DOUBLE DIFFUSION CONVECTION IN A TILTED SQUARE POROUS DOMAIN UNDER CROSS TEMPERATURE AND CONCENTRATION GRADIENTS

Nabil OUAZAA[1], Smail BENISSAAD[2], Mahmoud MAMOU[3]

The present study focuses on double diffusion natural convection in a tilted square porous cavity saturated with a binary fluid subjected to cross temperature and concentration gradients. The Darcy model with the Boussinesq approximation, energy and species transport equations are solved numerically using the classical finite difference method with a time-accurate scheme. The case of equal thermal and solutal buoyancy forces is considered. For this situation, an equilibrium state solution corresponding to the rest state is possible and the resulting onset of motion can be either supercritical or subcritical. The study is carried out for an inclination angle of 45°. The results are presented in terms of Nusselt and Sherwood numbers, and flow intensity as functions of the thermal Rayleigh number. In this study, the thresholds for the onset of convection are determined as function of the Lewis number.

Keywords: Double diffusion Convection, Porous medium, Supercritical Rayleigh number, Subcritical Rayleigh number.

1. Introduction

The dynamics of heat and mass transfer can be very different from those driven by the temperature field solely. Interest in coupled heat and mass transfer due to buoyancy forces in porous media has been motivated by such diverse engineering problems related to the dispersion of chemical contaminants through water-saturated soil, exploitation of continental geothermal reservoir, migration of moisture through the air contained in fibrous insulation, metallurgy, electrochemistry, geophysics, etc.

[1] Laboratoire d'Energétique Appliquée et de Pollution, Département de Génie Mécanique, Université des Frères Mentouri, Constantine, Algérie, e-mail: nabil.ouazaa5@gmail.com.
[2] Laboratoire d'Energétique Appliquée et de Pollution, Département de Génie Mécanique, Université des Frères Mentouri, Constantine, Algérie, e-mail: benissaad.smail@gmail.com.
[3]Aerodynamics Laboratory, Aerospace Research Centre, National Research Council Canada, Ottawa, Ontario, Canada, e-mail: mahmoud.mamou@nrc.ca.

A comprehensive review on the phenomena of heat and mass transfer and convection in porous media could be found in the book by Nield and Bejan [1]. Mamou et al. [2] examined the flow in a square cavity subjected to horizontal fluxes of heat and mass. In case where the volume forces are in opposite direction and same order of magnitude, the existence of multiple solutions was demonstrated. The existence of multiple solutions depended heavily on the thermal Rayleigh and Lewis numbers. Mansour et al. [3] studied numerically the Soret effect on multiple solutions in a square cavity. The authors concluded that the Soret parameter might have a strong effect on the convective flow. One, two or three solutions were found to be possible. Mohamad and Bennacer [4] obtained numerical results, on the basis of two- and three-dimensional flows, of heat and mass transfer in a horizontal enclosure with an aspect ratio of two and filled with a saturated porous medium. The enclosure was heated differentially and a stably stratified species concentration was imposed vertically. It was found that the difference in the rates of heat and mass transfer predicted by the two models was not significant. Mansour et al. [5] studied numerically the Soret effect on fluid flow and heat and mass transfer induced by double diffusive natural convection in a square porous cavity submitted to cross gradients of temperature and concentration. They concluded that the Soret effect might affect considerably the heat and mass transfer rates as it led to an enhancement or to a reduction of the mass transfer rate, depending on the flow structure and on the sign and magnitude of the Soret coefficient. Bourich et al. [6] studied analytically and numerically the Soret effect on thermal natural convection within a horizontal porous enclosure uniformly heated from below by a constant heat flux, using the Brinkman extended Darcy model. It was found that the Soret separation parameter had a strong effect on the thresholds of instabilities and on the heat and mass transfer characteristics. Saeid [7] studied the problem of natural convection in a two-dimensional square vertical porous cavity with the hot (left) wall temperature oscillating in time. The author found that during the heat transfer process, the hot wall temperature dropped and resulted in a temperature higher than the hot wall temperature at some locations inside the cavity. Also, it is observed that the average Nusselt number had a peak value at the non-dimensional frequency of 450 within the range considered (1–2000) for a Rayleigh number of 103, because the convection currents were stronger than those occurred at other frequencies. The transient free convection in a two-dimensional square cavity filled with a porous medium was considered by Saeid and Pop [8]. The flow was driven by considering the case when one of the cavity vertical walls was suddenly heated and the other one was suddenly cooled, while the horizontal walls were kept adiabatic. The results were obtained for the initial transient state up to the steady state for Rayleigh number values of 10^2–10^4. It was observed that the average Nusselt number showed an undershoot during the transient period and that the time required to reach the steady state is longer for low Rayleigh number and shorter for high Rayleigh number. Mansour, et al. [9] studied the transient MHD natural convection in an inclined cavity filled with a fluid saturated porous medium by including the effects of the magnetic field and heat source in the solid phase. The flow was driven by considering the case when one of the cavity vertical walls was suddenly heated and the other one was suddenly cooled, while the horizontal walls were adiabatic. The authors found that, in general, the temperature of the fluid could be increased by increasing both of the Magnetic field force and the inclination angle. Sezai and Mohamad [10] presented results for three-dimensional flow in a cubic cavity filled with a porous medium and subjected to opposing thermal and concentration gradients. Their results revealed that for a certain range of the controlling parameters, the flow became three-dimensional and multiple solutions were possible within the range. The stability of the flow structures was studied by Bergeon et al. [11] where the mechanisms by which the stable solutions lost stability or unstable solutions regained stability were determined. The authors also studied the influence of the cavity inclination on the stability and bifurcation of the solutions and found that the bifurcation at the critical Rayleigh number was either transcritical or pitchfork, depending on the aspect ratio and the inclination angle of the cavity. Vasseur et al. [12] studied analytically and numerically the flow in a tilted rectangular cavity and observed that the maximum heat transfer rate, for a given R_T, was obtained when the cavity was heated from below, with θ in the range of $90° < \theta < 180°$. They found that the maximum rate took place for values of θ approaching 90° whenever R_T increased. Trevisan and Bejan [13] used a numerical method and scale analysis to study double diffusion convection in a porous square cavity, with the vertical walls maintained at constant temperatures and concentrations. It was found that the fluid flow was possible beyond a critical Rayleigh number when $Le \neq 1$. However, the fluid motion disappeared completely for the $Le=1$ and $N=-1$. The results of this analysis were found in agreement with the numerical study.

In this work, a numerical study was conducted to examine the effect of the Rayleigh number on the heat and mass transfer rates in a porous square cavity tilted at 45°. We had examined the case where the thermal and mass buoyancy forces are equal and for different values of the Lewis number. Darcy's model is used to simulate the double diffusive convection inside the cavity. The existence of solution was demonstrated and the threshold for the onset of convection was obtained using a novel numerical method.

2. Mathematical formulation

The physical configuration considered in this work is a square (*A*=*L'*/*H'*=1) porous enclosure as shown in Fig. 1. The origin of the coordinates system is located at the cavity centre. Constant and crossed thermal and mass fluxes q' and j' were imposed on the walls of the cavity. The fluid saturating the porous matrix is incompressible and Newtonian, and obeys the Darcy law and the Boussinesq approximation.

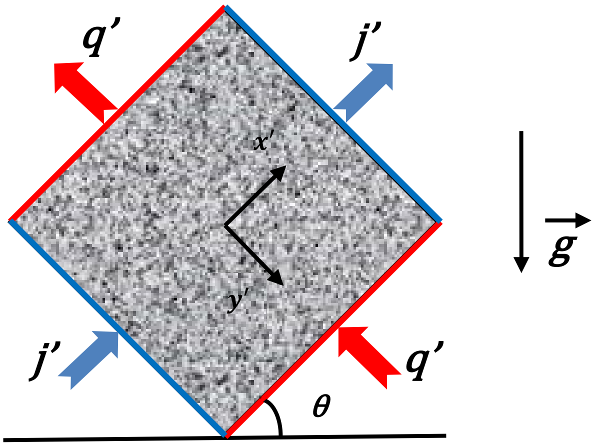

Fig. 1. Geometry of the problem.

The governing equations that describe the double-diffusive convection inside the enclosure are expressed in terms of the stream function, temperature and concentration in dimensionless form as:

$$\nabla^2\Psi = -R_T\mathcal{F}(T+NS) \tag{1}$$

$$\nabla^2 T = \frac{\partial T}{\partial t} - J(\Psi, T) \tag{2}$$

$$\frac{1}{Le}\nabla^2 S = \varepsilon\frac{\partial S}{\partial t} - J(\Psi, S) \tag{3}$$

where Ψ is the dimensionless stream function defined such that:

$$u = \frac{\partial\Psi}{\partial y}, \quad v = -\frac{\partial\Psi}{\partial x}$$

and the operators $\mathcal{F}$ and J are defined as follows:

$$\mathcal{F}(f) = \sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}, \qquad J(f,g) = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}$$

The variables u and v are the dimensionless velocity components, T and S are the dimensionless temperature and concentration, t is the dimensionless time, x

and *y* are the dimensionless coordinate axes, R_T is the thermal Rayleigh number, *N* is the ratio of the buoyancy forces, θ is the inclination angle, *Le* is the Lewis number and *ε* is the normalized porosity of the porous medium. In the Darcy model, the inertia and viscosity forces are assumed negligible and the Reynolds number based on the pores size is assumed to be very low.

The dimensionless boundary conditions are given by:

$$y = \pm\frac{1}{2} : \Psi = 0,\ \ \frac{\partial T}{\partial y} = 0\ \text{ and }\ \frac{\partial S}{\partial y} = -1 \tag{4}$$

$$x = \pm\frac{1}{2} : \Psi = 0,\ \ \frac{\partial T}{\partial x} = 1\ \text{ and }\ \frac{\partial S}{\partial x} = 0 \tag{5}$$

3. Numerical solution

The numerical solution of the equations governing the convective flows (1)-(3) with the boundary conditions (4)-(5) is obtained using a finite difference scheme. The entire domain, as shown in Figure 1, has been discredited with a uniform mesh (101x101). The solution includes the stream function, the temperature and concentration fields. A central finite difference scheme with second-order precision is used to transform the basic equations into a set of finite difference discretized equations. The energy and concentration equations, after having been written in a conservative form, are solved using the alternating direction implicit (A.D.I) method, while the stream function field is obtained from the Darcy equation using the successive over-relaxation (S.O.R) method with a convergence criterion based on the residual less or equal to 10^{-6}. The Nusselt and Sherwood numbers were integrated numerically using the Simpson scheme.

4. Calculation of the Critical Rayleigh Number

In what follows we present a new method for determining the onset of instabilities in natural convection. The calculation method of the supercritical Rayleigh number (R_{TC}^{sup}) requires at least two flow simulations above and below the threshold of instability. For *Le*=1 and by trial and error procedure, using the numerical code which solves the full governing equations, it was found that for R_T= 5, 10 and 15 the solution is purely conductive (rest state), however, for R_T=20, a convective solution is triggered and leads to a convective steady state solution. Even for R_T=18 the solution is convective. Thus, obviously the threshold for the onset of convection must be within the interval [15, 18]. As known, for infinitesimal amplitude convection, the time evolution of the flow intensity is exponential, according to the linear stability analysis, and can be correlated by $\boldsymbol{\Psi_{0max}(t)} = \boldsymbol{\Psi_0 e^{pt}}$, where $\boldsymbol{\Psi_{0max}}$ is the convective flow amplitude and $\boldsymbol{\Psi_0}$ is the initial flow amplitude at t=0. The parameter p represents the perturbation amplitude growth rate. When $p < 0$, the flow is decaying, and when p > 0 the flow is amplified in time. Then, $p < 0$ is obtained below the threshold of convection and $p > 0$ occurs above the threshold. By performing two simulations for two Rayleigh numbers below and above the threshold, the growth rate parameter can be computed numerically. Then the threshold of convection can be determined accurately by interpolation for p=0, the situation where the marginal stability occurs. Now, applying this procedure, starting first from a pure conductive state (instable), for R_T=18, a flow simulation is performed. The flow intensity time history is presented in Fig. 2. An excellent exponential curve fit is obtained $\boldsymbol{\Psi_{0max}} < 10^{-2}$. For this case, the solution is marched in time and converged to a steady state convective solution. Now, for R_T=15, using the converged solution as initial conditions, the flow simulation is carried again and as can be seen from Fig. 2, the flow decays towards the pure conductive state. Focusing only on the infinitesimal curve branch, $10^{-10} < \boldsymbol{\Psi_{0max}} < 10^{-2}$, an exponential curve fit is presented. The growth rate parameter, using the exponential curve fit, is obtained as p=1.117 for R_T=18 and p=-0.756 for R_T=15. Using linear interpolation for p=0, the threshold of marginal stability is obtained as R_{TC}^{sup}=16.21.

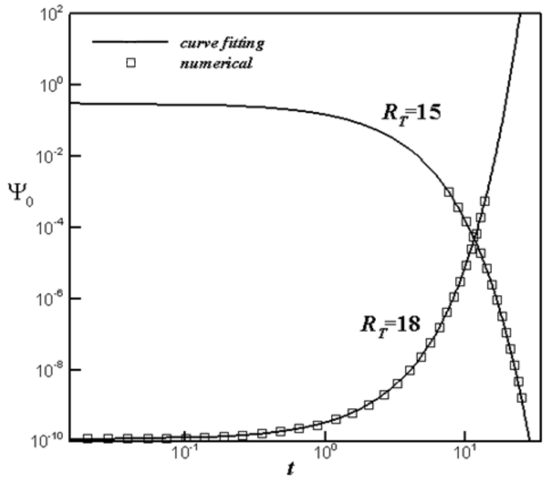

Fig. 2. Flow intensity time histories below and above the threshold of supercritical convection for Le=1.

5. Results and discussion

The present study is limited to the state of equilibrium where the thermal and solutal buoyancy forces are equal. Under these conditions, a solution of the rest state is possible and a threshold exists for the onset of convective flows. The effect of the Rayleigh and Lewis numbers on the flow behavior and on the heat and mass transfer rates is investigated and the convective flow instability thresholds are determined. The threshold of subcritical convection and the onset of natural convective were approximately determined from the numerical. The determination of the supercritical Rayleigh number is explained in section 4.

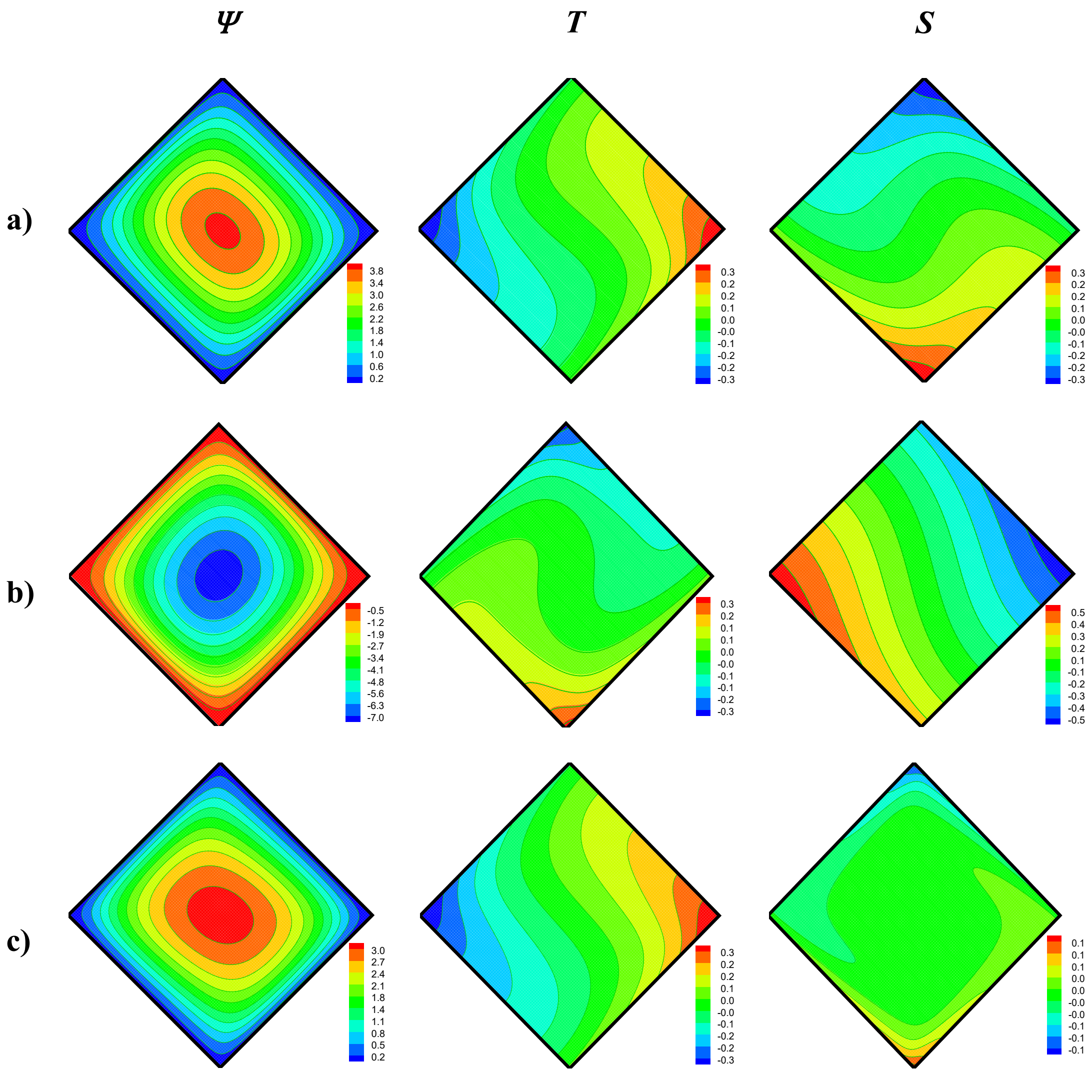

Fig.3. Stream function, temperature and concentration contours obtained for R_T=100:
a) Le =1: Ψ_{0min}= 0.00 Ψ_{0max}= 3.89 Nu_m= 3.19 Sh_m= 2.78
b) Le =0.1: Ψ_{0min}= -7.37 Ψ_{0max}= 0.00 Nu_m= 4.21 Sh_m= 1.13
c) Le =10: Ψ_{0min}= 0.00 Ψ_{0max}= 3.16 Nu_m= 3.07 Sh_m= 9.38.

Fig. 3 represents the contours of the stream function, temperature and concentration obtained for R_T =100 and Le = 10, 1 and 0.1. Fig. 4 (a) shows the intensity of the flow. The effect of the Rayleigh number on the heat and mass transfer rates, *Nu* and *Sh*, is presented in Fig. 4 (b) and (c), respectively, for various values of *Le*. The heat and mass transfer rates and the flow intensity are seen to increase monotonically with R_T. It is observed that, when R_T is relatively small, the flow intensity magnitude increases with the Lewis number. The same trend is observed for the Nusselt number. However, it is found that the Sherwood number increases monotonically with the Lewis number.

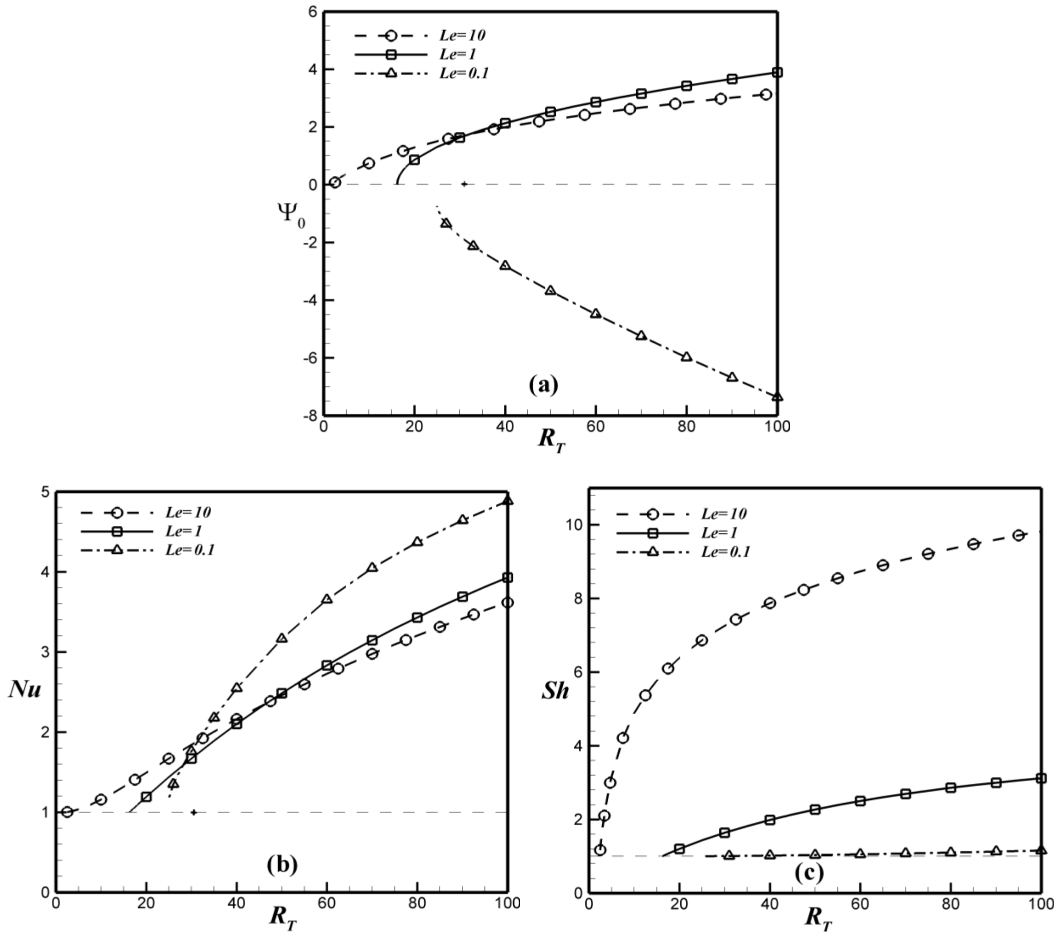

Fig. 4. Bifurcation diagram as a function of R_T and *Le*: (a) Flow intensity, (b) Nusselt number, and (c) Sherwood number.

As shown in Fig. 4 (a), natural convection is usually the preferable solution when launching the flow simulation using the rest state solution as initial conditions. As the mass diffusivity is greater than the thermal diffusivity (*Le*= 0.1), the heat transfer rate is higher than the mass transfer rate and conversely for the case where the thermal diffusivity exceeds the mass diffusivity (*Le* = 10). In our study, we have two types of bifurcations, a supercritical bifurcation for *Le* = 1, and a subcritical bifurcation for *Le* = 0.1 and 10 (see Table 1).

Table 1

Critical values of R_{TC}^{sup} and R_{TC}^{sub} and type of bifurcations.

Le	R_{TC}^{sup}	R_{TC}^{sub}	Bifurcation
10	2.86	2.50	Subcritical
1	16.21	…	Supercritical
0.1	29.34	25.07	Subcritical

Within a narrow range of the Rayleigh number near the critical point, the bifurcation diagrams are illustrated in Fig. 5 for various values of the Lewis number.

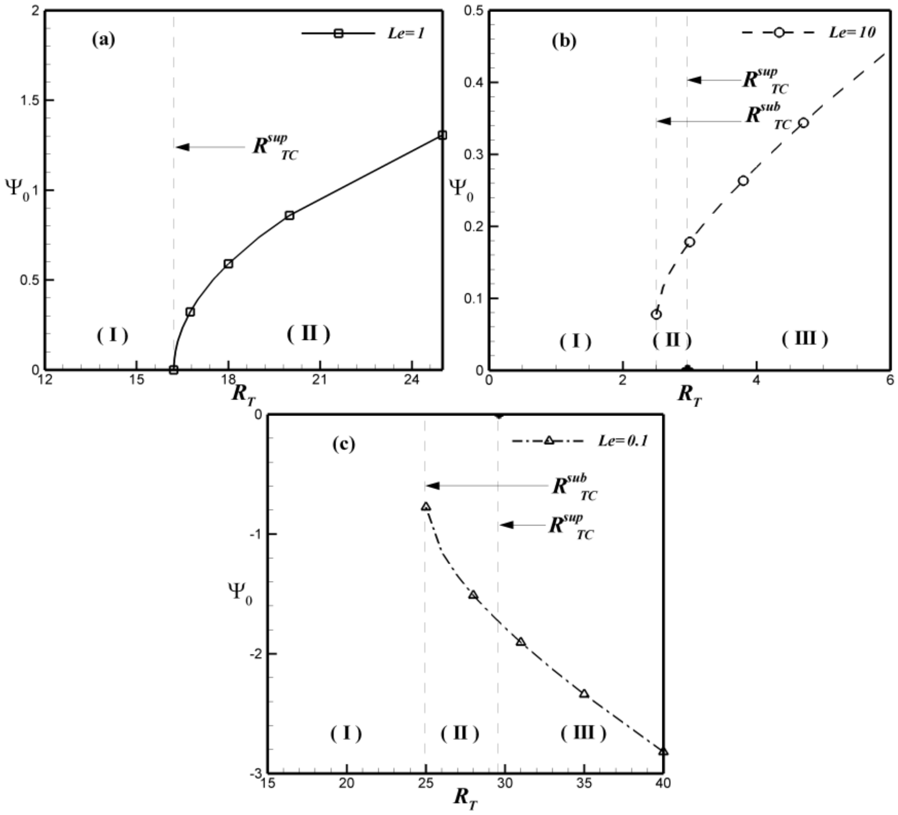

Fig. 5. Bifurcation diagrams in terms of Ψ_0 versus R_T for: (a) *Le*=1, (b) *Le*=10, and (c) *Le*=0.1.

Starting with the rest state solution as initial conditions, the numerical results presented in Fig. 5 (a)-(c) indicate that, below the subcritical or supercritical Rayleigh number (region I), the rest state prevails. In region (II), it is observed that the onset of steady motion is supercritical, as illustrated in Fig. 5 (a), and subcritical, as illustrated in Fig. 5 (b) and (c), occurring at Rayleigh numbers, R_{TC}^{sup} and R_{TC}^{sub}, above which the numerical solution bifurcates towards a steady finite amplitude convective regime. Upon increasing R_T above the supercritical Rayleigh number, R_{TC}^{sup}, (region III), the strength of the convection is promoted monotonically, as can be seen from Fig. 5.

After several tests, we can find a relationship that links the supercritical Rayleigh number to the Lewis number. Fig. 6 shows the supercritical Rayleigh number as a function of the Lewis number. A correlation giving a relationship between R_{TC}^{sup} and Le is obtained as follows:

$$\boldsymbol{R_{TC}^{sup} = \frac{32.37}{Le+1}} \tag{6}$$

Equation (6), illustrated in Fig. 6 by a solid line, is seen to be in good agreement with the numerical results depicted by square symbols.

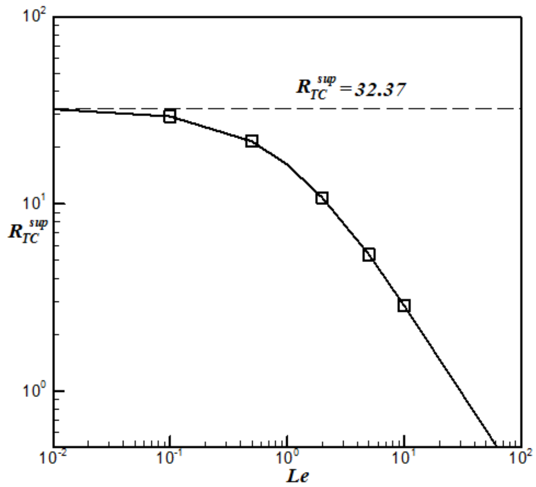

Fig. 6. Supercritical Rayleigh number according to Lewis number.

6. Conclusions

In the paper, double diffusion convection in a tilted porous square cavity, subjected to heat flow and cross-flow, was studied numerically. The conditions under which the thermal and solutal buoyancy forces are equal (N = 1) were considered. For this situation, we proved the existence of stable convective solutions for various values of the governing parameters. The existence of exchange stability was demonstrated; the threshold for the appearance of supercritical convection was obtained numerically and computed accurately using a novel numerical method. The main concluding remarks on the results are itemized below:

- When the thermal diffusivity is equal to the mass diffusivity (Le=1), we obtained one convective cell which circulates in the clockwise direction. The intensity of the flow increases by increasing the thermal Rayleigh number and the heat transfer rate is slightly larger compared to the mass transfer rate.

- When the mass diffusivity is larger than the thermal diffusivity (Le<1), we obtained a convective cell that circulates in counterclockwise direction with very a high flow intensity. Owing to fast solute diffusing component, we noticed that the heat transfer rate becomes more important than the mass transfer rate.

Nomenclature

A	cavity aspect, L'/H'
D	mass diffusivity of species
H'	height of the cavity
j'	constant mass flux per unit area
K	permeability of the porous medium
Le	Lewis number, α/D
N	buoyancy ratio, $\beta_T \Delta S'/\beta_T \Delta T'$
Nu	Nusselt number
q'	constant heat flux per unit area
R_T	thermal Darcy Rayleigh number, $g\beta_T KH'\Delta T'/\alpha\nu$
S	dimensionless concentration, $(S'-S'_0)/\Delta S'$
Sh	Sherwood number
S'_0	reference concentration at x'=0 and y'=0
$\Delta S'$	characteristic concentration, $j'H'/D$
ΔS	dimensionless wall-to-wall concentration difference
T	dimensionless temperature, $(T'-T'_0)/\Delta T'$
t	dimensionless time, $t'\alpha/\sigma H'^2$
$\Delta T'$	characteristic temperature, $q'H'/k$
ΔT	dimensionless wall-to-wall temperature difference
u	dimensionless velocity in x-direction, $u'H'/\alpha$
v	dimensionless velocity in y-direction, $v'H'/\alpha$
x	dimensionless coordinate axis, x'/H'
y	dimensionless coordinate axis, y'/H'

Greek symbols

α	thermal diffusivity, $k/(\rho C)_f$
β_S	concentration expansion coefficient
β_T	thermal expansion coefficient
θ	angle of inclination of the cavity
ν	kinematic viscosity of the fluid
μ	dynamic viscosity of fluid
ρ	density of the fluid
$(\rho C)_f$	heat capacity of fluid
$(\rho C)_P$	heat capacity of saturated porous medium
σ	heat capacity ratio $(\rho C)_p/(\rho C)_f$
ε	dimensionless porosity of the porous medium, ε'/σ=1
ε'	porosity of the porous medium
Ψ	dimensionless stream function, Ψ'/α
Ψ_0	stream function value at centre of the cavity

Superscripts

$'$	dimensional variable
sub	subcritical
sup	supercritical

Superscripts

c	critical value
m	average value
max	maximum value
min	minimum value
$\circ$	reference state

REFERENCES

[1]. *D. A. Nield and A. Bejan* 2006. Convection in porous media. Third edition, Springer.

[2]. *M. Mamou, P. Vasseur and E. Bilgen,* 1995. "Multiple solutions for double-diffusive convection in a vertical porous enclosure", Internat. J. Heat Mass Transfer 38: 1787-1798.

[3]. *A. Mansour, A. Amahmid, M. Hasnaoui and M. Bourich.* 2004. "Soret effect on double-diffusive multiple solutions in a square porous cavity subject to cross gradients of temperature and concentration". Int. Comm. Heat Mass Transfer, Vol. 31, No. 3, pp.431-440.

[4]. *A. A. Mohamad and R. Bennacer,* 2002. "Double diffusion, natural convection in an enclosure filled with saturated porous Medium subjected to cross gradients; stably stratified fluid". International Journal of Heat and Mass Transfer 45: 3725–3740.

[5]. *A. Mansour, A. Amahmid, M. Hasnaoui and M. Bourich,* 2006. "Numerical study of the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and submitted to horizontal concentration gradient in the presence of Soret effect". Numer. Heat transfer, Part A 49, 69–94.

[6]. *M. Bourich, A. Amahmid and M. Hasnaoui,* 2005. "Double diffusive convection in a porous enclosure submitted to cross gradients of temperature and concentration". Energy Convers. Manage. 45, 1655–1670.

[7]. *N.H. Saeid,* 2006. "Natural convection in a square cavity with an oscillating wall temperature". The Arabian Journal for Science and Engineering, Volume 31, Number 1A.

[8]. *N.H. Saeid and I. Pop.* 2004. "Transient free convection in a square cavity filled with a porous medium". International Journal of Heat and Mass Transfer 47: 1917–1924.

[9]. *M.A. Mansour, A.J. Chamkha, R.A. Mohamed, M.M. Abd El-Aziz and S.E. Ahmed,* 2010. "MHD natural convection in an inclined cavity filled with a fluid saturated porous medium with heat source in the solid phase. Nonlinear Analysis: Modelling and Control", Vol. 15, No. 1, 55–70.

[10]. *A.A. Sezai and R.A. Mohamad*, "Three-dimensional double-diffusive convection in a porous cubic enclosure due to opposing gradients of temperature and concentration", J. Fluid Mech. 400 (1999) 333–353.

[11]. *A. Bergeon, K. Ghorayeb and A. Mojtabi*, "Double diffusive instability in an inclined cavity," Phys. Fluids 11, 549 (1999).

[12]. *P. Vasseur, M. G. Satish and L. Robillard,* "Natural convection in a thin, inclined, porous layer exposed to a constant heat flux", Int. J. Heat Mass Tran. 30:3 (1987), 537–549.

[13]. *O.V. Trevisan, and A. Bejan,* 1985. "Natural convection with combined heat and mass transfer buoyancy effects in a porous medium", Int. J. Heat and Mass Transfer, 28, 1597-1611.