

A DETERMINISTIC INVENTORY MODEL WITH WEIBULL DETERIORATION RATE UNDER TRADE CREDIT PERIOD IN DEMAND DECLINING MARKET AND ALLOWABLE SHORTAGE USING FUZZY LOGIC

Pinki MAJUMDER¹, U.K. BERA¹ and M. MAITI²

In this paper, we introduce an EOQ (Economic order quantity) model under two level trade credit using fuzzy logic. Here fuzzy logic connects the profit with credit period and unit purchasing cost by verbal words. This paper develops a model to determine an optimal profit in fuzzy environment. Numerical example is given to demonstrate the optimal decision for the retailer. We solve these models by GRG method and using LINGO13.0. Finally, we obtain the optimal profit with respect to credit period and unit purchasing cost by Mamdani method (using Matlab2010). We also analysis the difference between the optimal results using GRG method and Mamdani method.

Keywords: Weibull Deterioration, lot size, trade credit, declining demand, fuzzy logic.

1. Introduction

Now-a-days, due to the advent of multinationals for globalization, a supplier/wholesaler offers credit period to attract more customers. In this system, a relaxed period for payment is available to the retailer if outstanding amount is paid within the given credit period. Goyal (1985)[4] established a single item inventory model under permissible delay in payment. Shah (1993)[10], Aggarwal and Jaggi (1995)[1] considered inventory models with exponential deterioration under trade credit policy. Huang (2007b)[5] determined optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy. In last few years, several researchers developed inventory models for deteriorating items where the demand rate is constant, time dependent, instantaneous inventory level or initial stock. Numerous relevant papers have been produced relating to trade credit such as Huang(2003)[6], Chen and Kang (2010a)[3]. Recently Majumder P.(2013)[8] introduce an EOQ model of deteriorating items with time dependent demand and allowable shortage under trade credit.

In this paper we establish an deterministic inventory model with allowable shortage, time dependent demand, weibull deterioration and two trade credit period. Here, an **alternative approach** of payment for the remaining inventory

¹ Department of Mathematics, National institution of Technology, Agartala, Tripura(west), India, e-mail: pinki.mjmdr@rediffmail.com, bera_uttam@yahoo.co.in

² Department of Mathematics, Vidyasagar University, West Midnapore-721102, mmaiti2005@yahoo.co.in

after the credit period is also proposed. As it is considered that the unit selling price is larger than the unit purchasing price, the retailer must have enough amounts before the end of business period and he may pay that to the wholesaler some time before the end of the business period and in that situation, he will have to pay less interest to the wholesaler. Moreover, the retailer can earn more interest after that time till the end of the business period. This new approach to calculate the interest earned by the retailer is given in this model.

Moreover, a supplier/wholesaler normally charges unit price of an item from a retailer depending upon the amount/total units purchased by the retailer. Generally, if the total purchased units are high, unit price is low and vice-versa. When the amount is moderate, unit price is also in between high and low. Instead of specifying this relation precisely, sometimes it is defined imprecisely i.e., in fuzzy sense with verbal words. Recently, fuzzy inference technique has been introduced in some supply-chain models to monitor and control supply chain variables expressed by words. The fuzzy inference process is used to handle the natural language expressions of the type

IF premise (antecedent) THEN conclusion (consequent)

There are two types of fuzzy inference systems that can be implemented in the Fuzzy logic Toolbox (2005): Mamdani - type [Mamdani (1974, 1975)][7,9] and Sugeno – type (Takagi and Sugeno, 1985). These two types differ in the way by which output is determined. Mamdani's effort was based on Bellman and Zadeh's (Bellman and Zadeh, 1970[2]) work developing fuzzy algorithms for complex systems and decision process. The main difference between Mamdani and Sugeno is that Sugeno output membership functions are either linear or constant whereas the Mamdani output is a fuzzy set. For an inventory model with trade credit, a Mamdani- type inference system is selected here for evaluating and aggregating the fuzzy rules. Till now, none has considered the above type EOQ model which is solve by fuzzy logic.

The features of the present investigation are

i) None has considered, the EOQ models allowing trade credit with verbal relations among the model parameters/ quantities though these types of relations are very much in vogue in real-life situations. No investigations have been made for trade credit models with fuzzy inferences. This vacuum has been filled up in this investigation.

ii) In reality, during the business, shortages occur depending upon the demand. Till now, none has considered the EOQ model with shortages in delay in payment in fuzzy environment using new approach of payment. This has been considered in the present investigation.

In this paper we derive the optimal value of cycle time which maximize the total profit of the retailer using fuzzy logic. Lastly numerical examples are set to illustrate all results obtained in this paper. Comparison between the optimal

results using GRG method & Mamdani method are carried out in this paper. The sensitivity analysis is carried out to observe the changes in the optimal solution.

2. Assumptions and notation

The following notations and assumptions are used for the development of proposed model.

2a. Notation

- (i) $D(t)=a(1-bt)$; the annual demand as a decreasing function of time where $a > 0$ is fixed demand and $b(0 < b < 1)$ denotes the rate of demand.
- (ii) c = The unit purchase cost.
- (iii) s = The unit selling cost with $(s > c)$.
- (iv) h = The inventory holding cost per year excluding interest charges.
- (v) A = The ordering cost per order.
- (vi) p = The unit shortage cost.
- (vii) Q = The order quantity at time $t = 0$.
- (viii) $\theta(t)$ = The deteriorating rate which is a weibull function of time as $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$; $\beta > 0$ and $t > 0$.
- (ix) M = Retailer's trade credit period offered by the supplier in years.
- (x) N = Customer's trade period offered by the retailer in years.
- (xi) I_c = Interest charges payable per \$ per year to the supplier.
- (xii) I_e = Interest earned per \$ per year.
- (xiii) $I(t)$ = Inventory level at time t .
- (xiv) T_1 = Length of the period with positive stock of the item.
- (xv) T_2 = Length of the period with negative stock of the item.
- (xvi) T = Length of the replenishment cycle. $T = T_1 + T_2$
- (xvii) $Z(T_1; T_2)$ = Total Profit when the length of period with positive stock of the item is T_1 and the length of the period with negative stock of the item is T_2 .
- (xviii) T' = Time for payment for the remaining quantities after the credit period (M) by the retailer to the wholesaler.

2b. Assumption

The planning horizon is infinite. Consider single item. Shortages are allowed.

$I_c \geq I_e$, $s \geq c$, $M \geq N$. The deteriorated units can neither be repaired nor replaced during the cycle time.

3. Mathematical Formulation

The inventory level $I(t)$ depletes to meet the demand and deterioration. The rate of change of inventory level is governed by the following differential equation

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = -a(1-bt), \quad 0 \leq t \leq T_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -a(1-bt), \quad T_1 \leq t \leq T \quad (2)$$

With the initial condition $I(0)=Q$ and the boundary condition $I(T_1) = 0$

Consequently, the solution of (1) is given by

$$I(t)=ae^{\alpha t^\beta} \left[\frac{\alpha}{\beta+1} (T_1^{\beta+1} - t^{\beta+1}) - \frac{b\alpha}{\beta+2} (T_1^{\beta+2} - t^{\beta+2}) - \frac{b}{2} (T_1^2 - t^2) + (T_1 - t) \right] \quad (3)$$

$$\text{The order quantity is } Q=I(0)=a \left[\frac{\alpha}{\beta+1} T_1^{\beta+1} - \frac{b\alpha}{\beta+2} T_1^{\beta+2} - \frac{bT_1^2}{2} + T_1 \right] \quad (4)$$

Consequently, the solution of (2) is given by

$$I(t)=a \left\{ (T_1 - t) - \frac{b}{2} (T_1^2 - t^2) \right\} \quad (5)$$

The total cost of inventory system per time unit include the following:

$$(a) \text{Total ordering cost(OC)} : A \quad (6)$$

$$(b) \text{Total deterioration cost (DC)} : c\alpha a \left[\frac{T_1^{\beta+1}}{\beta+1} - \frac{bT_1^{\beta+2}}{\beta+2} \right] \quad (7)$$

$$(c) \text{Total inventory holding cost (HC)} : ah \left[\frac{\alpha^2 b}{(\beta+1)(2\beta+3)} T_1^{(2\beta+3)} - \frac{\alpha^2}{2(\beta+1)^2} T_1^{(2\beta+2)} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)} T_1^{(\beta+3)} + \frac{\alpha\beta}{(\beta+1)(\beta+2)} T_1^{(\beta+2)} - \frac{bT_1^3}{3} + \frac{T_1^2}{2} \right] \quad (8)$$

$$(d) \text{Shortage cost(SHC)} = -p \int_{T_1}^T I(t) dt = -p \int_{T_1}^T a \left\{ (T_1 - t) - \frac{b}{2} (T_1^2 - t^2) \right\} dt \\ = -p \left[\left(T_1 \cdot T - \frac{T^2}{2} - \frac{T_1^2}{2} \right) - \frac{b}{2} \left(T_1^2 \cdot T - \frac{T^3}{3} - \frac{2T_1^2}{3} \right) \right] \quad (9)$$

Regarding interest charges and earned three cases may arise based on the length of $M; N; T_1; T$

The three cases are as follows

Case1: $N \leq M \leq T' \leq T_1 < T$; Case2: $N \leq T_1 \leq M < T$; Case3: $T_1 \leq N \leq M < T$

4. According to given assumption, there are three cases to occur in interest earned and interest charged.

4.1 Case1: $N \leq M \leq T' \leq T_1 < T$

4.1.a Conventional Approach:

Total interest earned TIE is given by

$$\text{TIE} = s I_e \left[(T - N) \int_0^N a(1 - bt) dt + (1 + T - M) \int_N^M a(1 - bt)(M - t) dt + (1 + T - T_1) \int_M^{T_1} a(1 - bt)(M - t) dt \right] \\ = s I_e \left[(T - N) \left(-\frac{a.N(bN-2)}{2} \right) - (1 + T - M) \left\{ \frac{a(M-N)^2(M.b+2.b.N-3)}{6} \right\} + (1 + T - T_1) \left\{ \frac{a(T_1-M)^2(T_1.b+2.b.M-3)}{6} \right\} \right] \quad (10)$$

Total interest payable TIP is given by

$$\text{TIP} = (c.i_c.a) \int_M^{T_1} I(t) dt = (c.i_c.a) \int_M^{T_1} \left[\frac{\alpha}{\beta+1} (T_1^{\beta+1} - t^{\beta+1}) - \frac{b\alpha}{\beta+2} (T_1^{\beta+2} - t^{\beta+2}) - \frac{b}{2} (T_1^2 - t^2) + (T_1 - t) \right] dt \\ = (c.i_c.a) \left[\frac{\alpha^2 b}{(\beta+1)(2\beta+3)} T_1^{(2\beta+3)} - \frac{\alpha^2}{2(\beta+1)^2} T_1^{(2\beta+2)} - \frac{b\alpha\beta}{(\beta+1)(\beta+2)} T_1^{(\beta+3)} + \frac{\alpha\beta}{(\beta+1)(\beta+2)} \right]$$

$$\begin{aligned}
& T_1^{(\beta+2)} - \frac{bT_1^3}{3} + \frac{T_1^2}{2} - \left(\frac{\alpha T_1^{(\beta+1)}}{(\beta+1)} - \frac{\alpha \beta T_1^{(\beta+2)}}{(\beta+2)} - \frac{bT_1^2}{2} + T_1 \right) \left(M - \frac{\alpha M^{(\beta+1)}}{(\beta+1)} \right) - \\
& \frac{\alpha \beta}{(\beta+1)(\beta+2)} M^{(\beta+2)} + \frac{\alpha \beta b}{2(\beta+2)(\beta+3)} M^{\beta+3} - \frac{\alpha^2}{(\beta+1)(2\beta+2)} M^{2\beta+2} + \\
& \frac{\alpha^2 b}{(\beta+2)(2\beta+3)} M^{2\beta+3} - \frac{bM^3}{6} + \frac{M^2}{2} \quad (11)
\end{aligned}$$

4.1.b New Approach

In new approach the retailer pay all his/her dues before the cycle. Here the retailer pay all the due amount to the supplier before T_1 (length of the period with positive stock of the item). Revenue earned during $(0, M)$ *REM* and interest earned during $(0, M)$ *IEM* are given by

$$REM = s \int_0^M a(1-bt)dt = sa \left[M - \frac{bM^2}{2} \right] \quad (12)$$

$$\begin{aligned}
IEM &= s I_e \left[(M-N) \int_0^N a(1-bt)dt + \int_N^M a(1-bt)(M-t)dt \right] \\
&= s I_e \left[(M-N) \left(-\frac{a.N(bN-2)}{2} \right) - \left\{ \frac{a(M-N)^2(M.b+2.b.N-3)}{6} \right\} \right] \quad (13)
\end{aligned}$$

Amount paid at the end of the credit period (M), total interest to be paid, *TIP*, revenue earned during (M, T') *RET'* and interest earned during (M, T') *IET'* are respectively given by

$$H = REM + IEM \quad (14)$$

$$TIP = (PC-H) \{1+I_c\} (T' - M) \quad (15)$$

$$RET' = s \int_M^{T'} a(1-bt)dt = sa \left[T' - M - \frac{b}{2} (T'^2 - M^2) \right] \quad (16)$$

$$IET' = s I_e \left[\int_M^{T'} a(1-bt)(T' - t)dt \right] = s I_e \left(-\left\{ \frac{a(T'-M)^2(T'.b+2.b.M-3)}{6} \right\} \right) \quad (17)$$

The condition to find T' , Revenue earned during (T', T) *TRE* and interest earned during (T', T) *TIE* are respectively given by,

$$RET' + IET' = TIP \quad (18)$$

$$TRE = s \int_{T'}^{T_1} a(1-bt)dt = sa \left[T_1 - T' - \frac{b}{2} (T_1^2 - T'^2) \right]$$

$$\begin{aligned}
TIE &= s I_e \left[(1+T-T_1) \int_{T'}^{T_1} a(1-bt)(T_1-t)dt \right] \\
&= s I_e \left[(1+T-T_1) \left(-\left\{ \frac{a(T_1-T')^2(T_1.b+2.b.T'-3)}{6} \right\} \right) \right] \quad (19)
\end{aligned}$$

4.2 Case2 : $N \leq T_1 \leq M < T$

In this case $T_1 \leq M$, so there is no need of T' .

Therefore for this case annual interest payable (*TIP*) = 0

Revenue earned during $(0, T)$ *TRE* and interest earned during $(0, T)$ *TIE* are respectively given by,

$$TRE = s \int_0^{T_1} a(1-bt)dt = sa \left[T_1 - \frac{bT_1^2}{2} \right] \quad (20)$$

$$TIE = s I_e \left[(T-N) \int_0^N a(1-bt)dt + (1+T-T_1) \int_N^{T_1} a(1-bt)(T_1-t)dt \right]$$

$$= sI_e[(T - N)(aN - \frac{abN^2}{2}) + (1 + T - T_1) (-\{\frac{a(T_1 - N)^2(T_1.b + 2.b.N - 3)}{6}\})] \quad (21)$$

4.3 Case3 : $T_1 \leq N \leq M < T$

In this case $T_1 \leq N \leq M$, so there is no need of T' .

Therefore for this case annual interest payable (TIP) = 0

Revenue earned during $(0, T)$ TRE and interest earned during $(0, T)$ TIE are respectively given by,

$$TRE = s \int_0^{T_1} a(1 - bt)dt = sa [aT_1 - \frac{abT_1^2}{2}] \quad (22)$$

$$TIE = sI_e[(T - N) \int_0^{T_1} a(1 - bt)dt] = sI_e[(T - N)[aT_1 - \frac{abT_1^2}{2}]] \quad (23)$$

Hence the required objective (profit) function, which is to be maximized for all cases described above is given by

$$Z = \frac{1}{T}(\text{TRE} + \text{TIE} - \text{HC} - \text{SHC} - \text{A} - \text{DC}) \quad (24)$$

In order to maximize the above profit function, the Generalized Reduced Gradient technique LINGO 14.0 optimizer code is used to find the optimum values of the decision variables.

5. EOQ model with fuzzy logic

In this investigation, the objective is to maximize the total profit function (z) using fuzzy logic. Here retailer purchases material from the wholesaler. Here unit purchasing cost c and credit period are fuzzy in nature.

If we denote the membership functions of the fuzzy numbers for the per unit purchasing cost by $\mu_{1low}(x), \mu_{1medium}(x), \mu_{1high}(x)$, the membership function of the fuzzy numbers for the credit period M by $\mu_{2low}(x), \mu_{2medium}(x), \mu_{2high}(x)$ and the membership functions of the fuzzy numbers for the total profit (Z) by $\mu_{3low}(x), \mu_{3very low}(x), \mu_{3medium}(x), \mu_{3high}(x), \mu_{3very high}(x)$. Now, the membership values of fuzzy purchasing cost c , credit period M and profit are represented as follows

$$\mu_{1low}(x) = \begin{cases} 0 & \text{if } x \leq a_0 \\ 1 & \text{if } a_0 \leq x \leq a_1 \\ \frac{a_2 - x}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 0 & \text{if } x \geq a_2 \end{cases}, \mu_{1medium}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x \geq a_3 \end{cases}, \mu_{1high}(x) = \begin{cases} 0 & \text{if } x \leq a_2 \\ \frac{x - a_2}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{if } x \geq a_4 \end{cases}$$

$$\begin{aligned}
\mu_{2low}(x) &= \begin{cases} 0 & \text{if } x \leq b_0 \\ 1 & \text{if } b_0 \leq x \leq b_1 \\ \frac{a_2-x}{a_2-a_1} & \text{if } b_1 \leq x \leq b_2 \\ 0 & \text{if } x \geq b_2 \end{cases} & \mu_{2medium}(x) &= \begin{cases} 0 & \text{if } x \leq b_1 \\ \frac{x-b_1}{b_2-b_1} & \text{if } b_1 \leq x \leq b_2 \\ \frac{b_2-x}{b_2-b_2} & \text{if } b_2 \leq x \leq b_3 \\ 0 & \text{if } x \geq b_3 \end{cases} & \mu_{2high}(x) &= \begin{cases} 0 & \text{if } x \leq b_2 \\ \frac{x-b_2}{b_3-b_2} & \text{if } b_2 \leq x \leq b_3 \\ 1 & \text{if } b_3 \leq x \leq b_4 \\ 0 & \text{if } x \geq b_4 \end{cases} \\
\mu_{3very low}(x) &= \begin{cases} 0 & \text{if } x \leq c_0 \\ 1 & \text{if } c_0 \leq x \leq c_1 \\ \frac{c_2-x}{c_2-c_1} & \text{if } c_1 \leq x \leq c_2 \\ 0 & \text{if } x \geq c_2 \end{cases} & \mu_{3low}(x) &= \begin{cases} 0 & \text{if } x \leq c_1 \\ \frac{x-c_1}{c_2-c_1} & \text{if } c_1 \leq x \leq c_2 \\ \frac{c_2-x}{c_2-c_2} & \text{if } c_2 \leq x \leq c_3 \\ 0 & \text{if } x \geq c_3 \end{cases} & \mu_{3medium}(x) &= \begin{cases} 0 & \text{if } x \leq c_2 \\ \frac{x-c_2}{c_3-c_2} & \text{if } c_2 \leq x \leq c_3 \\ \frac{c_3-x}{c_3-c_3} & \text{if } c_3 \leq x \leq c_4 \\ 0 & \text{if } x \geq c_4 \end{cases} \\
\mu_{3high}(x) &= \begin{cases} 0 & \text{if } x \leq c_3 \\ \frac{x-c_3}{c_4-c_3} & \text{if } c_3 \leq x \leq c_4 \\ \frac{c_4-x}{c_4-c_4} & \text{if } c_4 \leq x \leq c_5 \\ 0 & \text{if } x \geq c_5 \end{cases} & \mu_{3medium}(x) &= \begin{cases} 0 & \text{if } x \leq c_2 \\ \frac{x-c_2}{c_3-c_2} & \text{if } c_2 \leq x \leq c_3 \\ \frac{c_3-x}{c_4-c_3} & \text{if } c_3 \leq x \leq c_4 \\ 0 & \text{if } x \geq c_4 \end{cases} & \mu_{3high}(x) &= \begin{cases} 0 & \text{if } x \leq c_3 \\ \frac{x-c_3}{c_4-c_3} & \text{if } c_3 \leq x \leq c_4 \\ \frac{c_4-x}{c_5-c_4} & \text{if } c_4 \leq x \leq c_5 \\ 0 & \text{if } x \geq c_5 \end{cases} \\
\mu_{3very high}(x) &= \begin{cases} 0 & \text{if } x \leq c_4 \\ \frac{x-c_4}{c_5-c_4} & \text{if } c_4 \leq x \leq c_5 \\ 1 & \text{if } c_5 \leq x \leq c_6 \\ 0 & \text{if } x \geq c_6 \end{cases}
\end{aligned}$$

Using these membership functions, the following nine rules are proposed for total profit (Z), Credit period (M) and per unit purchasing cost (c)

- R_1 :if purchasing cost is low and credit period is low then total profit is medium.
 R_2 :if purchasing cost is low and credit period is medium then total profit is high.
 R_3 :if purchasing cost is low and credit period is high then total profit is very high.
 R_4 :if purchasing cost is medium and credit period is low then total profit is low.
 R_5 :if purchasing cost is medium and credit period is medium then total profit is medium.
 R_6 :if purchasing cost is medium and credit period is high then total profit is high.
 R_7 :if purchasing cost is high and credit period is low then total profit is very low.
 R_8 :if purchasing cost is high and credit period is medium then total profit is low.
 R_9 :if purchasing cost is high and credit period is high then total profit is medium.

Thus using these rules and corresponding membership function one can find the value of profit corresponding to values of unit purchasing cost and credit period by Mamdani method. We also compare the results obtained by Mamdani method with our numerical results which is obtained by solving the above models.

6. Numerical Experiment

To illustrate the EOQ model numerically, we consider the following crisp and imprecise input data (taken as triangular fuzzy number)

Fuzzy Data: Unit purchasing cost(c):Low = (5 , 7.5,13.5), medium= (7.5 , 13.5, 25.5), High= (13.5 , 25.5, 30) ,Credit period(M) : Low =(0.005,0.023,0.5) ,medium=(0.023,0.5,1.2), High=(0.5,1.2,1.5)

Crisp Data: Let $S = 20$; $P = 2$; $I_c = 2.5$; $I_e = 1.5$; $A = 30$; $a = 1000$; $b = 0.07$; $\alpha = 0.5$; $\beta = 2$; $h = 2$. Let $N=0.03$ for case 1, $N=0.4$ for case2, $N=0.9$ for case 3.

Out put: Let us defined the classification of output (profit Z) as follows

Profit (Z):Very low=(8000,8839,10000),low=(8839,10000,14000),medium(10000, 14000, 19000), High=(14000,19000,25000),very high=(19000,25000,40000)

Table 1

Optimal Results for Model 1

output Case				T_1	T'	T	Z
Case 1:Conventional Approach	6	15	03	7282003	—	9466604	2449.76
Case 1 :New Approach	6	15	03	7282003	175689	9466604	6429.93
Case2 : Conventional Approach and New App- roach are same as $M \geq T_1$	9	9	4	6086957	—	1 .4	6622.14
Case3:Conventio nal Approach and New App- roach are same as $M \geq T_1$	8	.3	9	7257143	—	2 .54	7431.72

Table 2

Comparison between the results obtained by GRG method (using LINGO 13) & Mamdani method (using Matlab 2010)

Conventional Approach Case 1: $N \leq M \leq T' \leq T_1 < T$		Case2 : $N \leq T_1 \leq M < T$		Case3 : $T_1 \leq N \leq M < T$	
(C=16, M=0.15)		(C= 19, M=0.90)		(C=18, M=1.3)	
Using GRG Method	Using Mandani Method	Using GRG Method	Using Manda- ni Method	Using GRG Method	Using Man- dani Method
Profit Z	Profit Z	Profit Z	Profit Z	Profit Z	Profit Z
12449.76	12465.00	16622.14	16626.00	17431.72	17741.00

Table 3:

Effect of p on profit function Z for the above three cases

p	Z_1 (Case 1. $N \leq M \leq T' \leq T_1 < T$)		Z_2 (Case2 : $N \leq T_1 \leq M < T$)	Z_3 (Case3: $T_1 \leq N \leq M < T$)
	Conventional	New		
2.3	13572.62	16422.80	16559.15	17255.43
2.5	13567.87	16418.04	16517.16	17137.91
2.7	13563.11	16413.28	16475.16	17020.39
3.1	13553.59	16403.77	16391.17	16785.34
3.3	13548.83	16399.01	16349.18	16667.82
3.5	13544.07	16394.25	16307.18	16550.30
3.7	13539.31	16388.32	16265.19	16432.77

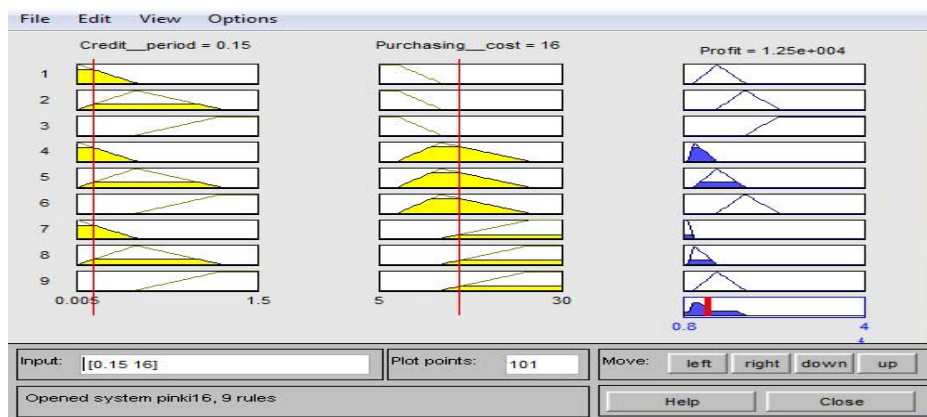
Fig. 1.Fuzzy rule for membership functions for Profit Z for case 1(conventional).Fig. 2.Fuzzy rule for membership functions for Profit Z for case 2

Table 4:

Effect of c on profit function Z for the above three cases

c	Z_1 (Case 1. $N \leq M \leq T' \leq T_1 < T$)		Z_2 (Case2 : $N \leq T_1 \leq M < T$)	Z_3 (Case3: $T_1 \leq N \leq M < T$)
	Conventional	New		
18	12488.62	16464.54	16648.13	17431.72
17	13034.19	16454.86	16674.12	17455.84
16	12449.76	16429.93	16700.11	17479.96
15	14125.34	16385.35	16726.10	17504.09
14	14670.91	16312.06	16752.10	17528.21
13	15216.48	16207.69	16778.09	17552.33

Fig. 3.Fuzzy rule for membership functions for Profit Z for case 3

Table 5

Effect of α on profit function Z for the above three cases

α	Z_1 (Case 1. $N \leq M \leq T' \leq T_1 < T$)		Z_2 (Case2 : $N \leq T_1 \leq M < T$)	Z_3 (Case3: $T_1 \leq N \leq M < T$)
	Conventional	New		
0.1	15145.04	16882.72	17028.89	17791.60
0.2	14743.18	16778.68	16927.09	17701.60
0.3	14348.34	16668.65	16825.36	17611.41
0.4	13960.54	16552.33	16723.71	17521.50
0.5	12449.76	16429.93	16622.14	17431.72

Table 6

Effect of h on profit function Z for the above three cases

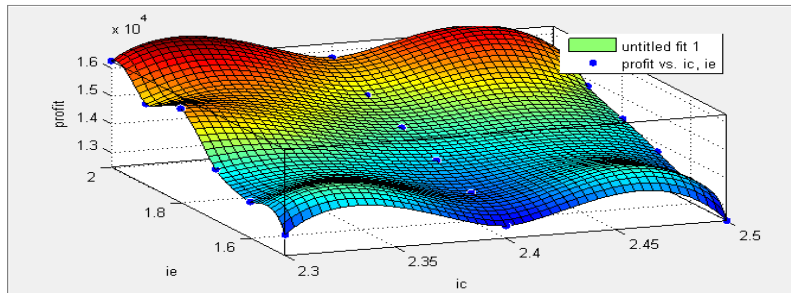
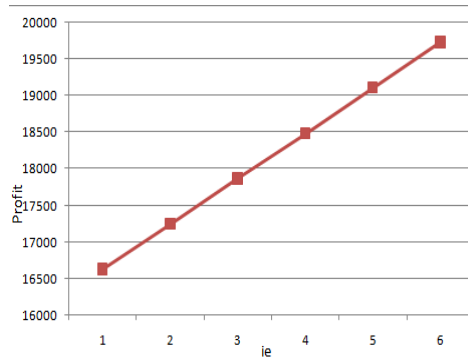
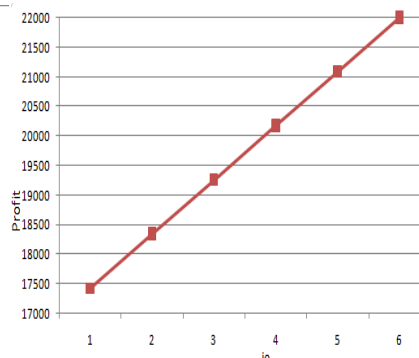
h	Z_1 (Case 1. $N \leq M \leq T' \leq T_1 < T$)		Z_2 (Case2 : $N \leq T_1 \leq M < T$)	Z_3 (Case3: $T_1 \leq N \leq M < T$)
	Conventional	New		
3	13287.55	16128.53	16486.17	17323.59
3.5	13141.45	15982.45	16418.18	17269.52

4	12995.34	15845.51	16350.20	17215.46
4.5	12849.24	15699.41	16282.21	17161.39
5	12703.13	15553.30	16214.22	17107.33
5.5	12557.03	15398.13	16146.24	17053.27

Table 7

Effect of b on profit function Z for the above three cases

b	Z_1 (Case 1: $N \leq M \leq T' \leq T_1 < T$)		Z_2 (Case 2: $N \leq T_1 \leq M < T$)	Z_3 (Case 3: $T_1 \leq N \leq M < T$)
	Conventional	New		
0.01	13391.99	16983.46	16812.47	17737.38
0.02	13223.28	16890.20	16780.75	17686.44
0.03	13054.58	16797.34	16749.03	17635.49
0.04	12885.87	16704.89	16717.31	17584.55
0.05	12617.17	16612.84	16685.59	17533.60
0.06	12558.47	16521.18	16653.86	17482.66
0.07	12449.76	16429.93	16622.14	17431.72
0.08	11911.06	16339.09	16590.42	17380.77

Fig4: Change of i_c & i_e on profit function Z for Case:1(conventional)Fig 5: Change of i_e on profit function Z for Case:2Fig6: Change of i_e on profit function Z for case 3

7. Result and Discussions

Table 1 describes the optimal results in different cases. From Table 1 we can observe that in case 1 profit for new approach is greater than the profit for conventional approach. Practically if the retailer pay all the due amount to the supplier before the cycle time, it is obvious that he have to pay less interest & in this condition he will get more profit. In Table 1(case 1) a comparison between conventional and new approach is carried out & we show that in new approach the retailer get more profit than the conventional approach. From Table 2 we observe that for case 1 the optimal profit solved by GRG method is **12449.76** which is near to the value **12465.00** obtained by Mamdani method. For case 2 we observe that the optimal profit solved by GRG method is **16622.14** which is near to the value **16626.00** obtained by Mamdani method. For case 3 we observe that the optimal profit solved by GRG method is **17431.72** which is near to the value **17741.00** obtained by Mamdani method. From Table 3 we observe that with the increase of p i.e with the increase of unit shortage cost, the shortage cost increases, so the profit Z decreases for three cases. From Table 4 we see that with the increase of C i.e with the increase of purchasing cost, the profit Z decreases for three cases. From Table 5 we observe that with the increase of α i.e with the increase of deterioration, the profit Z decreases for three cases. From Table 6 we observe that with the increase of h i.e with the increase of unit holding cost, the holding cost increases and in result the profit Z decreases for three cases. From Table 7 we see that with the increase of b , the demand decreases, so the profit Z decreases for three cases. From fig 4 we see that for case 1 when I_c (interest payable) increases then profit function decreases. If I_c increases then the retailer have to pay more interest to the supplier. For this retailer get less profit. Also from this figure we see that with the increase of I_e (interest earned) the total profit also increases. If I_e increases then the retailer have the opportunity to get more interest from the customer and consequently the retailer get more profit. From fig 5,6 (for case 2 and 3 respectively) we observe that with the increase of I_e (interest earned), the profit Z increases.

7.a Case study:

Case 1: In practical sense, increase in purchasing cost per unit (C) decreases the total Profit and decrease in credit period (M) also decreases the total profit.

Here, for the first case of Table 2, the purchasing cost per unit ($C=16$) lies between 15 to 17 (i.e., in Medium and High zones), the credit period ($M = 0.15$ units) lies between 0 and 1 (i.e., in Low and Medium zones) and the profit ($Z= 12449.76$ units) lies in Low and medium zone. Here, $\mu_{Low}(Z) = 0.387$, $\mu_{Medium}(Z) = 0.612$, $\mu_{Medium}(C) = 0.792$, $\mu_{High}(C) = 0.21$, $\mu_{Low}(M) = 0.734$, $\mu_{Medium}(M) = 0.266$

Case 2: Here, for the 2nd case of Table 2, the purchasing cost per unit ($C=19$) lies between 18 to 20 (i.e., in Medium and High zones), the credit period ($M = 0.9$

units) lies between 0 and 1 (i.e., in Medium and High zones) and the profit ($Z = 16622.14$ units) lies in medium and high zone. Here, $\mu_{High}(Z) = 0.524$, $\mu_{Medium}(Z) = 0.475$, $\mu_{Medium}(C) = 0.542$, $\mu_{High}(C) = 0.458$, $\mu_{High}(M) = 0.571$, $\mu_{Medium}(M) = 0.428$

Case 3: Here, for the 3rd case of Table 2, the purchasing cost per unit ($C=18$) lies between 17 to 19 (i.e., in Medium and High zones), the credit period ($M = 1.3$ units) lies between 1 and 1.5 (i.e., in High zone) and the profit ($Z = 17431.72$ units) lies in medium and high zone. Here, $\mu_{High}(Z) = 0.686$, $\mu_{Medium}(Z) = 0.314$, $\mu_{Medium}(C) = 0.625$, $\mu_{High}(C) = 0.375$, $\mu_{High}(M) = 0.5$

8. Practical/Business implication

Business of seasonal products like cold drinks, garments, rainy shoes etc always in finite quantity. In this case stocks always follow some fuzzy rule connecting two or more parameters as in section 5. These parameter may be purchasing cost, trade credit period, total profit. Here an EOQ model is formulated for the retailer for optimal profit using fuzzy logic & new approach of payment. This may be used for business management of developing countries like India, Nepal etc. where business deals are more imprecise with verbal words. For practical implementation numerical examples in different cases are given.

9. Managerial Decision

For EOQ model of a weibull deteriorating items with trade credit having fuzzy relation among profit, credit period, unit purchasing under new approach of payment, the managerial decisions are:

- (i) The optimal value of profit is highly sensitive to the credit period, purchasing cost, selling price, unit holding cost, unit shortage cost, deterioration, demand.
- (ii) An EOQ model have been developed allowing trade credit with verbal relation among model parameters which are very much vogue in real-life situation.
- (iii) A new approach of payment is introduced in this paper. In new approach the profit of the retailer is greater than the conventional approach.
- (iv) Longer credit period provides more profit.

10. Conclusion

The proposed model in-cooperates realistic phenomenon and practical features such as trade credit period. In keeping with this reality, these factors are incorporated into the present model. Here a new approach is introduced for getting the maximum profit with respect to retailer. Also in this paper fuzzy logic, more precisely fuzzy inference rules are introduced. Here the fuzzy logic connects the profit function with the credit period and unit purchasing cost by verbal words.

Some comparison is carried out in this paper between the optimal results obtained by GRG method & Mamdani method. GRG method is one of the class of techniques called reduced-gradient or gradient projection methods which is based on extending methods for linear constraints to apply to nonlinear constraints. They adjust the variables so the active constraints continue to be satisfied. In our research manuscript we use Hyper Lingo 13.0. Using this software we can solve a model having 8000 variable, 800 integers, 800 nonlinear variables, 20 global variable, 4000 constraints. In this paper we obtain the total profit by GRG method (using Lingo 14) & also by Mamdani method (using Mat Lab 2010). In both methods we get nearly similar profit. So we can conclude that GRG method is useful for solving nonlinear inventory problems. Numerical examples are presented to justify the claim of each case of the model analysis by obtaining the optimal inventory length, shortage time period and also calculated the total profit function. The proposed model can be extended in several ways. For instance, we may extend this model for partial trade credit period, quantity discount, taking selling price, ordering cost, and demand as a fuzzy number.

REFERENCES

- [1] S.P. Aggarwal, C.K. Jaggi, "Ordering policies of deteriorating items under permissible delay in payments", *Journal of the Operational Research Society*, **46** (5), 1995, 658–662.
- [2] R. E. Bellman, L. A. Zadeh, "Decision- making in a fuzzy environment", *Management Sciences*, **17**, 1970, 141-164.
- [3] L.H. Chen, F.S Kang, "Integrated inventory models considering permissible delay in payment and variant pricing strategy", *Appl. Math. Model*, 2010, **34**, 36-46.
- [4] S.K. Goyal, "Economic order quantity under conditions of permissible delay in payments", *J. Operat. Res. Soc.*, 1985, **36**, 335-338.
- [5] Y. F. Huang, "Supply Chain Model for the Retailer's ordering policy under two levels of delay payments derived algebraically", *Opsearch*, **44**, 2007b, 366-377.
- [6] Y.F. Huang, "Optimal retailer's ordering policies in the EOQ model under trade credit financing", *Journal of the Operational Research Society*, **54** (9), 2003, 1011–1015.
- [7] E. H. Mamdani, "Application of fuzzy algorithm for control of simple dynamic plant, *Prod. IEEE*", 1974, **12**, 1221: 1585-1888.
- [8] P. Majumder, U. K. Bera., "A deterministic inventory model for deteriorating items with time dependent demand and allowable shortage under trade credit", *Int. J Current science and technology*, 2013, **1**, 197-206.
- [9] E. H. Mamdani, S. Assilina, "An experiment in linguistic synthesis with a fuzzy logic controller", *Int. J. Man-Machine Studies*, 1975, **7**, 1-13.
- [10] N. H. Shah, "A lot size model for exponentially decaying inventory when delay in payments is permissible", *Cahiers du CERO*, 1993a, **35**, 115- 123.