

LATERAL-DIRECTIONAL OSCILLATORY-DEPARTURE CRITERIA FOR HIGH ANGLE-OF-ATTACK FLIGHT CONDITIONS

B.C. TEODORESCU*

Lucrarea prezentă abordează problema dezvoltării unor criterii analitice aproximative destinate predicției fenomenelor de pierdere a controlului avionului la incidențe mari de zbor. În mod concret, lucrarea tratează cazul pierderii controlului prin divergență oscilatorie a mișcării perturbate lateral-direcționale.

Spre deosebire de alte contribuții științifice, în care fenomenele de pierdere a controlului zborului sunt puse în relație cu apariția unei divergențe a modului de ruliu olandez, lucrarea de față evidențiază existența unui mecanism de destabilizare oscilatorie a mișcării lateral-direcționale generat de modul ruliu-spiral ("fugoida laterală"). Se arată, prin intermediul unui studiu de caz relevant, că o astfel de divergență poate fi prezisă, cu mare acuratețe, utilizând un nou criteriu algebric aproximativ.

The present work deals with the problem of developing approximate analytical criteria for predicting departure from controlled flight of airplanes at high angles of attack. Specifically, the paper is concerned with the case of departure caused by oscillatory divergence of the lateral-directional perturbed motion.

Unlike other scientific contributions, which relate departure phenomena to the occurrence of a Dutch roll divergence, this paper reveals the existence of a lateral-directional oscillatory-departure mechanism generated by the roll-spiral ("lateral phugoid") mode. By means of a relevant case study, it is shown that such a divergence can be predicted, with great accuracy, using a new approximate algebraic criterion.

Keywords: airplane dynamics, flight stability and control.

Introduction

The development of approximate analytical criteria for predicting the susceptibility of an aircraft to depart from controlled flight has both a theoretical, and a practical importance, especially in the context of current fighter airplanes designs, possessing high angle-of-attack maneuvering capabilities.

Among the lateral-directional departure modes, which are representative for moderate and high angle-of-attack flight conditions and have been intensely studied, one can mention different aperiodic and oscillatory phenomena, such as "wing drop", "nose slice", and "wing rock", [1]-[4].

* Lecturer, Dept. of Aerospace Engineering, University POLITEHNICA of Bucharest, Romania

The present paper is concerned with the problem of developing analytical criteria for predicting the occurrence of lateral-directional *oscillatory* departure from controlled flight as the airplane's angle-of-attack is increased.

Taking into account the dependence of the lateral-directional eigenvalue structure on the angle-of-attack, three approximate oscillatory-departure criteria are derived and their accuracy in predicting lateral-directional oscillatory divergences is investigated.

1. Mathematical model of the lateral-directional perturbed motion

The free lateral-directional airplane dynamics around a symmetric, wings-level, steady-state translational motion (taken as a reference flight condition) is mathematically modeled by the following fourth-order linear differential system

$$\frac{d}{dt} \Delta\beta = a_{11} \Delta\beta + a_{12} \Delta p + a_{13} \Delta r + a_{14} \Delta\phi , \quad (1)$$

$$\frac{d}{dt} \Delta p = a_{21} \Delta\beta + a_{22} \Delta p + a_{23} \Delta r , \quad (2)$$

$$\frac{d}{dt} \Delta r = a_{31} \Delta\beta + a_{32} \Delta p + a_{33} \Delta r , \quad (3)$$

$$\frac{d}{dt} \Delta\phi = \Delta p + a_{43} \Delta r , \quad (4)$$

where $\Delta\beta$, Δp , Δr , and $\Delta\phi$ represent, respectively, the perturbations (relative to the reference values) of the vehicle's sideslip angle, roll rate, yaw rate, and bank angle.

The constant coefficients a_{11} , a_{12} , \dots , a_{43} are given by the expressions

$$\begin{aligned} a_{11} &= \frac{Y_\beta}{mV_0}, \quad a_{12} = \sin\alpha_0 + \frac{Y_p}{mV_0}, \quad a_{13} = -\cos\alpha_0 + \frac{Y_r}{mV_0}, \quad a_{14} = \frac{g}{V_0} \cos\theta_0, \\ a_{21} &= \dot{L}_\beta, \quad a_{22} = \dot{L}_p, \quad a_{23} = \dot{L}_r, \\ a_{31} &= \dot{N}_\beta, \quad a_{32} = \dot{N}_p, \quad a_{33} = \dot{N}_r, \quad a_{43} = \tan\theta_0, \end{aligned} \quad (5)$$

where m is the airplane mass, α_0 - the reference angle-of-attack of the airplane, V_0 - the reference flight-speed, and the primed derivatives $\dot{L}_{(\cdot)}$ and $\dot{N}_{(\cdot)}$ are defined as

$$\dot{L}_{(\cdot)} = \frac{L_{(\cdot)}}{I_x'} + I_{xz}' N_{(\cdot)} = \frac{I_z L_{(\cdot)} + I_{xz} N_{(\cdot)}}{I_x I_z - I_{xz}^2} , \quad (6)$$

$$N'_{(\cdot)} = \frac{N_{(\cdot)}}{I_z} + I_{xz}' L_{(\cdot)} = \frac{I_x N_{(\cdot)} + I_{xz} L_{(\cdot)}}{I_x I_z - I_{xz}^2} . \quad (7)$$

The general solution of the linear differential system (1)-(4) can be determined by solving the following fourth-degree characteristic equation associated to the mentioned differential system,

$$P(\lambda) \equiv \lambda^4 + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 = 0 . \quad (8)$$

where the coefficients of the characteristic polynomial $P(\lambda)$ depend on the system coefficients $a_{11}, a_{12}, \dots, a_{43}$ in the form

$$c_3 = -a_{11} - a_{22} - a_{33} , \quad (9)$$

$$c_2 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32} , \quad (10)$$

$$\begin{aligned} c_1 = & a_{11}(a_{23}a_{32} - a_{22}a_{33}) + a_{12}(a_{21}a_{33} - a_{23}a_{31}) \\ & + a_{13}(a_{22}a_{31} - a_{21}a_{32}) - a_{14}(a_{21} + a_{31}a_{43}) , \end{aligned} \quad (11)$$

$$c_0 = a_{14}[a_{21}a_{33} - a_{23}a_{31} - a_{43}(a_{21}a_{32} - a_{22}a_{31})] . \quad (12)$$

2. Lateral-directional departure susceptibility

2.1. Critical stability conditions

According to the Routh-Hurwitz criterion, all the roots of the fourth-degree polynomial (8) have strictly negative real parts (corresponding to an asymptotically stable reference flight condition) if and only if

$$c_3 > 0 , \quad c_2 > 0 , \quad c_1 > 0 , \quad c_0 > 0 , \quad (13)$$

$$c_2 c_3 - c_1 > 0 , \quad (14)$$

$$R \equiv c_1 c_2 c_3 - c_1^2 - c_3^2 c_0 > 0 , \quad (15)$$

where R is the so-called Routh's discriminant.

As first pointed out by Duncan ([5], p.117-120), there are *two critical stability conditions*, namely

$$c_0 > 0 , \quad (16)$$

for preventing the occurrence of *an aperiodic divergence* of the perturbed motion (corresponding to a positive real root of the characteristic equation), and

$$R > 0 , \quad (17)$$

for preventing the occurrence of *an oscillatory divergence* (corresponding to a pair of complex-conjugate roots with positive real parts).

2.2. Lateral-directional oscillatory-departure criteria

The following investigation aims to develop approximate analytical expressions, which are *mathematically simpler than the critical stability condition* (15), for predicting fighter airplane susceptibility to oscillatory departure from controlled flight.

As known, the typical structure of the lateral-directional linear dynamics consists of two *aperiodic* modes -namely, the *spiral* (S) mode, with a very large time constant (as $\lambda_S \approx 0$), and the rapidly convergent *roll* (R) mode- and an *oscillatory*, usually lightly damped mode, called *Dutch roll* (D), which involves coupled banking, sideslipping, and yawing motions.

In this case, taking into account the corresponding factorization of the characteristic polynomial, i.e.

$$P(\lambda) = (\lambda - \lambda_S)(\lambda - \lambda_R)(\lambda^2 + 2\zeta_D \omega_{n_D} \lambda + \omega_{n_D}^2), \quad (18)$$

it follows, by coefficient identification, that

$$c_3 = 2\zeta_D \omega_{n_D} - \lambda_R - \lambda_S, \quad (19)$$

$$c_2 = \omega_{n_D}^2 + \lambda_R \lambda_S - 2(\lambda_R + \lambda_S) \zeta_D \omega_{n_D}, \quad (20)$$

$$c_1 = -(\lambda_R + \lambda_S) \omega_{n_D}^2 + 2\zeta_D \omega_{n_D} \lambda_R \lambda_S, \quad (21)$$

$$c_0 = \omega_{n_D}^2 \lambda_R \lambda_S. \quad (22)$$

On the oscillatory stability frontier, defined by zero Dutch roll damping ($\zeta_D = 0$), the expressions (19)-(22) become, *assuming* $\lambda_S \approx 0$,

$$c_3 \approx -\lambda_R, \quad c_2 \approx \omega_{n_D}^2, \quad c_1 \approx -\lambda_R \omega_{n_D}, \quad c_0 \approx 0, \quad (23)$$

so that, a *simplified oscillatory* (Dutch roll) *departure criterion* (R^*) can be used instead of R, i.e.

$$R^* = c_3 c_2 - c_1. \quad (24)$$

Obviously, oscillatory (Dutch roll) departure is avoided as long as $R^* > 0$, and is indicated by the relationship $R^* = 0$. Note that the derivation of the R^* oscillatory departure criterion is based on the hypothesis $\lambda_S \approx 0$ (that is, on the existence of a typical spiral mode, with a very large time-constant).

In the following, a completely oscillatory modal structure of the perturbed lateral-directional motion is taken into consideration, corresponding to the case when the roll and spiral modes couple and generate a single *roll-spiral* (RS) oscillation (which is, sometimes, referred to as the “lateral phugoid”). Hence, the

lateral-directional linear dynamics consists, in this case, of two oscillatory, usually lightly damped modes: the Dutch roll and roll-spiral modes.

Accordingly, the factorization of the characteristic polynomial is

$$P(\lambda) = (\lambda^2 + 2\zeta_D \omega_{n_D} \lambda + \omega_{n_D}^2)(\lambda^2 + 2\zeta_{RS} \omega_{n_{RS}} \lambda + \omega_{n_{RS}}^2), \quad (25)$$

so that

$$c_3 = 2(\zeta_D \omega_{n_D} + \zeta_{RS} \omega_{n_{RS}}), \quad (26)$$

$$c_2 = \omega_{n_D}^2 + \omega_{n_{RS}}^2 + 4\zeta_D \zeta_{RS} \omega_{n_D} \omega_{n_{RS}}, \quad (27)$$

$$c_1 = 2\omega_{n_D} \omega_{n_{RS}} (\zeta_D \omega_{n_{RS}} + \zeta_{RS} \omega_{n_D}), \quad (28)$$

$$c_0 = \omega_{n_D}^2 \omega_{n_{RS}}^2. \quad (29)$$

Consider the expression

$$R' = c_2^2 - 4c_0, \quad (30)$$

which can be written, using the above relationships, in the form

$$R' = \left(\omega_{n_D}^2 - \omega_{n_{RS}}^2 \right)^2 + 8\zeta_D \zeta_{RS} \omega_{n_D} \omega_{n_{RS}} \left(\omega_{n_D}^2 + \omega_{n_{RS}}^2 + 2\zeta_D \zeta_{RS} \omega_{n_D} \omega_{n_{RS}} \right). \quad (31)$$

If $\omega_{n_D} \approx \omega_{n_{RS}}$, R' may be used as an approximate oscillatory departure criterion. Specifically, as long as both the Dutch roll and the roll-spiral modes are convergent, $R' > 0$, while an oscillatory divergence (corresponding to the case when either ζ_D , or ζ_{RS} becomes negative) can be approximately predicted by the equality $R' = 0$.

It has to be remarked that R' represents a sufficiently accurate criterion for predicting lateral-directional oscillatory departure *only if* the difference between the values of the undamped circular frequencies ω_{n_D} and $\omega_{n_{RS}}$ is negligible.

If the mentioned difference cannot be neglected, another approximate oscillatory departure criterion can be identified by investigating the expression (28) of the coefficient c_1 . Thus, accounting for the fact that the Dutch roll and roll-spiral damping ratios are typically small, it follows that

$$R'' = c_1 \quad (32)$$

can be interpreted as a simplified oscillatory-departure criterion.

In this context, Dutch roll or roll-spiral departures can be approximately predicted by

$$R'' = 0. \quad (33)$$

The accuracy of the previously presented criteria in predicting lateral-directional departure of high angle-of-attack maneuvering airplanes has been tested by means of the following numerical example.

3. Numerical example

A highly maneuverable fighter airplane, flying horizontally at low altitude ($H = 1000$ m.; $\rho = 0.907$ kg/m³), is considered. The aircraft's mass is $m = 12000$ kg and its inertia moments are $I_x = 11000$ kgm², $I_z = 88000$ kgm², $I_{xz} \approx 0$.

The dependence of the nondimensional stability derivatives on the airplane's angle-of-attack (α) has been modeled in a polynomial form and is illustrated in figures 1-9.

As known (e.g. [6], p.38), the represented nondimensional derivatives are related to their dimensional counterparts as follows:

$$\begin{aligned} Y_\beta &= \frac{\rho}{2} V_0^2 S C_{Y_\beta}, \quad L_\beta = \frac{\rho}{2} V_0^2 S b C_{l_\beta}, \quad N_\beta = \frac{\rho}{2} V_0^2 S b C_{n_\beta}, \\ Y_p &= \frac{\rho}{4} V_0 S b C_{Y_p}, \quad L_p = \frac{\rho}{4} V_0 S b^2 C_{l_p}, \quad N_p = \frac{\rho}{4} V_0 S b^2 C_{n_p}, \\ Y_r &= \frac{\rho}{4} V_0 S b C_{Y_r}, \quad L_r = \frac{\rho}{4} V_0 S b^2 C_{l_r}, \quad N_r = \frac{\rho}{4} V_0 S b^2 C_{n_r}. \end{aligned} \quad (34)$$

The angle-of-attack dependence of each of the discussed approximate criteria (R^* , R' , R''), as well as the α -dependence of the exact criterion (the Routh's discriminant R), are illustrated in figures 10 and 11.

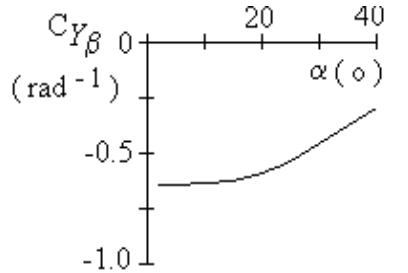


Fig. 1. $C_{Y_\beta}(\alpha)$

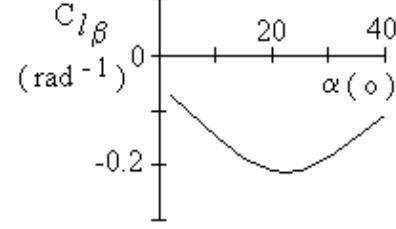
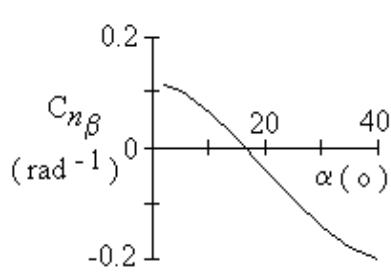
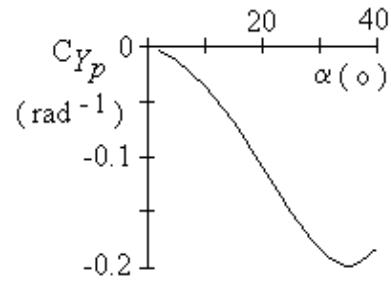
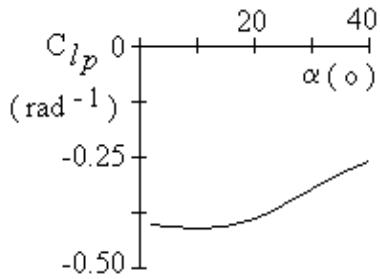
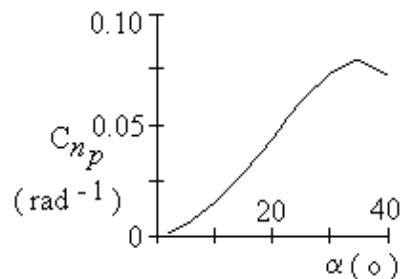
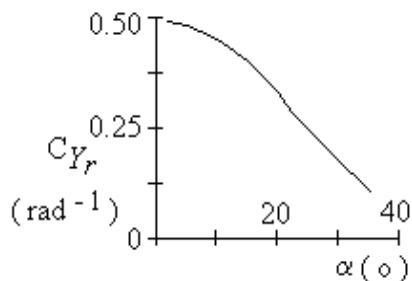
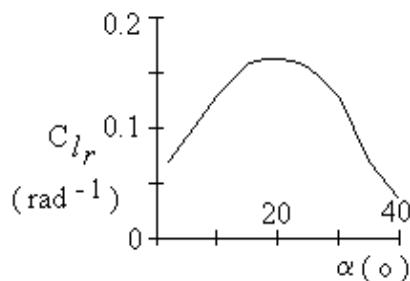
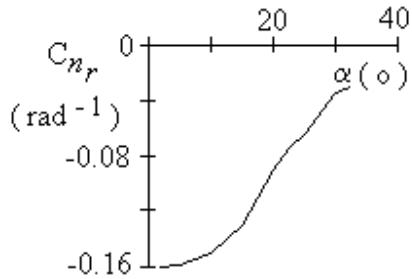
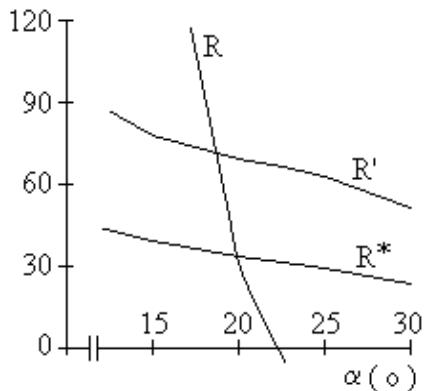
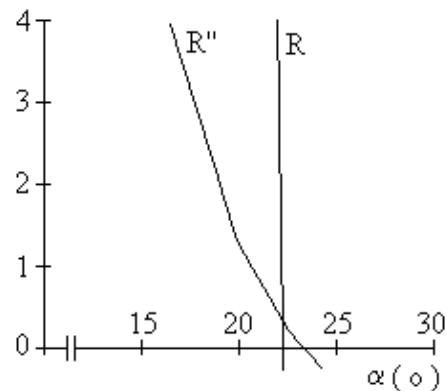


Fig. 2. $C_{l_\beta}(\alpha)$


 Fig. 3. $C_{n\beta}(\alpha)$

 Fig. 4. $C_{Yp}(\alpha)$

 Fig. 5. $C_{lp}(\alpha)$

 Fig. 6. $C_{np}(\alpha)$

 Fig. 7. $C_{Yr}(\alpha)$

 Fig. 8. $C_{lr}(\alpha)$

Fig.9. $C_{n_r}(\alpha)$ Fig. 10. $R^*(\alpha)$, $R'(\alpha)$, $R(\alpha)$ Fig. 11. $R''(\alpha)$, $R(\alpha)$

A lateral-directional oscillatory departure occurs at $\alpha = 22.2^\circ$ ($R = 0$ at $\alpha = 22.2^\circ$). Obviously, this oscillatory departure phenomenon can be reliably predicted on the basis of the simplified criterion R'' ($R'' = 0$ at $\alpha = 23.2^\circ$), while the other two approximate criteria are unable to sense it.

In order to distinguish the nature of the oscillatory departure (Dutch roll divergence or roll-spiral divergence), the dependence of the lateral-directional eigenvalues on the aircraft's angle of attack has been studied, some of the obtained eigenvalues being listed below:

$$\alpha = 5^\circ, \lambda_{1,2} = -0.501 \pm 2.494i; \lambda_3 = -7.698; \lambda_4 = -0.005;$$

$$\alpha = 10^\circ, \lambda_{1,2} = -0.712 \pm 1.554i; \lambda_3 = -4.756; \lambda_4 = -0.028;$$

$$\alpha = 15^\circ, \lambda_{1,2} = -2.044 \pm 0.995i; \lambda_3 = -0.746; \lambda_4 = -0.148;$$

$$\alpha = 20^\circ, \lambda_{1,2} = -2.007 \mp 1.973i; \lambda_{3,4} = -0.059 \mp 0.286i;$$

$$\alpha = 25^\circ, \lambda_{1,2} = -1.810 \mp 2.266i; \lambda_{3,4} = +0.051 \mp 0.307i;$$

$$\alpha = 30^\circ, \lambda_{1,2} = -1.626 \mp 2.331i; \lambda_{3,4} = +0.120 \mp 0.306i;$$

$$\alpha = 35^\circ, \lambda_{1,2} = -1.529 \mp 2.276i; \lambda_{3,4} = +0.190 \mp 0.278i;$$

$$\alpha = 40^\circ, \lambda_{1,2} = -1.560 \mp 2.146i; \lambda_{3,4} = +0.273 \mp 0.222i.$$

As the angle of attack is increased, *the roll eigenvalue shifts toward the spiral one* and, before α reaches 20° , the two real eigenvalues couple and generate a single convergent *roll-spiral* (RS) oscillatory mode.

As illustrated in figure 12, the oscillatory departure is due to the above-mentioned roll-spiral mode, which becomes divergent for $\alpha > 22,2^\circ$.

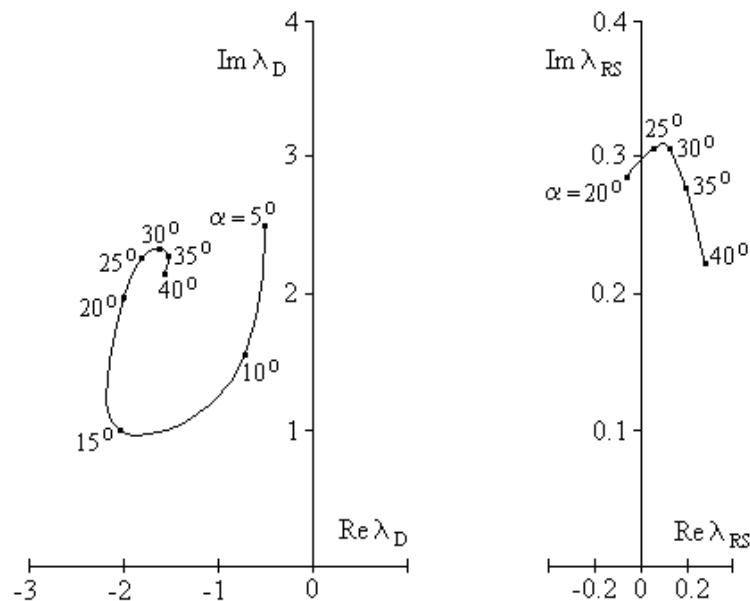


Fig. 12. Dutch roll (D) and roll-spiral (RS) root loci

It is important to note that $\text{Re} \lambda_D$ and $\text{Im} \lambda_D$ differ substantially from $\text{Re} \lambda_{RS}$ and, respectively, $\text{Im} \lambda_{RS}$. A similar difference (of, approximately, one order of magnitude) exists between ω_{n_D} and $\omega_{n_{RS}}$, which explains, according to Eq. (31), the inability of the R' criterion to predict the observed oscillatory

departure. Also, this departure phenomenon cannot be predicted by the R^* criterion, since the derivation of R^* is based on the existence of a classical spiral mode, with $\lambda_S \approx 0$.

Conclusions

Three approximate analytical criteria (referred to as R^* , R' , R'') for predicting lateral-directional oscillatory departure from controlled flight in high angle-of-attack conditions have been analyzed.

The R^* criterion is based on the assumption $\lambda_S \approx 0$, related to a classical lateral-directional eigenvalue structure, with one oscillatory mode (Dutch roll), and two aperiodic modes (roll and spiral). The other two criteria (R' and R'') have been derived on the basis of an entirely oscillatory eigenvalue structure, consisting of a Dutch roll mode and a roll-spiral ("lateral phugoid") mode. As shown numerically, the latter eigenvalue structure evolves from the former one at increasing angle-of-attack values.

Unlike previous contributions (see, e.g., [1]-[3]), which relate lateral-directional oscillatory departure to Dutch roll divergence, the present paper reveals the existence of an oscillatory departure mechanism generated by the roll-spiral mode. It is shown that the proposed R'' approximate analytical criterion (i.e. the c_1 coefficient of the characteristic polynomial) represents, in this case, an accurate predictor of the oscillatory departure phenomenon.

R E F E R E N C E S

1. *N. Ananthkrishnan, P. Shah, S. Unnikrishnan*, Approximate Analytical Criterion for Aircraft Wing Rock Onset, *Journal of Guidance, Control, and Dynamics*, **vol.27**, no.2, March-April 2004, pp. 304-307.
2. *G. Mengali, F. Giulietti*, Unified Algebraic Approach to Approximation of Lateral-Directional Modes and Departure Criteria, *Journal of Guidance, Control, and Dynamics*, **vol.27**, no.4, July-August 2004, pp. 724-728.
3. *G. Leng*, Reduced-Order Nonlinear Analysis of Aircraft Dynamics, *Journal of Guidance, Control, and Dynamics*, **vol.18**, no.2, March-April 1995, pp. 361-364.
4. *F.H. Lutze, W.C. Durham, W.H. Mason*, Unified Development of Lateral-Directional Departure Criteria, *Journal of Guidance, Control, and Dynamics*, **vol.19**, no.2, March-April 1996, pp. 489-493.
5. *W.J. Duncan*, *The Principles of the Control and Stability of Aircraft*, University Press, Cambridge, 1952.
6. *B.C. Teodorescu*, *Dinamica avionului-Stabilitatea zborului*, Editura Printech, Bucureşti, 2005.