

## MATHEMATICAL MODELING OF SHARON BIOLOGICAL WASTEWATER TREATMENT PROCESS

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*The paper presents a mathematical model for kinetics of the Sharon biological process used for the ammonium removal from the wastewater. It takes into account two substrates, ammonium and nitrite, and two groups of microorganisms, ammonia oxidizing microorganisms and nitrite oxidizing microorganisms. A simulation study was conducted for predicting substrates removal and microorganisms growth in the system, using a Simulink model. The theoretical results are in good agreement with experimental ones conducted on a laboratory installation.*

**Keywords:** wastewater treatment, Sharon biological process, modeling, simulation

### 1. Introduction

Nitrogen removal from wastewater has became a very important problem due to eutrophication of water bodies. During the last decade new biological nitrogen removal techniques have been developed for reject water from sludge dewatering, digestion or drying facilities and other ammonia-rich wastewater streams, based on partial nitrification. In the SHARON process (Single reactor High activity Ammonia Removal Over Nitrite) the nitrification process is stopped at nitrite and the nitrate production is prevented. This is achieved under some specific operation conditions such as high temperature, approximately 35°C and neutral pH, [3], [7]. In this way the maximum specific growth rate of the ammonium oxidizers will increase faster than the maximum specific growth rate of the nitrite oxidizers, which can be washed out. The hydraulic retention time is

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equal with cells retentions time so that the process operates without sludge retention.

There are previous studies that have been devoted to modelling of Sharon process, [1], [2], [4], [5], [6], [7], but in this paper the theoretical model is developed in dimensionless form and it is verified with experimental data.

## 2. Mathematical modeling

It is considered a schematic representation consider the schematic representation of Sharon process, which is typically a continuous stirred tank reactor CSTR, and a control volume V with enter point 0 and outlet point 1 (see Fig. 1).

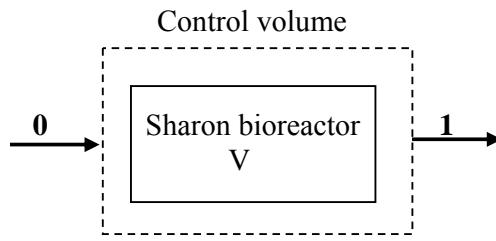


Fig.1 Scheme of SHARON process

The multiphase fluid consisting of water and pollutants is considered quantitatively and qualitatively homogenous.

Two substrates are taking into account, ammonium and nitrite, and two groups of microorganisms, ammonium oxidizers and nitrite oxidizers.

In non-steady-state conditions, taking into account complete mixing in the equations for the mass balance of the microorganisms, respectively of the substrates in control can be written as:

$$\frac{d(X_a V)}{dt} = Q_0 X_{a0} - Q_1 X_{a1} + \mu_a(t) X_{a1} V - K_{da} X_{1a} V \quad (1)$$

$$\frac{d(X_n V)}{dt} = Q_0 X_{n0} - Q_1 X_{n1} + \mu_n(t) X_{n1} V - K_{dn} X_{1n} V \quad (2)$$

$$\frac{d(S_{a1} V)}{dt} = Q_0 S_{a0} - Q_1 S_{a1} - \frac{1}{Y_a} \mu_a(t) X_{a1} V - \left( \frac{1}{Y_a} - f_N \right) \mu_n(t) X_{n1} V \quad (3)$$

$$\frac{d(S_{n1} V)}{dt} = Q_0 S_{n0} - Q_1 S_{n1} - \frac{1}{Y_n} \mu_n(t) X_{n1} V + \left( \frac{1}{Y_n} - f_N \right) \mu_a(t) X_{a1} V \quad (4)$$

where:  $X_a$  – concentration of the ammonium oxidizing microorganisms, [mg/l];  $S_a$  – ammonium concentration, [mg/l];  $X_n$  – concentration of the nitrite oxidizing microorganisms, [mg/l];  $S_n$  – nitrite concentration, [mg/l];  $Q$  – volumetric flowrate, [ $m^3/d$ ];  $K_{da}$  – rate decay coefficient of the ammonium oxidizing microorganisms, [ $h^{-1}$ ];  $K_{dn}$  - rate decay coefficient of nitrite oxidizing microorganisms, [ $h^{-1}$ ];  $Y_a$  – yield coefficient of the biomass during ammonium oxidation;  $Y_n$  - yield coefficient of the biomass during nitrite oxidation;  $f_N$  – nitrogen fraction in biomass.

Taking into account that the volume  $V$  is constant, denoted the dilution rate with  $\theta=1/t_r=Q_0/V$ , where  $t_r$  – hydraulic retention time and consider that the influent doesn't contain biomass, the equations become:

$$\frac{dX_{a1}}{dt} = -\theta \cdot X_{a1} + \mu_a(t)X_{a1} - k_{da}X_{a1} \quad (5)$$

$$\frac{dX_{n1}}{dt} = -\theta \cdot X_{n1} + \mu_n(t)X_{n1} - k_{dn}X_{n1} \quad (6)$$

$$\frac{dS_{a1}}{dt} = \theta(S_{a0} - S_{a1}) - \frac{1}{Y_a} \mu_a(t)X_{a1} - \left( \frac{1}{Y_a} - f_N \right) \mu_n(t)X_{n1} \quad (7)$$

$$\frac{dS_{n1}}{dt} = \theta(S_{n0} - S_{n1}) - \frac{1}{Y_n} \mu_n(t)X_{n1} + \left( \frac{1}{Y_n} - f_N \right) \mu_a(t)X_{a1} \quad (8)$$

In Sharon reactor the growth of the ammonium oxidizing microorganisms is favored and the growth of the nitrite oxidizing microorganisms is limited. This model take into account the inhibition of the ammonium oxidation by nitrite and the inhibition of the nitrite oxidation by ammonium and nitrite. Also, the decreasing of dissolved oxygen concentration is a limiting factor of the process.

The expressions for growth rate of the two kind of microorganisms are:

$$\mu_a(t) = \mu_{ma} \cdot \frac{S_{a1}}{K_a + S_{a1} + \frac{S_{a1}^2}{K_{ia}}} \cdot \frac{K_{i,na}}{K_{i,na} + S_{n1}} \cdot \frac{OD}{K_{ODa} + OD} \quad (9)$$

$$\mu_n(t) = \mu_{mn} \cdot \frac{S_{n1}}{K_n + S_{n1}} \cdot \frac{K_{i,nn}}{K_{i,nn} + S_{n1}} \cdot \frac{K_{i,an}}{K_{i,an} + S_{a1}} \cdot \frac{OD}{K_{ODn} + OD} \quad (10)$$

where:  $\mu_{ma}$  – net specific maximum growth rate of the ammonium oxidizing microorganisms at the infinite substrate concentration,  $[h^{-1}]$ ;  $\mu_{mn}$  – net specific maximum growth rate of the nitrite oxidizing microorganisms at the infinite substrate concentration,  $[h^{-1}]$ ;  $K_a$  – affinity constant for ammonium of the ammonium oxidizing microorganisms,  $[mgN/l]$ ;  $K_n$  – affinity constant for nitrite of nitrite oxidizing microorganisms,  $[mgN/l]$ ;  $K_{i, na}$  – inhibition constant of the ammonium oxidizing microorganisms by nitrite,  $[mgN/l]$ ;  $K_{i, nn}$  – inhibition constant of the nitrite oxidizing microorganisms by nitrite,  $[mgN/l]$ ;  $K_{i, an}$  – inhibition constant of the nitrite oxidizing microorganisms by ammonium,  $[mgN/l]$ ;  $K_{ODa}$  – affinity constant for oxygen of the ammonium oxidizing microorganisms,  $[mgO_2/l]$ ;  $K_{ODn}$  – affinity constant for oxygen of the nitrite oxidizing microorganisms,  $[mgO_2/l]$ ; OD – dissolved oxygen concentration,  $[mgO_2/l]$ ;

In order to generalize, the model is written using dimensionless terms, considering the following terms:

$$\begin{aligned} X_{a1}^* &= Y_a \cdot S_{a0}; t^* = \frac{1}{\theta}; S_{a1}^* = S_{a0}; X_{n1}^* = Y_n \cdot S_{n0}; S_{n1}^* = S_{n0} \\ X_{a1} &= X_{a1}^* \cdot X_{a1}'; t = t^* \cdot t'; S_{a1} = S_{a1}^* \cdot S_{a1}'; X_{n1} = X_{n1}^* \cdot X_{n1}'; S_{n1} = S_{n1}^* \cdot S_{n1}' \end{aligned} \quad (11)$$

Replacing these expressions in the equations will lead to:

$$\frac{dX_{a1}'}{dt} = X_{a1}' \cdot (\mu_a' - k_{da}' - 1) \quad (12)$$

$$\frac{dX_{n1}'}{dt} = X_{n1}' \cdot (\mu_n' - k_{dn}' - 1) \quad (13)$$

$$\frac{dS_{a1}'}{dt} = 1 - S_{a1}' - \mu_a' \cdot X_{a1}' - \mu_n'' \cdot X_{n1}' \quad (14)$$

$$\frac{dS_{n1}'}{dt} = 1 - S_{n1}' - \mu_n' \cdot X_{n1}' + \mu_a'' \cdot X_{a1}' \quad (15)$$

The following notation are made:

$$\mu_a' = \mu_{ma}' \cdot \frac{S_{a1}'}{K_a' + S_{a1}' + \frac{S_{a1}'}{K_{ia}'}} \cdot \frac{K_{i,na}'}{K_{i,na}'+S_{n1}'} \cdot \frac{OD}{K_{ODa}'+OD} \quad (16)$$

$$\mu_n' = \mu_{mn}' \cdot \frac{S_{n1}'}{K_n' + S_{n1}'} \cdot \frac{K_{i,nn}'}{K_{i,nn}'+S_{n1}'} \cdot \frac{K_{i,an}'}{K_{i,an}'+S_{a1}'} \cdot \frac{OD}{K_{ODn}'+OD} \quad (17)$$

$$\mu_a'' = \mu_a' \cdot Y_a \cdot \frac{S_{a0}}{S_{n0}} \cdot \left( \frac{1}{Y_a} - f_N \right) \quad (18)$$

$$\mu_n'' = \mu_n' \cdot Y_n \cdot \frac{S_{n0}}{S_{a0}} \cdot \left( \frac{1}{Y_a} - f_N \right) \quad (19)$$

$$\begin{aligned} \mu_{ma}' &= \mu_{ma}/\theta, K_a' = K_a/S_{a0}, K_{ia}' = K_{ia}/S_{a0}, K_{i,na}' = K_{i,na}/S_{n0} \\ \mu_{mn}' &= \mu_{mn}/\theta, K_n' = K_n/S_{n0}, K_{i,nn}' = K_{i,nn}/S_{n0}, K_{i,an}' = K_{i,an}/S_{a0} \\ k_{da}' &= k_{da}/S_{a0}, k_{dn}' = k_{dn}/S_{n0} \end{aligned} \quad (20)$$

### 3. Simulation study

Theoretical simulation study was conducted for predicting substrates removal and microorganisms growth in the system. A Simulink model was made starting from equations (12) - (15), (fig.2). The kinetics parameters from the literature were used for the simulations, [4], [5], [6]. For every set of data was taking into account the change of the dilution rate depending on the influent flowrate and the experimental data of the influent concentration of ammonium and nitrite and dissolved oxygen concentration.

Some of the theoretical results are presented in fig. 3, 4, 6 and 7.

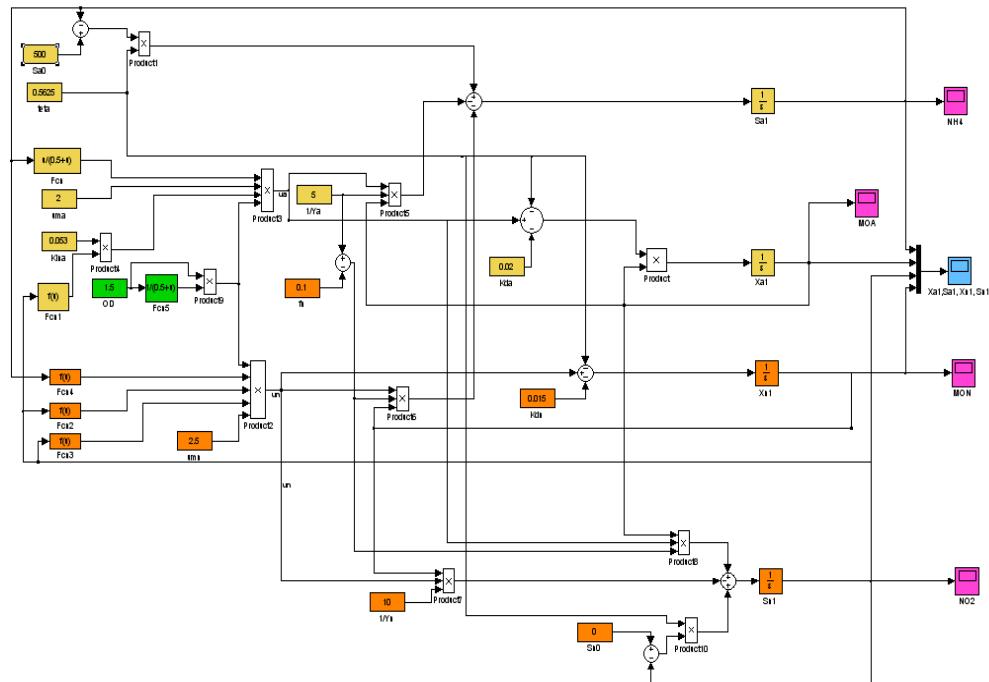


Fig. 2 Simulink model

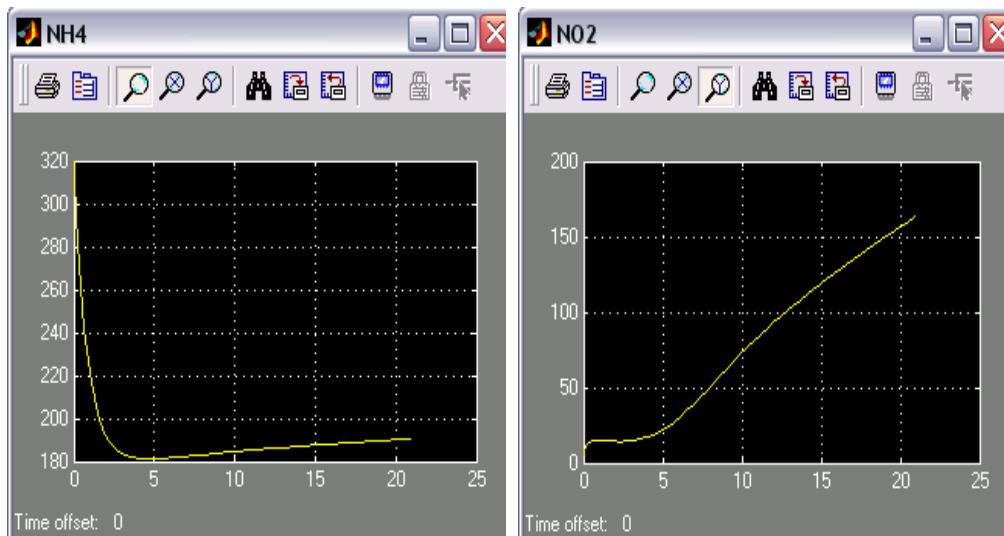


Fig. 3. The time variation of the ammonium concentration during 10.01 – 21.02

Fig. 4. The time variation of the nitrite concentration during 10.01 – 21.02

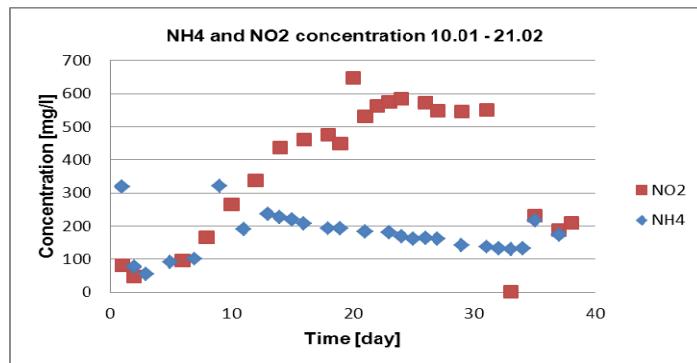


Fig. 5. Experimental data of the time variation of the ammonium and nitrite concentrations during 10.01 – 21.02

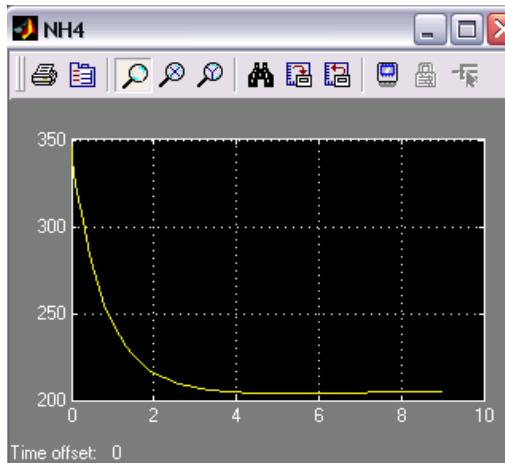


Fig. 6. The time variation of the ammonium concentration during 07.03 – 4.04

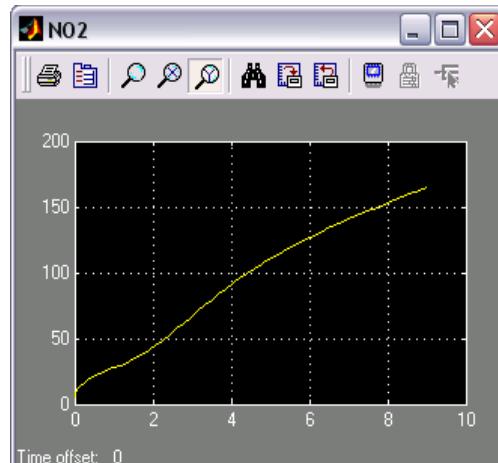


Fig. 7. The time variation of the nitrite concentration during 07.03 – 4.04

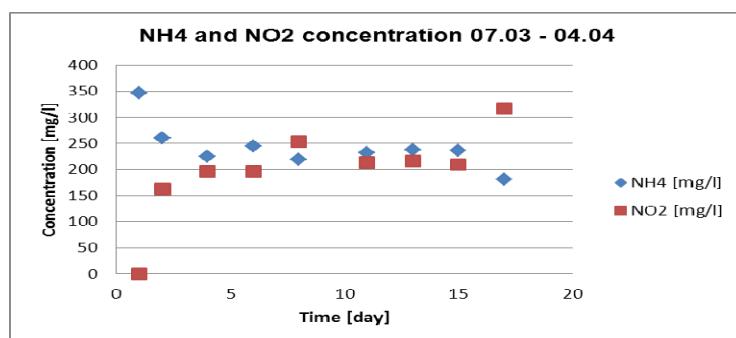


Fig. 8. Experimental data of the time variation of the ammonium and nitrite concentrations during 07.03 – 04.04

The theoretical results were compared with the experimental ones. The experimental installation for study of the Sharon process was made at National Institute for Research and Development in Industrial Ecology, Bucharest, Romania. The experimental data are presented in fig. 5 and 8. The results shows a good correlation between the theoretical model and experimental results.

## 6. Conclusions

In this paper the kinetic model of the Sharon biological wastewater treatment process was investigated. A mathematical model of the process was developed and theoretically simulated using a Matlab Simulink model. A good agreement between theoretical and experimental results is obtained. Further works have been done to couple Sharon with Anammox process.

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