of information and uncertain data are presented in various domains. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [30]. In our daily life, we usually want to seek opinions from professional persons with the best qualifications, for examples, the best medical doctors can provide the best diagnostics, the best pilots can provide the best navigation suggestions for airplanes, etc. It is therefore desirable to incorporate the knowledge of these experts into some automatic systems so that it would become helpful for other people to make appropriate decisions which are (almost) as good as the decisions made by the top experts. With this aim in mind, our task is to design a system that would provide the best advice from the best experts in the field. However, one of the main hurdles of this incorporation is that the experts are usually unable to describe their knowledge by using precise and exact terms. For example, in order to describe the size of certain type of a tumor, a medical doctor would rarely use the exact numbers. Instead he would say something like the size is between 1.4 and 1.6 cm. Also, an expert would usually use some words from a natural language, e.g., the size of the tumor is approximately 1.5 cm, with an error of about 0.1 cm. Thus, under such circumstances, the way

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to formalize the statements given by an expert is one of the main objectives of fuzzy logic.

Besides several generalizations of fuzzy sets, the intuitionistic fuzzy sets introduced by Atanassov [1, 2, 3] have been found to be highly useful to cope with imperfect and/or imprecise information. Atanassovs intuitionistic fuzzy sets are an intuitively straightforward extension of Zadehs fuzzy sets: while a fuzzy set gives the degree of membership of an element in a given set, an Atanassovs intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L-fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy logics, the degree of similarity between intuitionistic fuzzy sets, intuitionistic fuzzy rough sets. Atanassovs intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision making problems, particularly in the case of medial diagnosis, sales analysis, new product marketing, nancial services, etc. there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. To be more precise - intuitionistic fuzzy sets let us express e.g., the fact that the temperature of a patient changes, and other symptoms are not quite clear.

The topic of investigations about fuzzy semigroups belongs to the theoretical soft computing (fuzzy structures). Indeed, it is well known that semigroups are basic structures in many applicative branches like automata and formal languages, coding theory, finite state machines and others. Due to these possibilities of applications, semigroups and related structures are presently extensively investigated in fuzzy settings.

A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. In 1981 Sen [22] introduced the notion of Γ -semigroup as a generalization of semigroup and ternary semigroup. We call this Γ -semigroup a *both sided Γ -semigroup*. In 1986, Sen and Saha [16, 25] modified the definition of Sen's Γ -semigroup. This newly defined Γ -semigroup is known as *one sided Γ -semigroup*. Γ -semigroups have been analyzed by a lot of mathematicians, for instance by Chattopadhyay [4, 5], Dutta and Adhikari [8], Hila [9, 10], Chinram [6], Saha [16], Sen et al. [23, 24, 26], Seth[27]. Dutta and Adhikari [8] mostly worked on *both sided Γ -semigroups*. They defined operator semigroups of such type of Γ -semigroups and established many results and found out many correspondences. In this paper we have considered *both sided Γ -semigroups*.

After the introduction of fuzzy sets by Zadeh [30], reconsideration of the concept of classical mathematics began. On the other hand, because of the importance of group theory in mathematics, as well as its many areas of application, the notion of fuzzy subgroups was defined by Rosenfeld [15] and its structure was investigated. Das characterized fuzzy subgroups by their level subgroups in [7]. Nobuaki Kuroki [12, 13, 14] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki [12, 14]. In [13], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as Xie [29], Jun [11], are mentioned in the bibliography.

In 2007, Uckun, Öztürk and Jun [28] introduced the notions of intuitionistic fuzzy ideals in Γ -semigroups. Motivated by Kuroki [12, 13, 14], Uckun et al. [28], Sardar et al. [17, 18, 19, 20] have initiated the study of Γ -semigroups in terms of fuzzy sets. In this paper, we introduce the notion of intuitionistic fuzzy interior ideals and intuitionistic fuzzy characteristic interior ideals of a Γ -semigroup and obtain some of their properties such as the characteristic function criterion and level subset criterion. Finally, we obtain a characterization of simple Γ -semigroup in terms of intuitionistic fuzzy interior ideal.

2. Intuitionistic fuzzy interior ideals

Let $S = \{x, y, z, \dots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \dots\}$ be two non-empty sets. Then, S is called a Γ -semigroup if there exists a mapping from $S \times \Gamma \times S$ to S , written as $(a, \alpha, b) \longrightarrow a\alpha b$ satisfying: (i) $x\gamma y \in S \forall x, y \in S, \forall \gamma \in \Gamma$, (ii) $(x\beta y)\gamma z = x\beta(y\gamma z) \forall x, y, z \in S, \forall \beta, \gamma \in \Gamma$.

Example 2.1. Let $\Gamma = \{5, 7\}$. For any $x, y \in \mathbb{N}$ and $\gamma \in \Gamma$, we define $x\gamma y = x \cdot \gamma \cdot y$, where \cdot is the usual multiplication on \mathbb{N} . Then, \mathbb{N} is a Γ -semigroup.

Example 2.2. If M is the set of $m \times n$ matrices and Γ is a set of some $n \times m$ matrices over the field of real numbers, then we can define $A_{m,n}\alpha_{n,m}B_{m,n}$ such that

$$(A_{m,n}\alpha_{n,m}B_{m,n})\beta_{n,m}C_{m,n} = A_{m,n}\alpha_{n,m}(B_{m,n}\beta_{n,m}C_{m,n}),$$

where $A_{m,n}, B_{m,n}, C_{m,n} \in M$ and $\alpha_{n,m}, \beta_{n,m} \in \Gamma$. An algebraic system that satisfying the associativity property of the above type is a Γ -semigroup.

Example 2.3. Let $M = [0, 1]$ and $\Gamma = \{\frac{1}{n} \mid n \text{ is a positive integer}\}$. Then, M is a Γ -semigroup under the usual multiplication. Next, let $K = [0, 1]$. We have that K is a non-empty subset of M and $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$. Then, K is a sub Γ -semigroup of M .

A non-empty subset A of a Γ -semigroup S is said to be a *subsemigroup* of S if $A\Gamma A \subseteq A$. A *subsemigroup* A of a Γ -semigroup S is called an *interior ideal* of S if $S\Gamma A\Gamma S \subseteq A$. A *left (right) ideal* of a Γ -semigroup S is a non-empty subset I of S such that $S\Gamma I \subseteq I$ ($I\Gamma S \subseteq I$). If I is both a *left* and a *right ideal* of a Γ -semigroup S , then we say that I is an *ideal* of S .

After the introduction of fuzzy sets by Zadeh [30], several researches were conducted on the generalization of fuzzy sets. As an important generalization of the notion of fuzzy sets on a non-empty set X , Atanassov introduced in [1, 2], the concept of intuitionistic fuzzy sets defined on a non-empty set X as an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Such defined objects are studied by many authors and have many interesting applications not only in mathematics (see [3]).

Let A and B be two intuitionistic fuzzy subsets of a set X . Then, the following expressions are defined in [1, 2].

- (1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,
- (2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- (3) $A^C = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$,
- (4) $A \cap B = \{ \langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\} \rangle : x \in X \}$,
- (5) $A \cup B = \{ \langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\} \rangle : x \in X \}$,
- (6) $\Box A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$,
- (7) $\Diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle : x \in X \}$.

Example 2.4. Consider the universe $X = \{10, 100, 500, 1000, 1200\}$. An intuitionistic fuzzy set “Large” of X denoted by L and may be defined by

$$L = \{ \langle 10, 0.01, 0.9 \rangle, \langle 100, 0.1, 0.88 \rangle, \langle 500, 0.4, 0.5 \rangle, \langle 1000, 0.8, 0.1 \rangle, \langle 1200, 1, 0 \rangle \}.$$

One may define an intuitionistic fuzzy set “Very Large” (denoted by VL) as follows:

$$\mu_{VL}(x) = (\mu_L(x))^2 \quad \text{and} \quad \nu_{VL}(x) = 1 - (1 - \nu_L(x))^2,$$

for all $x \in X$. Thus,

$$VL = \{ \langle 10, 0.0001, 0.99 \rangle, \langle 100, 0.01, 0.9856 \rangle, \langle 500, 0.16, 0.75 \rangle, \langle 1000, 0.64, 0.19 \rangle, \langle 1200, 1, 0 \rangle \}.$$

Example 2.5. Consider the universe $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. Let A and B be two intuitionistic fuzzy sets of X given by

$$A = \{ \langle a_1, 0.2, 0.6 \rangle, \langle a_2, 0.3, 0.7 \rangle, \langle a_3, 1, 0 \rangle, \langle a_4, 0.8, 0.1 \rangle, \langle a_5, 0.5, 0.4 \rangle \}$$

and

$$B = \{ \langle a_1, 0.4, 0.4 \rangle, \langle a_2, 0.5, 0.2 \rangle, \langle a_3, 0.6, 0.2 \rangle, \langle a_4, 0.1, 0.7 \rangle, \langle a_5, 0, 1 \rangle \}.$$

Then,

$$\begin{aligned}
 A^c &= \{ \langle a_1, 0.6, 0.2 \rangle, \langle a_2, 0.7, 0.3 \rangle, \langle a_3, 0, 1 \rangle, \\
 &\quad \langle a_4, 0.1, 0.8 \rangle, \langle a_5, 0.4, 0.5 \rangle \}, \\
 A \cap B &= \{ \langle a_1, 0.2, 0.6 \rangle, \langle a_2, 0.3, 0.7 \rangle, \langle a_3, 0.6, 0.2 \rangle, \\
 &\quad \langle a_4, 0.1, 0.7 \rangle, \langle a_5, 0, 1 \rangle \}, \\
 A \cup B &= \{ \langle a_1, 0.4, 0.4 \rangle, \langle a_2, 0.5, 0.2 \rangle, \langle a_3, 1, 0 \rangle, \\
 &\quad \langle a_4, 0.8, 0.1 \rangle, \langle a_5, 0.5, 0.4 \rangle \}, \\
 \Box A &= \{ \langle a_1, 0.2, 0.8 \rangle, \langle a_2, 0.3, 0.7 \rangle, \langle a_3, 1, 0 \rangle, \\
 &\quad \langle a_4, 0.8, 0.2 \rangle, \langle a_5, 0.5, 0.5 \rangle \}, \\
 \Diamond B &= \{ \langle a_1, 0.6, 0.4 \rangle, \langle a_2, 0.8, 0.2 \rangle, \langle a_3, 0.8, 0.2 \rangle, \\
 &\quad \langle a_4, 0.3, 0.7 \rangle, \langle a_5, 0, 1 \rangle \}.
 \end{aligned}$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.

Unless or otherwise stated throughout this paper S stands for a one sided Γ -semigroup.

Definition 2.1. A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is called an intuitionistic fuzzy subsemigroup of S if it satisfies:

- (1) $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$,
- (2) $\nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$.

Definition 2.2. A intuitionistic fuzzy subsemigroup $A = (\mu_A, \nu_A)$ of S is called an intuitionistic fuzzy interior ideal of S if it satisfies:

- (1) $\mu_A(x\alpha a\beta y) \geq \mu_A(a)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$,
- (2) $\nu_A(x\alpha a\beta y) \leq \nu_A(a)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$.

Definition 2.3. A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is called an intuitionistic fuzzy left ideal of S if it satisfies:

- (1) $\mu_A(x\gamma y) \geq \mu_A(y)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$,
- (2) $\nu_A(x\gamma y) \leq \nu_A(y)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$.

A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is called an intuitionistic fuzzy right ideal of S if it satisfies:

- (1) $\mu_A(x\gamma y) \geq \mu_A(x)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$,
- (2) $\nu_A(x\gamma y) \leq \nu_A(x)$, $\forall x, y \in S$ and $\forall \gamma \in \Gamma$.

A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is called an intuitionistic fuzzy ideal of S if it is an intuitionistic fuzzy left and intuitionistic fuzzy right ideal of S .

Example 2.6. Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers. Then, S is a Γ -semigroup where $a\gamma b$ denote the usual multiplication of integers a, γ, b with $a, b \in S$ and $\gamma \in \Gamma$. Let $A = (\mu_A, \nu_A)$

be an intuitionistic fuzzy subset of S , defined as follows

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0.1 & \text{if } x = -1, -2 \\ 0 & \text{if } x = -3, -6 \\ 0.4 & \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0 & \text{if } x = 0 \\ 0.8 & \text{if } x = -1, -2 \\ 1 & \text{if } x = -3, -6 \\ 0.5 & \text{otherwise.} \end{cases}$$

Then, the intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is an intuitionistic fuzzy interior ideal of S which is not an intuitionistic fuzzy ideal of S .

Proposition 2.1. *If $\{A_i\}_{i \in \Lambda}$ is a family of intuitionistic fuzzy interior ideals of a Γ -semigroup S , then $\bigcap_{i \in \Lambda} A_i$ is an intuitionistic fuzzy interior ideal of S , provided it is non-empty.*

Proof. Let $B = \bigcap_{i \in \Lambda} A_i$ and $x, a, y \in S, \alpha, \beta, \gamma \in \Gamma$. Then,

$$\begin{aligned} \mu_B(x\gamma y) &= \inf_{i \in \Lambda} \{\mu_{A_i}(x\gamma y)\} \\ &\geq \inf_{i \in \Lambda} \{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\} \\ &= \min \left\{ \inf_{i \in \Lambda} \{\mu_{A_i}(x)\}, \inf_{i \in \Lambda} \{\mu_{A_i}(y)\} \right\} \\ &= \min\{\mu_B(x), \mu_B(y)\} \end{aligned}$$

Also, we have

$$\begin{aligned} \nu_B(x\gamma y) &= \sup_{i \in \Lambda} \{\nu_{A_i}(x\gamma y)\} \\ &\leq \sup_{i \in \Lambda} \{\max\{\nu_{A_i}(x), \nu_{A_i}(y)\}\} \\ &= \max \left\{ \sup_{i \in \Lambda} \{\nu_{A_i}(x)\}, \sup_{i \in \Lambda} \{\nu_{A_i}(y)\} \right\} \\ &= \max\{\nu_B(x), \nu_B(y)\}. \end{aligned}$$

Hence, $\bigcap_{i \in \Lambda} A_i$ is an intuitionistic fuzzy subsemigroup of S . Again,

$$\mu_B(a) = \inf_{i \in \Lambda} \{\mu_{A_i}(a)\} \leq \inf_{i \in \Lambda} \{\mu_{A_i}(x\alpha a\beta y)\} = \mu_B(x\alpha a\beta y).$$

Also, we have

$$\nu_B(a) = \sup_{i \in \Lambda} \{\nu_{A_i}(a)\} \geq \sup_{i \in \Lambda} \{\nu_{A_i}(x\alpha a\beta y)\} = \nu_B(x\alpha a\beta y).$$

Therefore, $\bigcap_{i \in \Lambda} A_i$ is an intuitionistic fuzzy interior ideal of S . □

Lemma 2.1. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , then so is $\Box A = (\mu_A, \mu_A^c)$.*

Proof. It is sufficient to show that μ_A^c satisfies the conditions (2) of Definitions 2.1 and 2.2. For $x, y \in S$ and $\alpha, \beta \in \Gamma$ we have

$$\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$$

which implies that

$$1 - \mu_A(x\gamma y) \leq 1 - \min\{\mu_A(x), \mu_A(y)\}$$

and so

$$\mu_A^c(x\gamma y) \leq \max\{\mu_A^c(x), \mu_A^c(y)\}.$$

Hence, $\Box A = (\mu_A, \mu_A^c)$ is an intuitionistic fuzzy subsemigroup of S . Again, for $x, y, a \in S$ and $\alpha, \beta \in \Gamma$

$$\mu_A(x\alpha a\beta y) \geq \mu_A(a)$$

which implies that

$$1 - \mu_A(x\alpha a\beta y) \leq 1 - \mu_A(a)$$

and so

$$\mu_A^c(x\alpha a\beta y) \leq \mu_A^c(a).$$

Therefore, $\Box A = (\mu_A, \mu_A^c)$ is an intuitionistic fuzzy interior ideal of S . \square

Lemma 2.2. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , then so is $\Diamond A = (\nu_A^c, \nu_A)$.*

Proof. The proof is similar to the proof of Lemma 2.1. \square

Combining Lemmas 2.1 and 2.2, we have following theorem.

Theorem 2.1. *$A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S if and only if $\Box A$ and $\Diamond A$ are intuitionistic fuzzy interior ideals of S .*

Corollary 2.1. *An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is an intuitionistic fuzzy interior ideal of S if and only if μ_A and ν_A^c are fuzzy interior ideals of S .*

Definition 2.4. *For any $t \in [0, 1]$ and a fuzzy subset μ of S , the set*

$$U(\mu; t) = \{x \in S : \mu(x) \geq t\} \text{ (respectively, } L(\mu; t) = \{x \in S : \mu(x) \leq t\})$$

is called an upper (respectively, lower) t -level cut of μ .

Example 2.7. *Let $S = \{a, b, c\}$ and $\Gamma = \{\gamma, \delta\}$, where γ, δ is defined on S with the following caley table:*

γ	a	b	c		δ	a	b	c
a	a	a	a	and	a	a	a	a
b	b	b	b		b	b	b	b
c	c	c	c		c	c	c	c

Then, S is a Γ -semigroup. Let us consider a fuzzy subset $\mu : S \rightarrow [0, 1]$, by $\mu(a) = 0.8, \mu(b) = 0.7, \mu(c) = 0.6$. For $t = 0.7$, $U(\mu; t) = \{a, b\}$ and $L(\mu; t) = \{b, c\}$.

Theorem 2.2. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , then the upper and lower level cuts $U(\mu_A; t)$ and $L(\nu_A; t)$ are interior ideals of S for every $t \in Im(\mu_A) \cap Im(\nu_A)$.

Proof. Let $t \in Im(\mu_A) \cap Im(\nu_A)$ and let $x, y \in U(\mu_A; t)$ and $\gamma \in \Gamma$. Then, $\mu_A(x) \geq t$ and $\mu_A(y) \geq t$. Now, since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , it is an intuitionistic fuzzy subsemigroup of S , hence $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$. Consequently, $x\gamma y \in U(\mu_A; t)$. Hence, $U(\mu_A; t)$ is a subsemigroup of S .

Again, let $x, y \in S, a \in U(\mu_A; t)$ and $\alpha, \beta \in \Gamma$. Then, $\mu_A(a) \geq t$. Now, since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , hence $\mu_A(x\alpha a\beta y) \geq \mu_A(a) \geq t$. Consequently, $x\alpha a\beta y \in U(\mu_A; t)$. Hence, $U(\mu_A; t)$ is an interior ideal of S .

Similarly, let $x, y \in L(\nu_A; t)$ and $\gamma \in \Gamma$. Then, $\nu_A(x) \leq t$ and $\nu_A(y) \leq t$. Now, since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , it is an intuitionistic fuzzy subsemigroup of S , hence $\nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq t$. Consequently, $x\gamma y \in L(\nu_A; t)$. Hence, $L(\nu_A; t)$ is a subsemigroup of S .

Again, let $x, y \in S, a \in L(\nu_A; t)$ and $\alpha, \beta \in \Gamma$. Then, $\nu_A(a) \leq t$. Now, since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , hence $\nu_A(x\alpha a\beta y) \leq \nu_A(a) \leq t$. Consequently, $x\alpha a\beta y \in L(\nu_A; t)$. Hence, $L(\nu_A; t)$ is an interior ideal of S . \square

Theorem 2.3. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of S such that the non-empty sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are interior ideals of S for all $t \in [0, 1]$. Then, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S .

Proof. For $t \in [0, 1]$, let us assume that the non-empty sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are interior ideals of S . Now, we shall show that $A = (\mu_A, \nu_A)$ satisfies the conditions of Definitions 2.1 and 2.2. Let $x, y \in S$ and $\gamma \in \Gamma$. Let $t_0 = \min\{\mu_A(x), \mu_A(y)\}$ and $t_1 = \max\{\nu_A(x), \nu_A(y)\}$. Then, $x, y \in U(\mu_A; t_0)$ and $x, y \in L(\nu_A; t_1)$. So $x\gamma y \in U(\mu_A; t_0)$ and $x\gamma y \in L(\nu_A; t_1)$ which implies that $\mu_A(x\gamma y) \geq t_0 = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x\gamma y) \leq t_1 = \max\{\nu_A(x), \nu_A(y)\}$. Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of S .

Again, let $x, a, y \in S$ and $\alpha, \beta \in \Gamma$. Let $t_2 = \mu_A(a)$ and $t_3 = \nu_A(a)$. Then, $a \in U(\mu_A; t_2)$ and $a \in L(\nu_A; t_3)$. So $x\alpha a\beta y \in U(\mu_A; t_2)$ and $x\alpha a\beta y \in L(\nu_A; t_3)$ which implies that $\mu_A(x\alpha a\beta y) \geq t_2 = \mu_A(a)$ and $\nu_A(x\alpha a\beta y) \leq t_3 = \nu_A(a)$. Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S . \square

Corollary 2.2. *Let I be an interior ideal of a Γ -semigroup S . If two fuzzy subsets μ and ν are defined on S by*

$$\mu(x) := \begin{cases} \alpha_0 & \text{if } x \in I \\ \alpha_1 & \text{if } x \in S - I \end{cases}$$

and

$$\nu(x) := \begin{cases} \beta_0 & \text{if } x \in I \\ \beta_1 & \text{if } x \in S - I, \end{cases}$$

where $0 \leq \alpha_1 < \alpha_0, 0 \leq \beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$. Then, $A = (\mu, \nu)$ is an intuitionistic fuzzy interior ideal of S and $U(\mu; \alpha_0) = I = L(\nu; \beta_0)$.

Corollary 2.3. *Let χ_I be the characteristic function of an interior ideal I of S . Then, $I = (\chi_I, \chi_I^c)$ is an intuitionistic fuzzy interior ideal of S .*

Example 2.8. *Let S denote the set of all integers of the form $4n + 1$ and Γ denote the set of all integers of the form $4n + 3$ where n is an integer. Then, S is a regular Γ -semigroup where $a + \alpha + b$ denote the usual sum of integers a, α, b with $a, b \in S$ and $\alpha \in \Gamma$. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S , defined as follows*

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = 4n + 1 \\ 0.3 & \text{if } x = 4n + 3 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0 & \text{if } x = 4n + 1 \\ 0.6 & \text{if } x = 4n + 3 \end{cases}$$

Then, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S which is also an intuitionistic fuzzy ideal of S .

By routine verification we can have the following theorem.

Theorem 2.4. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of a Γ -semigroup S , then for all $x \in S$ we have $\mu_A(x) = \sup\{\alpha \in [0, 1] : x \in U(\mu_A : \alpha)\}$ and $\nu_A(x) = \inf\{\alpha \in [0, 1] : x \in L(\nu_A : \alpha)\}$.*

Let Φ be a mapping from a set X to a set Y . Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X and $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy subset of Y . Then, the *inverse image* $\Phi^{-1}(B) = (\Phi^{-1}(\mu_B), \Phi^{-1}(\nu_B))$ is the intuitionistic fuzzy subset of X defined by

$$\Phi^{-1}(B)(x) = (\Phi^{-1}(\mu_B)(x), \Phi^{-1}(\nu_B)(x)) = (\mu_B(\Phi(x)), \nu_B(\Phi(x))).$$

The *image* $\Phi(A) = (\Phi(\mu_A), \Phi(\nu_A))$ is the intuitionistic fuzzy subset of Y defined by

$$\Phi(\mu_A)(x) = \begin{cases} \sup\{\mu_A(t) : t \in \Phi^{-1}(y)\} & \text{if } \Phi^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

and

$$\Phi(\nu_A)(x) = \begin{cases} \inf\{\nu_A(t) : t \in \Phi^{-1}(y)\} & \text{if } \Phi^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

Definition 2.5. A mapping Φ from a Γ -semigroup S to another Γ -semigroup T is called a homomorphism if

$$\Phi(x\gamma y) = \Phi(x)\gamma\Phi(y) \text{ for all } x, y \in S \text{ and } \gamma \in \Gamma.$$

Proposition 2.2. Let $\Phi : S \rightarrow T$ be a homomorphism of Γ -semigroups. Then,

- (1) If $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy interior ideal of T , then $\Phi^{-1}(B) = (\Phi^{-1}(\mu_B), \Phi^{-1}(\nu_B))$ is an intuitionistic fuzzy interior ideal of S .
- (2) If Φ is onto and $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , then $\Phi(A) = (\Phi(\mu_A), \Phi(\nu_A))$ is an intuitionistic fuzzy interior ideal of T .

Proof. (1) Let $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy interior ideal of T . Then, $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy subsemigroup of T . Let $x, y \in S$ and $\gamma \in \Gamma$. Then,

$$\begin{aligned} \Phi^{-1}(\mu_B)(x\gamma y) &= \mu_B(\Phi(x\gamma y)) = \mu_B(\Phi(x)\gamma\Phi(y)) \\ &\quad (\text{since } \Phi \text{ is a homomorphism of } \Gamma\text{-semigroups}) \\ &\geq \min\{\mu_B(\Phi(x)), \mu_B(\Phi(y))\} \\ &= \min\{\Phi^{-1}(\mu_B)(x), \Phi^{-1}(\mu_B)(y)\} \end{aligned}$$

and

$$\begin{aligned} \Phi^{-1}(\nu_B)(x\gamma y) &= \nu_B(\Phi(x\gamma y)) = \nu_B(\Phi(x)\gamma\Phi(y)) \\ &\quad (\text{since } \Phi \text{ is a homomorphism of } \Gamma\text{-semigroups}) \\ &\leq \max\{\nu_B(\Phi(x)), \nu_B(\Phi(y))\} \\ &= \max\{\Phi^{-1}(\nu_B)(x), \Phi^{-1}(\nu_B)(y)\}. \end{aligned}$$

Thus, $\Phi^{-1}(B) = (\Phi^{-1}(\mu_B), \Phi^{-1}(\nu_B))$ is an intuitionistic fuzzy subsemigroup of S . Again, let $x, y, a \in S$ and $\alpha, \beta \in \Gamma$. Then,

$$\begin{aligned} \Phi^{-1}(\mu_B)(x\alpha a\beta y) &= \mu_B(\Phi(x\alpha a\beta y)) \\ &= \mu_B(\Phi(x)\alpha\Phi(a)\beta\Phi(y)) \\ &\quad (\text{since } \Phi \text{ is a homomorphism of } \Gamma\text{-semigroups}) \\ &\geq \mu_B(\Phi(a)) = \Phi^{-1}(\mu_B)(a) \end{aligned}$$

and

$$\begin{aligned} \Phi^{-1}(\nu_B)(x\alpha a\beta y) &= \nu_B(\Phi(x\alpha a\beta y)) \\ &= \nu_B(\Phi(x)\alpha\Phi(a)\beta\Phi(y)) \\ &\quad (\text{since } \Phi \text{ is a homomorphism of } \Gamma\text{-semigroups}) \\ &\leq \nu_B(\Phi(a)) = \Phi^{-1}(\nu_B)(a). \end{aligned}$$

Therefore, $\Phi^{-1}(B) = (\Phi^{-1}(\mu_B), \Phi^{-1}(\nu_B))$ is an intuitionistic fuzzy interior ideal of S .

(2) Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy interior ideal of S . Then, it is an intuitionistic fuzzy subsemigroup of S . Since $(\Phi(\mu_A))(x') = \sup_{\Phi(x)=x'} \{\mu_A(x)\}$ for $x' \in T$ and $(\Phi(\nu_A))(x') = \inf_{\Phi(x)=x'} \{\nu_A(x)\}$ for $x' \in T$, so $\Phi(A) = (\Phi(\mu_A), \Phi(\nu_A))$ is non-empty. Let $x', y' \in T$ and $\gamma \in \Gamma$. Then,

$$\begin{aligned} (\Phi(\mu_A))(x' \gamma y') &= \sup_{\Phi(z)=x' \gamma y'} \{\mu_A(z)\} \\ &\geq \sup_{\Phi(x)=x', \Phi(y)=y'} \{\mu_A(x \gamma y)\} \\ &\geq \sup_{\Phi(x)=x', \Phi(y)=y'} \{\min\{\mu_A(x), \mu_A(y)\}\} \\ &= \min \left\{ \sup_{\Phi(x)=x'} \{\mu_A(x)\}, \sup_{\Phi(y)=y'} \{\mu_A(y)\} \right\} \\ &= \min \left\{ (\Phi(\mu_A))(x'), (\Phi(\mu_A))(y') \right\} \end{aligned}$$

and

$$\begin{aligned} (\Phi(\nu_A))(x' \gamma y') &= \inf_{\Phi(z)=x' \gamma y'} \{\nu_A(z)\} \\ &\leq \inf_{\Phi(x)=x', \Phi(y)=y'} \{\nu_A(x \gamma y)\} \\ &\leq \inf_{\Phi(x)=x', \Phi(y)=y'} \{\max\{\nu_A(x), \nu_A(y)\}\} \\ &= \max \left\{ \inf_{\Phi(x)=x'} \{\nu_A(x)\}, \inf_{\Phi(y)=y'} \{\nu_A(y)\} \right\} \\ &= \max \left\{ (\Phi(\nu_A))(x'), (\Phi(\nu_A))(y') \right\}. \end{aligned}$$

Therefore, $\Phi(A) = (\Phi(\mu_A), \Phi(\nu_A))$ is an intuitionistic fuzzy subsemigroup of T . Again, let $x', y', a' \in T$ and $\alpha, \beta \in \Gamma$. Then,

$$\begin{aligned} (\Phi(\mu_A))(x' \alpha a' \beta y') &= \sup_{\Phi(z)=x' \alpha a' \beta y'} \{\mu_A(z)\} \\ &\geq \sup_{\Phi(x)=x', \Phi(a)=a', \Phi(y)=y'} \{\mu_A(x \alpha a \beta y)\} \\ &\geq \sup_{\Phi(a)=a'} \{\mu_A(a)\} \\ &= (\Phi(\mu_A))(a') \end{aligned}$$

and

$$\begin{aligned}
\Phi(\nu_A)(x' \alpha a' \beta y') &= \inf_{\Phi(z)=x' \alpha a' \beta y'} \{\nu_A(z)\} \\
&\leq \inf_{\Phi(x)=x', \Phi(a)=a', \Phi(y)=y'} \{\nu_A(x \alpha a \beta y)\} \\
&\leq \inf_{\Phi(a)=a'} \{\nu_A(a)\} \\
&= (\Phi(\nu_A))(a').
\end{aligned}$$

Therefore, $\Phi(A) = (\Phi(\mu_A), \Phi(\nu_A))$ is an intuitionistic fuzzy interior ideal of T . \square

Proposition 2.3. *Let S be a Γ -semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S . Then, A is an intuitionistic fuzzy interior ideal of S .*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S . Let $x, y \in S$ and $\gamma \in \Gamma$. Then, $\mu_A(x\gamma y) \geq \mu_A(x)$ and $\mu_A(x\gamma y) \geq \mu_A(y)$, which implies that $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$. Again, we have $\nu_A(x\gamma y) \leq \nu_A(x)$ and $\nu_A(x\gamma y) \leq \nu_A(y)$, which implies $\nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$. Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of S .

Now, let $x, a, y \in S$ and $\alpha, \beta \in \Gamma$. Then, $\mu_A(x\alpha a\beta y) = \mu_A(x\alpha(a\beta y)) \geq \mu_A(a\beta y) \geq \mu_A(a)$ and $\nu_A(x\alpha a\beta y) = \nu_A(x\alpha(a\beta y)) \leq \nu_A(a\beta y) \leq \nu_A(a)$.

Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S . \square

Regarding the converse we have the following result.

An element $x \in S$ is called *regular* in S if $x \in x\Gamma S\Gamma x$, where $x\Gamma S\Gamma x = \{x\alpha a\beta x \mid a \in S, \alpha, \beta \in \Gamma\}$. A Γ -semigroup S is called *regular* if every element is regular.

Proposition 2.4. *Let S be a regular Γ -semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy interior ideal of S . Then, A is an intuitionistic fuzzy ideal of S .*

Proof. Let $x, y \in S, \gamma \in \Gamma$. Since S is regular, for any $x \in S$ there exist $a \in S$ and $\alpha, \beta \in \Gamma$ such that $x = x\alpha a\beta x$. Then, $\mu_A(x\gamma y) = \mu_A(x\alpha a\beta x\gamma y) \geq \mu_A(x)$ and $\nu_A(x\gamma y) = \nu_A(x\alpha a\beta x\gamma y) \leq \nu_A(x)$. So $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy right ideal of S .

Similarly, we can prove that $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy left ideal of S . Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of S . \square

Remark 2.1. *From the above two propositions it is clear that in regular Γ -semigroups the concept of intuitionistic fuzzy ideals and intuitionistic fuzzy interior ideals coincide.*

Definition 2.6. *An interior ideal M of a Γ -semigroup S is called a characteristic interior ideal of S if $f(M) = M$ for all $f \in \text{Aut}(S)$.*

Example 2.9. Let S be the set of all 2×3 matrices and Γ be the set of all 3×2 matrices over the ring of integers. Then, S is a Γ -semigroup with respect to usual matrix multiplication. Let us define $f : S \rightarrow S$ by $f(A_{2,3}) = 2A_{2,3}$. Then, $f \in \text{Aut}(S)$. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S defined as follows:

$$\mu_A(x) = \begin{cases} 0.3 & \text{if } x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ 0.2 & \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.6 & \text{if } x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ 0.8 & \text{otherwise} \end{cases}$$

Then, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy characteristic interior ideal of S .

Definition 2.7. An intuitionistic fuzzy interior ideal $A = (\mu_A, \nu_A)$ of a Γ -semigroup S is called an intuitionistic fuzzy characteristic interior ideal of S if $\mu_A(f(x)) = \mu_A(x)$ and $\nu_A(f(x)) = \nu_A(x) \forall x \in S$ and $\forall f \in \text{Aut}(S)$.

Theorem 2.5. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy characteristic interior ideal of S then the upper and lower level cuts $U(\mu_A; t)$ and $L(\nu_A; t)$ are characteristic interior ideals of S for every $t \in \text{Im}(\mu_A) \cap \text{Im}(\nu_A)$.

Proof. Let $t \in \text{Im}(\mu_A) \cap \text{Im}(\nu_A)$. Then, $U(\mu_A; t)$ and $L(\nu_A; t)$ are interior ideals of S . Let $f \in \text{Aut}(S)$ and $x \in U(\mu_A; t)$. Then, $\mu_A(x) \geq t$ and so $\mu_A(f(x)) \geq t$. Hence, $f(x) \in U(\mu_A; t)$. This implies that $f(U(\mu_A; t)) \subset U(\mu_A; t)$. Again, let $x \in U(\mu_A; t)$ and $y \in S$ such that $f(y) = x$. Then, $\mu_A(y) = \mu_A(f(y)) = \mu_A(x) \geq t$, so $y \in U(\mu_A; t)$. Consequently, $f(y) \in f(U(\mu_A; t))$, whence $x \in f(U(\mu_A; t))$. Hence, $U(\mu_A; t) \subset f(U(\mu_A; t))$. So, we have $U(\mu_A; t) = f(U(\mu_A; t))$. Hence, $U(\mu_A; t)$ is a characteristic interior ideal of S . By using similar argument we can show that $L(\nu_A; t)$ is a characteristic interior ideal of S . \square

Theorem 2.6. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of S such that the non-empty sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are characteristic interior ideals of S for all $t \in [0, 1]$. Then, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy characteristic interior ideal of S .

Proof. Let $U(\mu_A; t)$ and $L(\nu_A; t)$ are characteristic interior ideals of S for all $t \in [0, 1]$. Let $f \in \text{Aut}(S)$, $x \in S$ and $\mu_A(x) := t_0$ and $\nu_A(x) := t_1$. Then, $x \in U(\mu_A; t_0)$ and $x \in L(\nu_A; t_1)$. Since, by hypothesis, $U(\mu_A; t_0) = f(U(\mu_A; t_0))$ and $L(\nu_A; t_1) = f(L(\nu_A; t_1))$, so we see that $f(x) \in U(\mu_A; t_0)$ and $f(x) \in L(\nu_A; t_1)$. Hence, $\mu_A(f(x)) \geq t_0$ and $\nu_A(f(x)) \leq t_1$. Let $t_2 = \mu_A(f(x))$ and $t_3 = \nu_A(f(x))$. Then, $t_2 \geq t_0$ and $t_3 \leq t_1$ and $f(x) \in U(\mu_A; t_2) = f(U(\mu_A; t_2))$, $f(x) \in L(\nu_A; t_3) = f(L(\nu_A; t_3))$. Since f is one-one, we have $x \in U(\mu_A; t_2)$ and $x \in L(\nu_A; t_3)$. This implies that $\mu_A(x) \geq t_2$ and $\nu_A(x) \leq t_3$. Hence, $t_0 \geq t_2$

and $t_1 \leq t_3$. Thus, we obtain $\mu_A(f(x)) = \mu_A(x)$ and $\nu_A(f(x)) = \nu_A(x)$. Hence, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy characteristic interior ideal of S . \square

Using Corollary 2.3 we obtain the following corollary.

Corollary 2.4. *Let χ_I be the characteristic function of a characteristic interior ideal I of S . Then, $I = (\chi_I, \chi_I^c)$ is an intuitionistic fuzzy characteristic interior ideal of S .*

Definition 2.8. *A Γ -semigroup is said to be simple if it does not contain any proper ideal.*

Example 2.10. *Let S be the set of all integers of the form $4n + 1$ and Γ be the set of all integers of the form $4n + 3$ where n is any integer. Then, S is a simple Γ -semigroup.*

Definition 2.9. *A Γ -semigroup is said to be intuitionistic fuzzy simple if every intuitionistic fuzzy ideal of S is a constant function.*

Lemma 2.3. *A Γ -semigroup S is simple if and only if it is intuitionistic fuzzy simple.*

Proof. Let S be a Γ -semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy interior ideal of S . Let $a, b \in S$. Then, by Theorem 3.8 [21], $U(\mu_A, \mu_A(a))$, $U(\mu_A, \mu_A(b))$, $L(\nu_A, \nu_A(a))$ and $L(\nu_A, \nu_A(b))$ are ideals of S . Since S is simple, we have $U(\mu_A, \mu_A(a)) = S = U(\mu_A, \mu_A(b))$ and $L(\nu_A, \nu_A(a)) = S = L(\nu_A, \nu_A(b))$. Consequently, it follows that $a \in U(\mu_A, \mu_A(b))$, $L(\nu_A, \nu_A(b))$ and $b \in U(\mu_A, \mu_A(a))$, $L(\nu_A, \nu_A(a))$, which implies that $\mu_A(a) \geq \mu_A(b)$, $\nu_A(a) \leq \nu_A(b)$ and $\mu_A(b) \geq \mu_A(a)$, $\nu_A(b) \leq \nu_A(a)$. So, $\mu_A(a) = \mu_A(b)$ and $\nu_A(a) = \nu_A(b)$ for all $a, b \in S$. Hence, S is intuitionistic fuzzy simple.

Conversely, suppose that S is intuitionistic fuzzy simple. Let I be an ideal of S . Then, (χ_I, χ_I^c) is an intuitionistic fuzzy ideal of S (cf. Corollary 3.11 [21]). Let $x \in S$. Since S is intuitionistic fuzzy simple, the intuitionistic fuzzy ideal (χ_I, χ_I^c) is a constant function, and so $\mu_I(x) = \mu_I(a)$ and $\nu_I(x) = \nu_I(a) \forall a \in I$. Hence, $(\chi_I(x), \chi_I^c(x)) = (1, 0)$, whence $x \in I$. Thus, we have $S = I$. Hence, the Γ -semigroup is simple. \square

Theorem 2.7. *A Γ -semigroup is simple if and only if every intuitionistic fuzzy interior ideal of S is constant.*

Proof. Let S be a simple Γ -semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy interior ideal of S . Let $a, b \in S$. Since S is simple, each of $\langle a \rangle$ and $\langle b \rangle$, the principal ideals generated by a and b , respectively, is equal to S . Hence, there exist $x, y, u, v \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $a = x\alpha b\beta y$ and $b = u\gamma a\delta v$. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S , so we have $\mu_A(x\alpha b\beta y) \geq \mu_A(b)$, $\mu_A(u\gamma a\delta v) \geq \mu_A(a)$ and $\nu_A(x\alpha b\beta y) \leq \nu_A(b)$, $\nu_A(u\gamma a\delta v) \leq \nu_A(a)$. Thus, we see that $\mu_A(a) \geq \mu_A(b)$, $\mu_A(b) \geq \mu_A(a)$ and $\nu_A(a) \leq \nu_A(b)$, $\nu_A(b) \leq \nu_A(a)$. hence $\mu_A(a) = \mu_A(b)$ and $\nu_A(a) = \nu_A(b)$. Consequently, $A = (\mu_A, \nu_A)$ is constant.

Conversely, suppose that every intuitionistic fuzzy interior ideal of S is constant. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S . Then, by Proposition 2.3, $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of S . Hence, by hypothesis $A = (\mu_A, \nu_A)$ is constant. Thus, S is intuitionistic fuzzy simple. Therefore, by Lemma 2.3, S is a simple Γ -semigroup. \square

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