

## DIMENSIONAL ANALYSIS OF THE PHOTONIC CRYSTALS

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*Aceasta lucrare prezintă structurile de benzi fotonice obținute pentru o serie de cristale 1D, 2D și 3D formate dintr-o succesiune de 3 straturi subțiri de  $SiO_2$  și  $TiO_2$  depuse într-o ordine dată de seria aperiodică Thue-Morse. Combinăriile alese de noi prezintă benzi fotonice interzise largi pentru cristalul fotonice unidimensional în cazul modurilor TM, TE și cuplate, benzi fotonice interzise înguste pentru modul TM al cristalului fotonice bidimensional și nu prezintă benzi pentru structura tridimensională.*

*This paper presents the photonic bands structures for 1D, 2D and 3D photonic crystals built as a stack of eight thin layers of  $SiO_2$  and  $TiO_2$  deposited in an order given by Thue-Morse aperiodic sequence. Our combinations give large photonic band gaps for 1D photonic crystal for each TM, TE and coupled modes, small photonic band gaps for TM mode of 2D photonic crystal and no photonic band gaps for the 3D structure.*

**Keywords:** photonic crystals, Thue-Morse sequence, band gaps

### 1. Introduction

Photonic crystals are artificial structures created by analogy with solid crystals and displaying band gaps for photons in the same manner as semiconductor materials display for electrons. A photonic crystal is made by a lattice, photonic atoms and photons. As in solid crystals, the lattice is described by its Brillouin zone. We call « photonic atoms » material domains having different dielectric constant comparing to the background which are arranged in a periodic structure controlled by the lattice type. Photons are propagating within this structure in the same manner as electrons in solid crystal only if the photonic lattice dimension is of the same order of magnitude as the wavelength of the incident light.

Over the last years, a lot of work was done in this field, in order to investigate the photonic bands structure of different materials in different geometries [1,2]. One of the most important parameter that influences the band gap width of the photonic crystal is the lattice geometry. It is already known that

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lattice geometry with reduced symmetry gives the larger band gap. In this context, more and more aperiodic structures, like Thue-Morse sequence, are in study [3].

In this paper, we investigate the structure of photonic bands for photonic crystals with a square lattice of square photonic atoms made of stacks of 8 layers arranged in an aperiodic structure. The analysis will be made for 1D, 2D and 3D photonic crystals generated from 2 kinds of layers, one of  $\text{TiO}_2$  and the other of  $\text{SiO}_2$  ordered using Thue-Morse sequence.

The computation of the photonic bands is made by means of frequency-domain planewave method [4]. In the following, we consider that  $xy$  axes are in the same plane as the 2D crystal (see figure 2a) and so, the TM mode has the electric field in the  $Oz$  direction.

## 2. The 1D photonic crystal

The 1D photonic crystal is made by a stack of two kinds of thin layers: the type A, a thin film of  $\text{SiO}_2$  with the refractive index  $n_1=1.45$  and type B, a layer of  $\text{TiO}_2$  (rutile variety) having the refractive index  $n_2=2.65$ . The order of deposited layers is given by Thue-Morse sequence [3], which is recurrent but not periodic. For our analysis we choose an eight-layer stack with the following structure: ABBABAAB, called Thue-Morse structure.

Taking the wavelength of incident radiation,  $\lambda_{\text{ref}}=1.55\mu\text{m}$ , the width of each kind of layer satisfies the following equations:

$$n_1 \cdot dA = n_2 \cdot dB = \frac{\lambda_{\text{ref}}}{4} \quad (1)$$

$$4dA + 4dB = a \quad (2)$$

where  $a$  is the height of the stack,  $dA$  and  $dB$  are the widths of layers A and B respectively. Solving equations (1) and (2), one obtains  $dA=0.1625a$  and  $dB=0.0875a$ .

The photonic bands of the 1D photonic crystal are presented in figure 1.

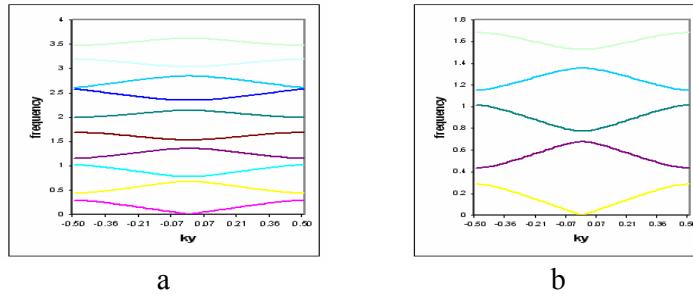


Fig. 1: Photonic bands structure for 1D photonic crystal:a) TM mode; b) coupled modes

From fig. 1 and table 1, one can remark for 1D Thue-Morse structure, 6 important band gaps having a width larger than 10%. The structure presents band gaps for the TM mode.

Table 1

The band Gaps of 1D Photonic crystals in  $k_x$  direction

	TM Mode		Coupled mode	
	Gap width (%)	Midgap frequency ( $2\pi c/a$ )	Gap width (%)	Midgap frequency ( $2\pi c/a$ )
<b>BG 1-2</b>	41.38	0.360502	-	-
<b>BG 2-3</b>	13.01	0.724741	41.38	0.360502
<b>BG 3-4</b>	12.64	1.08127	-	-
<b>BG 4-5</b>	12.06	1.440715	13.01	0.724747
<b>BG 5-6</b>	16.95	1.836636	-	-
<b>BG 6-7</b>	9.35	2.240297	12.64	1.08127

There are less band gaps for coupled mode, but also important as width.

## 2. The 2D photonic crystal

The 2D photonic crystal is made by a square lattice of Thue-Morse photonic atoms like that presented in figure 2a. These atoms have square geometry with dimension of  $h=0.4a$ , where  $a$  is the photonic lattice constant. They are made of a stack of 8 layers of  $\text{SiO}_2$  and  $\text{TiO}_2$  and the layers are infinite in  $z$  direction, nonhomogenous and asymmetric in respect with  $y$ -axis. The order of the deposited layers is also given by Thue-Morse sequence: ABBABABAAB.

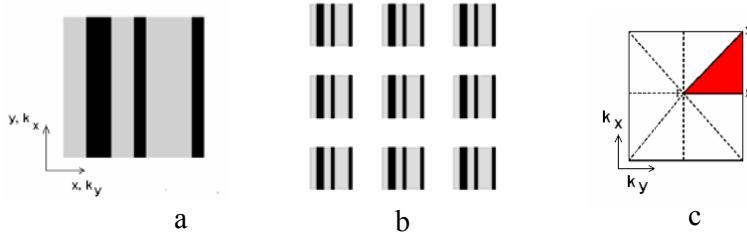


Fig. 2:a) photonic atom of 2D photonic crystal; b) square lattice of 2D photonic crystals; c) irreducible Brillouin zone of 2D photonic crystal; The gray color represents the type A thin film of  $\text{SiO}_2$  and the black, type B layer of  $\text{TiO}_2$

The band structure for the 2D photonic crystal is presented in figure 3a. The obtained bands will be compared with ones obtained from a square lattice of square atoms of rutile presented in figure 3b. The presented bands are calculated for square atoms having the edge  $h=0.4a$ .

Using the values given in table 2 and figure 3, one can observe that the first TM band gap of nonhomogenous atoms is displaced to higher frequency and

it has a smaller width due to a lower filling factor, 6.98%, as compared to 14.85% for homogeneous ones.

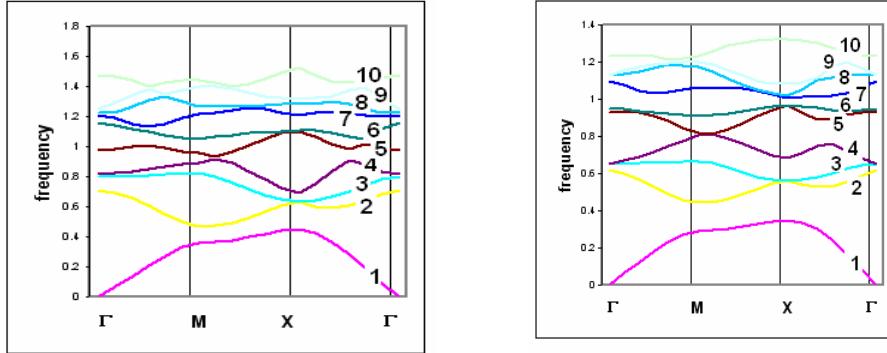


Fig. 3: Band gaps structure for 2D photonic crystal: a) with Thue-Morse type photonic atoms; b) with homogeneous atoms

The second TM band gap of Thue-Morse structure is displaced towards lower frequency than the corresponding one of the square lattice of rutile square photonic atoms.

Table 2:

**The Band Gap of 2D photonic crystals**

		Band Gap Width (%)	Midgap frequency ( $2\pi c/a$ )
Thue-Morse atoms	BG TM 1-2	5.24	0.46159
	BG TM 4-5	1.71	0.929982
rutile square atoms	BG TM 1-2	24.15	0.393979
	BG TM 6-7	4.27	1.205902

There are no band gaps for TE or coupled modes for 2D Thue-Morse structure. According to figure 4, both the width and the midgap frequency of first band gap of the 2D Thue-Morse structure diminish with increasing the relative dimension of the photonic atom.

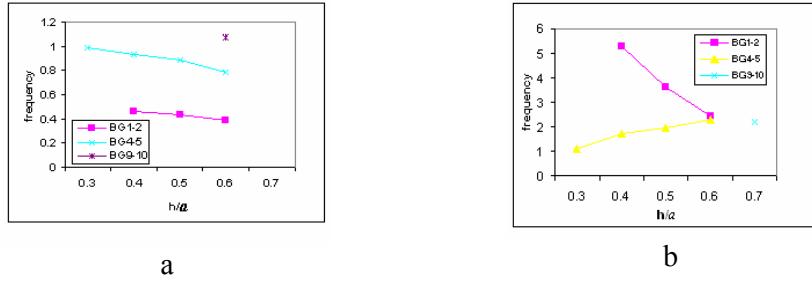


Fig. 4: a) The width of the band gaps as a function of dimension of atom; b) the midgap frequency as a function of dimension of atom

The width of the second band gap increases for  $h$  rising from  $0.3a$  to  $0.6a$ . For  $h$  greater than  $0.6a$ , these 2 band gaps disappear and a new band gap appears to a midgap frequency of 1.072863 in  $2\pi c/a$  units.

### 3. The 3D photonic crystal

The unit cell of the 3D photonic crystal is presented in figure 5 and is made of a stack of  $\text{SiO}_2$ ,  $\text{TiO}_2$  layers deposited in a Thue-Morse sequence order. Each layer is orthogonal on x-axis. The 3D photonic atom is a cube with the edge  $h=0.4a$  and the lattice geometry is cubic.

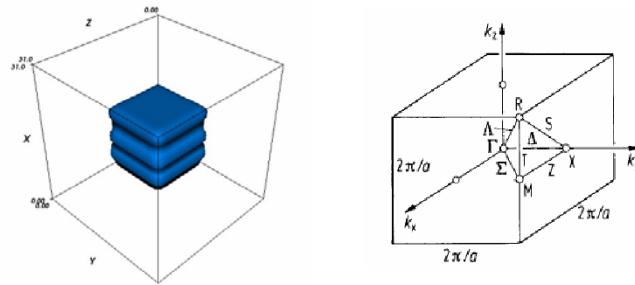


Fig. 5: Primitive cell and irreducible Brillouin zone of 3D photonic crystal

Fig. 6 presents the band structure for a 3D geometry of Thue-Morse cube atoms having the edges of  $h=0.4a$ , where  $a$  is the photonic lattice constant. The photonic bands structure for 3D Thue-Morse atoms is compared with a cubic lattice of cubic atoms of rutile having the edge  $h=0.4a$ . For 3D geometry, only the coupled modes are possible, but there are no band gaps for it.

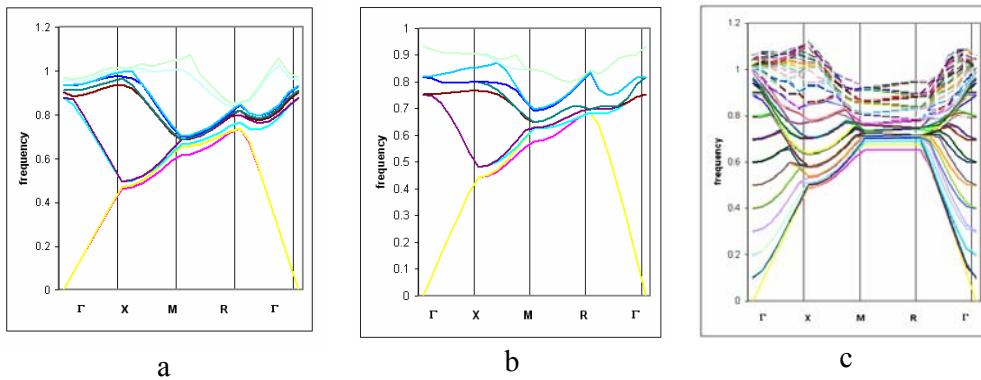


Fig. 6: Structure of 3D photonic crystals: a) square lattice of Thue-Morse atoms; b) square lattice of square atoms; c) square lattice of Thue-Morse atoms in a finite slab.

To simulate a slab with finite dimensions and the same geometry like the 2D photonic crystal presented in figure 2, we use a supercell having normalized dimensions of (1 1 10). The background of the supercell is air. The photonic bands for this structure are presented in figure 6c and one can observe that there are no band gaps for coupled modes.

## 6. Conclusions

The paper presents the bands structures for 1D, 2D and 3D photonic crystals made of eight  $\text{SiO}_2$  ( $n_1=1.45$ ) and  $\text{TiO}_2$  ( $n_2=2.65$ ) thin layers arranged according to Thue-Morse sequence. For a stack of infinite plane layers of  $\text{TiO}_2$ ,  $\text{SiO}_2$  representing 1D photonic crystal, we obtain large band gaps even for coupled modes. A 2D photonic crystal made by a square lattice of Thue-Morse atoms has only 2 small band gaps for TM mode that are more stable to the variations of atom dimensions than that made of homogeneous atoms of rutile.

The 3D Thue-Morse photonic crystal has no band gaps for any dimension of the atoms. The contrast index of this structure is smaller than for one of homogeneous atoms of rutile and is not large enough to allow band gaps for coupled modes in 3D, but it is large enough to have TM band gaps in 2D.

Using a 2D photonic crystal with presented geometry, one can obtain a photonic band gap centered at  $\lambda=1.55\mu\text{m}$  if we use a Thue-Morse sequence of  $\text{SiO}_2$  and  $\text{TiO}_2$  layers having 100.8nm and 54.20nm width respectively. The constant of square lattice have to be  $a=620\text{nm}$  and the dimension of square atoms,  $h=341\text{nm}$ .

## R E F E R E N C E S

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