

## MATHEMATICAL MODELS USED IN QUALITY MANAGEMENT OF THE ELECTRICAL ENERGY

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*În cadrul acestei lucrări, principalul scop urmărit este de a prezenta metodologia de elaborare a unor programe în domeniul consumului de energie, cu ajutorul unor modele matematice.*

*Programele consumului de energie am realizat-o pe o perioadă scurtă cu un model matematic de tip probabilist, explorativă, deoarece analiza are loc din trecut în viitor, variabila independentă este timpul, considerându-se programele directă.*

*Datele estimate și programele în serie de timp este făcută folosind metodele de modelare. Am elaborat folosind programul Matlab, modele matematice pentru programele consumului de energie electrică.*

*As part of this paper, the main purpose is to present the elaboration methodology of some forecasts in the energy consumptions area, using few mathematical models.*

*The forecast for energy consumptions I realised for a short period of time with a mathematical model, probabilist type, because the analysis is taken place from past to future and the independent variable is the time, and we consider that the prognosis is direct.*

*The dates estimation and the forecast in a time series is made using the modeling methods will be discussed in this paper. We have elaborate using Matlab, the mathematical model for the forecast of the electrical energy consumption.*

**Keywords:** Mathematical model, forecast, electrical energy, consumption, additive model.

### 1. Introduction

The importance of the forecast in management is very important. The initiatory of the private process of the electricity distribution, the generalization of the dealing on the market, the substantiation of a new mechanisms and instruments for the market risk management and the bigger decentralization of the dealing with electric energy are some of the most important aspects in which the forecast

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studies on short term, are very important. [1] In this context, the paper shows a lots of aspects conected with the forecast on short term of the electric energy consumption.[2] The powerfull industrial development have brought important changes in all areas, and this were reflected in the environment, and also also at the society level. The only possibility for mentain the control on the fast and important changes is the adaptive behavior against this changes. This means firstly to establish by forecast the future development and the corect appreciation of the factors impact and the decizions on the future, and secondly, the decizions phase, are introduced the castigations needed for the purposes.[3]

Is self-understood that because of the forecast and taking decizions, are processes which are developed in time, in conditions of aleatory perturbation, the adaptation process must be continuous, showed in forecasts and rehearses castigations which will mentain the evolution on the target. More, once we are close to the specific targets, in the future are new targets, who presumes new forecast horizons and new decisions.[4]

As part of this paper, the main purpose is to present the elaboration methodology of some forecasts in the energy consumptions area, using few mathematical models.

Following this idea, the next definition for the forecast of the energy consumption and power also, comes like:

The forecast for the electrical energy consumption and power also is the scientific activity with the main purpose: the forecast for the energy consumption and power based on calculations analysis and based on the interpretation of a different dates, so we will obtain a more precise concordance between the estimated consumptions and the one effectively realised.[5]

We can see that a batch of parameters (reasons) with aleatory character leads to the energy consumption: climatic factors, demographical factors, economical factors and another factors.

The methodology of elaboration of a forecast study for the energy consumption has few main steps:

- collecting, selection and analyse the initial dates;
- establishing the mathematical model for the consumption;
- the analyse for the variance which has been obtained for the forecast and establishing the final decision. [6]

## 2. Components of the mathematical model for the energy consumption

Consumption curve, represents the energy fluctuation in time (or taking into consideration another parameter) and it can be split in more components. The forecast experience of the energy consumption shows four main components which determine the consumption curve ( $W$ ) (See example Fig. 1)[7]:

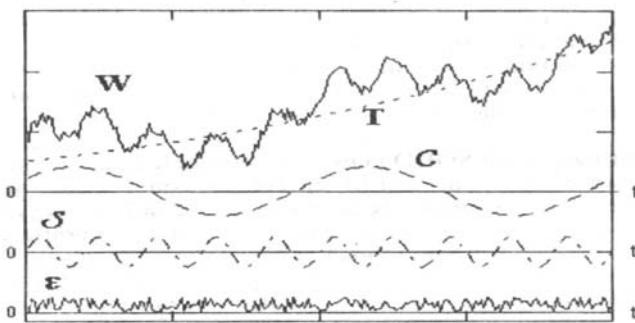


Fig.1. The components for the mathematical model of the energy consumption curve

1. the trend (T) represents the consumption's main compound, establishing the modification essential form of energy consumption.
2. the cyclic component (C) it's due to the existence of some fluctuating causes with a slow effect like the request-supply correlation with a period over a year.
3. the seasonal component (S) it is caused by certain parameters which present seasonal fluctuations (especially climatic elements). This component has a few months variation period and a similar shape for all years.[8]
4. the aleatory component ( $\varepsilon$ ) it is due to perchance causes, that has been previously specified.

As a conclusion, the energy consumption results, totaling the elements that have been specified above:

$$W(t) = T(t) + C(t) + S(t) + \varepsilon(t) \quad (1)$$

## 3 The extrapolative methods principle

The direct forecast methods are supposing the assumption that the causes, the factors and the trends which established the energy consumption in the past are also maintaining in the future, without appearing any dramatic and sudden changes during the forecast which to affect the consumption evolution.[9]

This assumption justifies the energy consumptions evolution trend extrapolation from the past for the future period and brings the forecast problem to the analysis of the energy consumption variation law from the past to the future.[10]

The mooted forecast methods are supposing the establishment of a mathematical model likeness a one or more variables function (generally a single variable, time) who fairly estimates the trend on the last period. The estimation of the functions coefficients is making by solving an equations system where the coefficients are calculating means the energy consumptions from the last period.[11]

### 3.1 The estimation for the model components

It is considered a value set  $y_t$  observed, of a chronological serie.

Mathematical shaping can be made using an additive model:

$$y_t = T_t + C_t + S_t + R_t \quad (2)$$

We consider the additive model:  $y_t = T_t * C_t * S_t * R_t$  where:  $T_t$  represents the trend (continuous component),  $C_t$  represents cyclical component,  $S_t$  represents seasonal components,  $R_t$  represents the component due to aleatory variations.

The additive model is merged in additiv model by logarithmical way.[12]

**a)The trend  $T_t$**  is determined by using liniar model:

$y_t = b_0 + b_1 * t + \varepsilon_t$ , where finding the parameters  $b_0, b_1$  is made with matrix method.

Are noted the following matrixs:

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \dots & \dots \\ 1 & x_n \end{pmatrix}, Y = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix}, B = \begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \end{pmatrix} \Rightarrow B = (X'X)^{-1}(X'Y) \quad (3)$$

$\Rightarrow \hat{b}_0$  and  $\hat{b}_1$  parameters which determine the regressive right line :

$$y_t = \hat{b}_0 + \hat{b}_1 \cdot x_t. \quad [13] \quad (4)$$

The advantaje for this method is that it can be applied successfully in case of multiple regressive and non right line regressive. [14]

**b) The cyclical component  $C_t$**  is aquired using the additive model

$$y_t = T_t * C_t * S_t * R_t$$

**Graphical method**

1. is established the trend (regressive right line )
2. for each period of time is evaluated by calculations the trend value  $\hat{y}_t$

3. the percent of the trend is  $\frac{y_t}{\hat{y}_t} * 100$ . Is graphically represented, the points  $\left( t, \frac{y_t}{\hat{y}_t} * 100 \right)$ ,  $t=1,..,n$  and the line 100%. If we see a cyclic phenomenon, we can consider the cycle with the length  $T$ .[15]

***The methods for development in Fourier series***

We have the following steps: In the simple cases,  $y_t$  can be represented using the mathematical formula:  $y_t = \alpha + \beta \cdot \sin \frac{2\pi t}{T} + \gamma \cdot \cos \frac{2\pi t}{T} + \varepsilon_t$ ,  $T$ =cycle period,  $(\varepsilon_t)_t$  is the aleatory component.[16]

If  $T$  is known and  $n$  (number of observations) is a  $T$  multiple:  $n = m*T$ , than  $m$  is the complete cycle number involved in our analysis. The unknown parameters  $\alpha, \beta, \gamma$  are calculated using the method of the smallest squares. So we can obtain the calculations:

$$\hat{\alpha} = \frac{1}{n} \sum_{i=1}^n y_t = \frac{1}{T} \sum_{u=1}^T \bar{z}_u, \hat{\beta} = \frac{1}{T} \sum_{u=1}^T \bar{z}_u \cdot \sin\left(\frac{2\pi u}{T}\right), \quad (5)$$

$$\hat{\gamma} = \frac{1}{T} \sum_{u=1}^T \bar{z}_u \cdot \cos\left(\frac{2\pi u}{T}\right), \quad (6)$$

where  $\bar{z}_u = \frac{1}{m} \sum_{v=0}^{m-1} y_{u+vT}$ ,  $u = 1, 2, \dots, T$ . With  $T$  estimated in this way, are graphically represented the points  $(t, \hat{y}_t), t=1,2,..,n$

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} \cdot \sin \frac{2\pi t}{T} + \hat{\gamma} \cdot \cos \frac{2\pi t}{T}. \quad [17] \quad (7)$$

**c) The seasonal component  $S_t$**

The seasonal parameter is used to compare with periodical fluctuations on short term between seasons (in our paper: months). The method showed below is applied for the additive model:  $y_t = T_t * C_t * S_t * R_t$  and assuming that doesn't exists a cyclic effect:

We calculate  $MA(T)$ ; is determined the parameter of the time serie  $y/MA$ ; we calculate the means on each month; we calculate the sum of this means and we will obtain in this way the seasonal paramaters.[18]

#### d) The forecast

The forecast can be obtained by smoothing. We will consider the exponential smoothing using the formula:  $s_1 = y_1$ ;  $s_t = \alpha \cdot y_t + (1 - \alpha) \cdot s_{t-1}$ ,  $t \geq 2$ ,  $\alpha \in (0,1)$   $\alpha$  is picked up like we want it.

$$\begin{aligned}
 s_2 &= \alpha \cdot y_2 + (1 - \alpha) \cdot y_1 \\
 s_3 &= \alpha \cdot y_3 + (1 - \alpha) \cdot y_2 = \alpha \cdot y_3 + \alpha(1 - \alpha) \cdot y_2 + (1 - \alpha)^2 \cdot y_1 = \\
 &\quad \alpha(y_3 + (1 - \alpha) \cdot y_2 + (1 - \alpha)^2 y_1) \\
 s_t &= \alpha(y_t + (1 - \alpha) \cdot y_{t-1} + (1 - \alpha)^2 y_{t-2} + \dots), t \geq 2 \quad [19], [20]. \tag{8}
 \end{aligned}$$

#### 4. Case study

It is considered a data base (60 dates) which represents the electrical energy consumption from University Politehnica of Bucharest, during January and February 2007. The registrations from the data base represents a real data base concerning the energy consumptions which allows to locate, with a certain trust level, the consumptions on intervals obtained by proportional division principle. The safety of the forecasts is directly proportional with the number of the available registrations and with their precision, and the dates are renewed daily.

The dates estimation and the forecast in a time series is made using the modeling methods which have been discussed earlier. We have elaborate using Matlab, the mathematical model for the forecast of the electrical energy consumption.

Realising a forecast for the energy consumption on short term is made with Matlab, reaching the following steps:

- a). It is realised a data base;
- b). We will make the calculations for the geometrical trend and we will see that concordant with the graphical method;

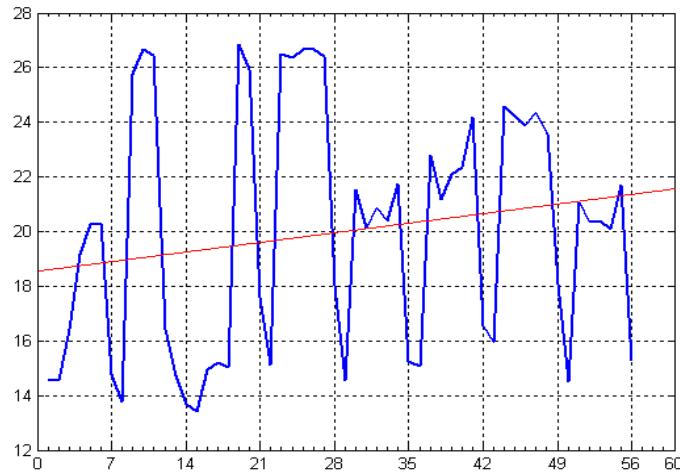


Fig 2. The evolution in time of the energy consumption

c). We've made the calculations for determining the cyclical component and this is shown like below;

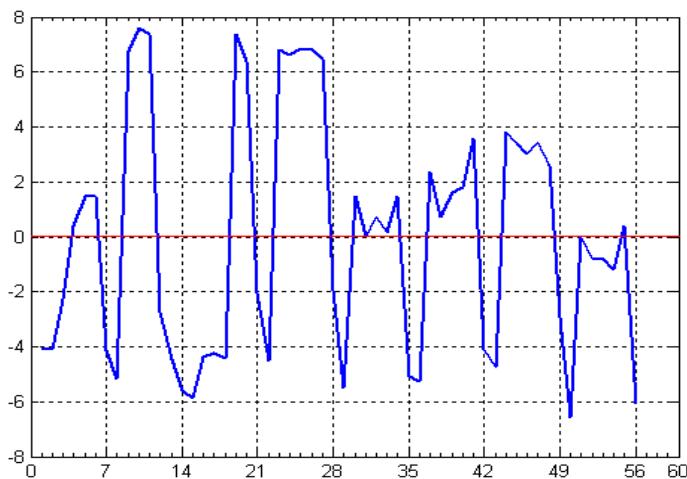


Fig.3 The estimative result  $100 * y/y$  for determine the cyclical effect.

d). We've made the calculations for the seasonal effect and we have obtained the graphic:

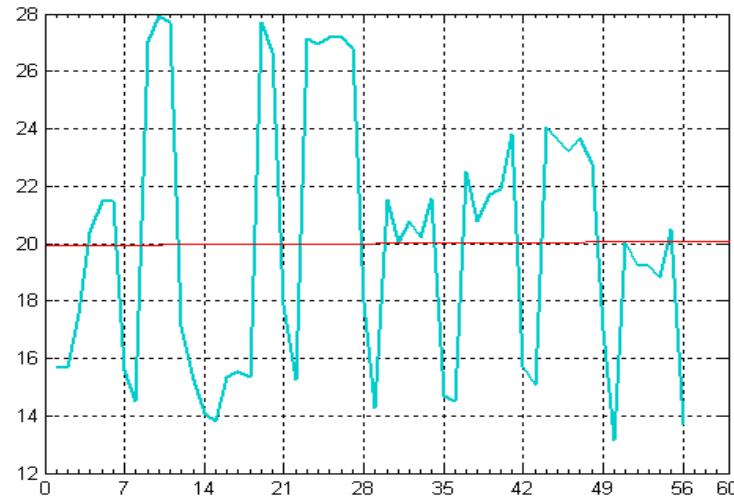


Fig.4 The produced energy evolution in time, after the dismission of the seasonal effect

e).It is realised the forecast for the next year using the exponential straightening and we will obtain the graphic.

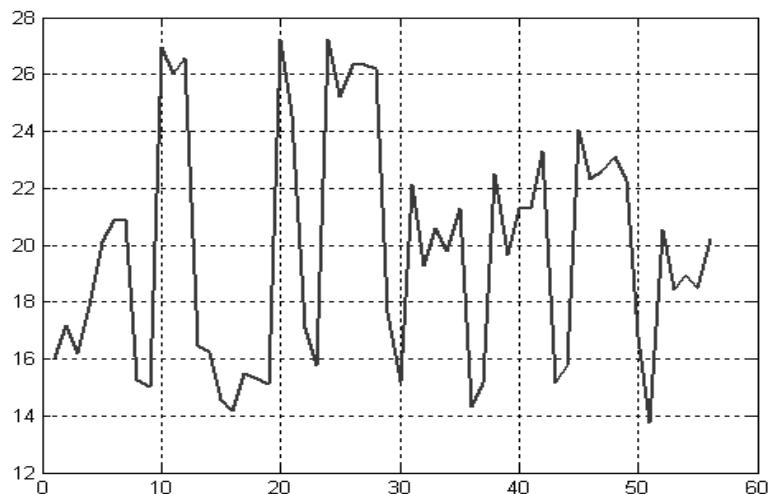


Fig.5 Estimative forecast of the energy consumption for March and April 2007

## 5. Conclusions

The forecasts for consumption represents the main elements for analysis in the elaboration/ modification of some decisions in different stages of the supply electric energy service management. In this case, is need to make some consume forecasts on short and medium term, very precise, in this way we want to obtain the contract on the competitive market of a eighth quantities and implicit the cost reduction connected to the electrical energy acquisition.

Using a procedure of recursive approximation gave us some good results, and so in the conditions of large variations, to develop a model which takes into considerations the previous dates in reduced number. In conformity with the graphics, the forecast shows that the energy consumption in March and April 2007, at University Politehnica of Bucharest is almost the same with the one realised in January and February 2007.

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