

ON THE MODELING OF DRY FRICTION DAMPERS IN VEHICLES SUSPENSIONS

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Widely used in vehicles suspensions, dry friction dampers are much cheaper and more rugged than hydraulic shock absorbers, due to their mechanical simplicity. Their modeling and simulation however, proves to be far from being trivial. Regularized and non-smooth approaches are the two main choices for modeling mechanical systems with unilateral contacts and friction. The latter one is best fitted for the studied phenomena. The present paper describes a switch model for suspensions with dry friction. Systems oscillations are analyzed by means of a two degree-of-freedom (DOF) switch model with two friction contacts. Stick-slip intervals and friction forces of the dampers are determined. The model is completely described and can be integrated with any standard ordinary differential equation (ODE) solver.

Keywords: stick-slip; vertical oscillations; dry friction; discontinuous systems; switch model, vehicle suspension.

1. Introduction

Vehicles often use friction elements because they are cheap and require only low maintenance. The dry friction dampers are much cheaper and more rugged than hydraulic dampers, due to their mechanical simplicity. However, the presence of dry friction in the suspension leads to low performance of the vehicle as their parameter design is non-trivial. The available solutions are to use old empirical rules to obtain a desired degree of damping. More than that, the degree of damping can be easily adjusted but it is spoiled by weather, wear and dirt.

The main encumbrance in the system study is the presence of the dry friction. The laws of the dry friction are usually different for static and for kinetic friction (sometimes called sliding friction or dynamic friction), see for instance [1, 2]. Vehicles with dry friction dampers feature alternative stick-slip phases wherein the masses linked by the suspension elements stick, respectively slip, relative to each other. As a consequence that the stick-slip motion phases involve different mechanisms, the differential equations which describe it are discontinuous.

Such equations can be solved by means of a regularization (or penalty) method which replaces the discontinuous equations by a smooth adjoint system.

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In reverse, the equations are stiff, determining high integration costs and, the worst thing, not only the obtained values are inexact, but also the solution behaviour. For instance, the occurrence of stick-slip may not be observed.

The alternative method to solve the problem is to deal with the nonlinear characteristic of the dry friction force. This choice is more suitable but also more difficult to apply. The non-smooth approach faces disjoint subintervals of the computation time.

In spite of the difficulties in modeling the friction phenomena, there are several contributions about forced vibrations [3-8] of dry friction systems. Depending on the dry friction level, a two DOF oscillator may exhibit full-stick, full-slip or stick slip behavior, see for instance [6]. The stick-slip forced oscillations may be stable (symmetric or asymmetric), they may swing periodically between two different limit cycles or they may be chaotic.

Previous studies like [9] investigated the effect of friction forces upon systems dynamics or vehicle comfort. The present paper analyzes the influence of the dry friction upon the vertical oscillations.

Although full slip solutions can be found analytically, this does not seem to be possible in the case of the stick-slip behavior which involves transcendental equations [8]. This paper proposes an original model for the forced vibrations of two DOF oscillators in presence of two dry friction contacts.

The model may be integrated by any standard ODE solver. The integration does not need any supplementary calculus. Viscous damping may be easily added to the model. The algorithm is simple, accurate and it is intended to be used for the proper design and evaluation of the friction dampers. There is no restriction concerning the friction laws or the track vertical irregularities form.

2. Dry friction set valued law and relative rest detection

Friction appears between two surfaces in contact in all mechanical systems. Its characteristics are strongly influenced by contaminations and it includes a wide range of physical phenomena. Friction is both important for mechanical engineering and for control. The friction effects are highly nonlinear and they may induce steady state errors, limit cycles, and poor performance [10, 11]. Therefore, it is important to have a proper model of the phenomena.

The laws of the dry friction are usually different for *static friction* between surfaces which are not moving one in respect to the other and for *kinetic friction* (sometimes called sliding friction or dynamic friction) between surfaces with relative displacement, see for instance [12].

For the static friction, the stick force is exerted in a direction that opposes potential moving (practically, it opposes to the resultant of all the other exerted forces, including inertia force, if applicable) and takes any value from zero up to

maximum stick force which is known as traction. The static friction force is a function of the external force and it exactly cancels it. When the other applied forces overcome this threshold value, the motion would commence.

In the case of the kinetic friction, the slip force opposes the relative movement between the contact points (which may differ from the relative movement between the bodies). There have been proposed many laws for its value (see for instance [7, 8]) most of them velocity dependent. Hence, a general description of friction may be Equation 1, where v_{rel} denotes the relative velocity in the contact point.

$$F_f = \begin{cases} F_{slip}(v_{rel}) & \text{if } v_{rel} \neq 0 \text{ (slip)} \\ -F_{ext} & \text{if } v_{rel} = 0 \text{ and } |F_{ext}| < F_{stick} \\ -F_{stick} \text{sign}(F_{ext}) & \text{otherwise} \end{cases} \quad (1)$$

The slip force, $F_{slip}(v_{rel})$, may be constant or it may be an arbitrary function of the relative velocity v_{rel} which best fits the studied system – constant or linear expressions of the slip force are used in models known as the Coulomb friction law, Figure 1a. The static value of the friction force F_f is set-valued instead. It equals the external and inertia forces resultant, F_{ext} , if this is smaller than the limit value F_{stick} (also known as traction); otherwise, its limit value opposes to the external forces but a slip phase will begin. These rules are exemplified by the Equation 1.

Such a friction law doesn't explicitly specify the friction force at zero velocity; the stick force counteracts the external resultant below the traction level and thus keeps objects in contact not to move relative to each other. But the main problem posed by the simulation of a model such as described by Equation 1 is to detect when the velocity is zero.

A solution is given by the Karnopp model, Figure 1b, which considers that the stick occurs when the relative velocity is “small enough”; this condition is formulated as $|v_{rel}| < \eta$, (instead of $v_{rel} = 0$), where η should be much smaller than the average speed values of the system elements. The slip mode is defined by the complementary relative velocities which lie outside the narrow stick band, $|v_{rel}| > \eta$, (instead of $v_{rel} \neq 0$). As a result, one obtains a discontinuous system, non-stiff in the stick interval.

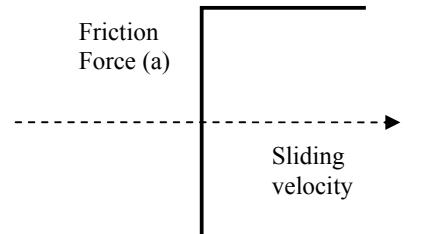


Fig. 1a Coulomb model

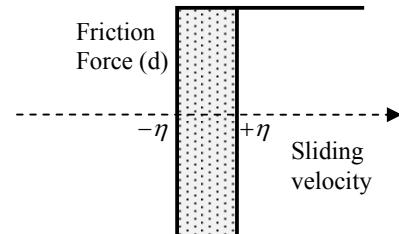


Fig. 1b Karnopp model

This method overcomes the problems of the zero velocity detection and allows efficient simulations; the stick band may be quite coarse but the stick and slip periods are nevertheless distinguished. On the other hand, because the relative acceleration is put to zero in the stick phase, the constant offset of the relative velocity causes a drift-off effect for large intervals and can cause numerical instability of the ODE integrator.

In order to overcome the numerical instabilities involved by this method in the stick phase, R.I.Leine et al. [13] proposed and improved version called the switch model which analyses the stick phases of a system distinguishing between *transitions* and *attractive or repulsive sliding modes*. In the case of a continuous stick (attractive mode), the system is “guided” to the middle of the stick band, where the relative velocity is exactly zero. The mathematical fundamentals of this method may be found in [15].

3. The suspension switch model

The mechanical model of the studied system is illustrated in Figure 2. The sprung masses are denoted by m_i , their displacements by y_i ($i = 1$, for the primary suspension and 2 for the secondary) and the track irregularities by x . This approach for the study of the vertical oscillations is usually known as quarter car model and it is used both for railway and road vehicles. In the case of railway vehicles, the first index is used for the bogie frame and the second for the car body, with their suspensions. When used for road vehicles, the first index is for the wheel masses and tires rigidities.

The force exerted by the first spring over the first mass was denoted by F_{el1} and by F_{el2} , the force exerted by the second spring over the second mass. The forces values are given by the products of the springs stiffnesses k_i and the relative displacements, Equation 2.

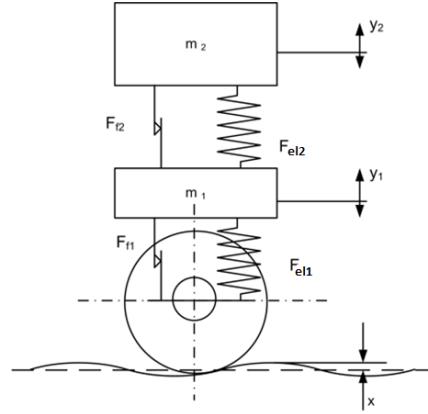


Fig. 2. Model for the vehicle vertical oscillations

The characteristics of the vehicle suspension are analytic functions only within certain intervals and the sliding contact may give rise to changes in the systems degrees of freedom during operation [15]. These features indicate the switch model as the most fitted to represent the suspension system.

More than that, dry friction may generate stick-slip oscillations which have to be described by an appropriate set of equations. In our case, the stiction may occur in both suspension stages.

$$F_{el1} = -k_1(y_1 - x) \quad \text{and} \quad F_{el2} = -k_2(y_2 - y_1) \quad (2)$$

In the general case, F_{fi} , dry friction forces occur in both suspension stages and their kinematic and static values are denoted by F_{slip1} or F_{slip2} and F_{stick1} or F_{stick2} , respectively.

The equations of the above model may be written as follows:

$$\dot{Y} = \begin{bmatrix} \dot{y}_1 \\ 1/m_1(F_{el1} - F_{f1} + F_{f2} - F_{el2}) \\ \dot{y}_2 \\ 1/m_2(-F_{f2} + F_{el2}) \end{bmatrix} \quad (3)$$

The state vector contains the vertical displacements and velocities of the two bodies.

$$\dot{Y} = [y_1 \quad \dot{y}_1 \quad y_2 \quad \dot{y}_2]^T \quad (4)$$

In the switch model, the logical complexity exponentially increases with the number of contact points (see, for instance [15]) and the evaluation of the inequalities (inclusions) becomes difficult.

However, as the present system has only two friction contacts (and due to its particular features) its inequalities may be evaluated without involving any special techniques. The friction force may be written as follows:

$$\begin{aligned} & \text{if } \text{abs}(v_{rel1}) > \eta \text{ and } \text{abs}(v_{rel2}) > \eta \\ & F_{f1} = F_{slip1}(v_{rel1}) \\ & F_{f2} = F_{slip2}(v_{rel2}) \\ & \text{else if } \text{abs}(v_{rel1}) > \eta \\ & F_{f1} = F_{slip1}(v_{rel1}) \\ & F_{f2} = -\min(\text{abs}(F_{el2} - m_2\ddot{y}_2), F_{stick2}) \cdot \text{sign}(F_{el2} - m_2\ddot{y}_2) \\ & \text{else if } \text{abs}(v_{rel2}) > \eta \\ & F_{f2} = F_{slip2}(v_{rel2}) \\ & F_{f1} = -\min(\text{abs}(F_{el1} - F_{el2} - m_1\ddot{x}), F_{stick1}) \cdot \text{sign}(F_{el1} - F_{el2} - m_1\ddot{x}) \end{aligned} \quad (5)$$

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else
 $F_{f1} = -\min(\text{abs}(F_{el1} - F_{el2} - m_1\ddot{x}), F_{stick1}) \cdot \text{sign}(F_{el1} - F_{el2} - m_1\ddot{x})$ 
 $F_{f2} = -\min(\text{abs}(F_{el2} - m_2\ddot{x}), F_{stick2}) \cdot \text{sign}(F_{el2} - m_2\ddot{x})$ 

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Dry friction may be present in both stages of a vehicle (for instance, some of the Diesel locomotives). The model of a system where there is only one dry friction contact may be derived as a particular case of the general Equations 5, where one of the two friction forces is assumed to be zero.

4. Numerical simulation

Stick-slip solutions are given by transcendental equations which cannot be solved analytically. Besides, track irregularities usually feature a stochastic character. Hence, numerical simulation is a basic tool in the study of dry friction dynamics.

Simulations were carried out considering that the friction force is decreasing when slip commences, as in Equation 6. This assumption is a better model of the real phenomenon. Such characteristic also implies a considerable lengthening of the transient time.

$$F_{slip}(v_{rel}) = \frac{F_{stick} \cdot \tanh(k \cdot v_{rel})}{1 + \delta |v_{rel}|} \quad (6)$$

Minimum values for the integration tolerances should be assigned considering the value of η . Considering a reference value $\Omega = 1\text{rad/s}$, the simulations were carried out using the dimensionless ratios below. The same ratios are used to plot the results.

$$\begin{aligned}
m_1 &= 1*m_1, m_2 = 2*m_1, \\
k_1 &= 2*m_1 \cdot \Omega^2, k_2 = 2*m_1 \cdot \Omega^2, \\
F_{stick1}/m_1 \cdot g &= F_{stick2}/m_1 \cdot g, \text{ between 0 and 1, for Figure 4} \\
x &= A \cdot \sin(\omega/\Omega t)/(m_1 \cdot g/k_1), \\
F_{stick1} &= F_{stick2} = 0.2125*m_1 \cdot g \text{ for Figures 3, 5 and 6} \\
A &= 0.1*m_1 \cdot g/k_1 \text{ for Figure 5 and } A = 0.5 \text{ for Figure 6,} \\
\omega/\Omega &= 1.1, \\
\eta &= 10^{-6}, \delta = 3.
\end{aligned}$$

4.1. Steady state solutions

Steady state solutions, obtained for sinusoidal perturbations, are important for the design of dry friction dampers as they indicate the most probable operation characteristics of the suspension. Each oscillation period may consist in stick or slip modes or in a mix of them, known as stick-slip. Figure 3 illustrates an

oscillation comprising alternate sticking modes of suspension stages and phases of concomitant slip.

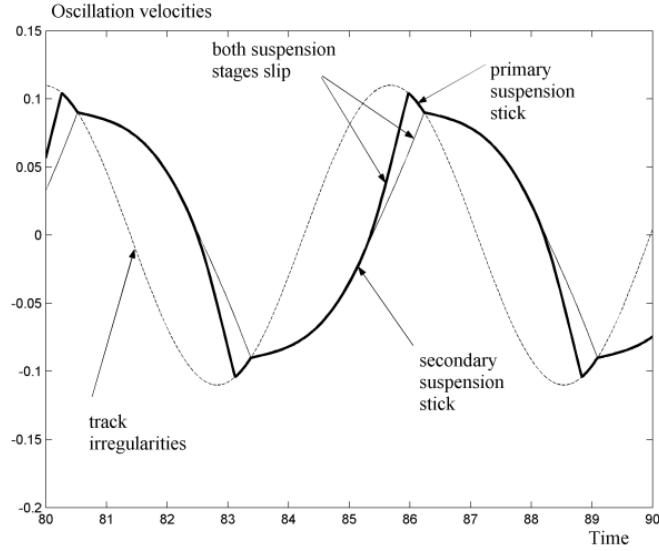


Fig. 3. Dimensionless displacements ratios to deflection $m_1 \cdot g/k_1$. Steady state oscillations may be combinations of alternate stick or slip phases in the suspension stages.

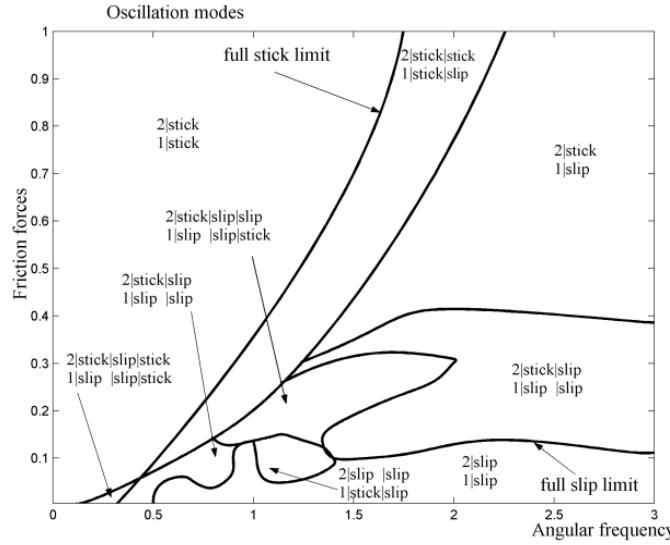


Fig. 4. Friction forces ratios to $m_1 \cdot g$, versus dimensionless angular frequency ω/Ω . Oscillation mode boundaries

A two parameter bifurcation diagram offers a detailed picture of the suspension working regime. The map of the frequency behavior can be traced in the plane $F_{\text{stick}}-\omega$, (Figure 4), where boundaries between different oscillation modes of the suspension stages are plotted. Each solution configuration is explained through suspension stage index and the dampers phases. This diagram was computed with a Monte Carlo simulation.

For example, the oscillation plotted in (Figure 3), corresponding to the coordinates $A = 0.2125$, $\omega = 1.1$ in (Figure 4), is described by 2|stick|slip|slip, 1|slip|slip|stick, which means that there are three phases: secondary suspension damper blocked (stick) and first stage slipping, both slipping and first blocked with second slipping.

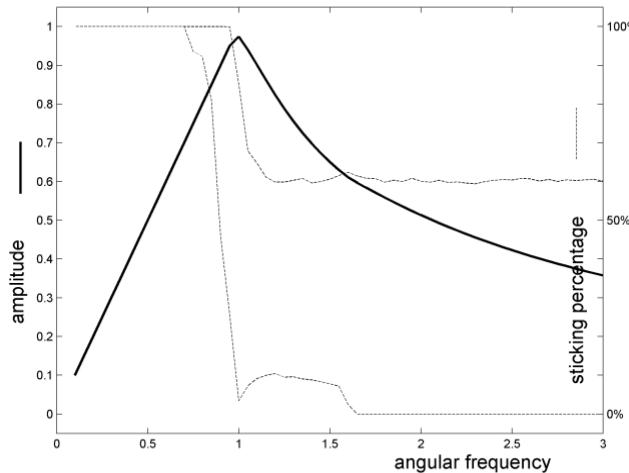


Fig. 5. Car body velocity transfer functions and sticking time percentage of each suspension stage. ($A=0.1$). A section at $F_{\text{stick}1} = F_{\text{stick}2} = 0.2125$ through the plot represented in Figure 3

Sections of the previous plot may reveal more details of the solution, (Figure 5). As the main variable state of a vehicle is speed, oscillations velocity amplitude (solid line) was plotted versus perturbation angular time frequency (which is directly related to vehicle speed). Dotted curves represent the stiction time percentage of each suspension stage. A 100% value means full stick; the referred stage is completely blocked. The stiction intervals alter the suspension efficiency and the energy dissipation.

Slight parameter variations may imply completely different responses of the system. Compared to (Figure 5), the only difference in the parameters of the solutions computed in (Figure 6) is a greater value for the track irregularities amplitude. It may be seen that the system response is essentially altered. Similar changes occur when other parameters vary, for instance the friction forces values or ratio.

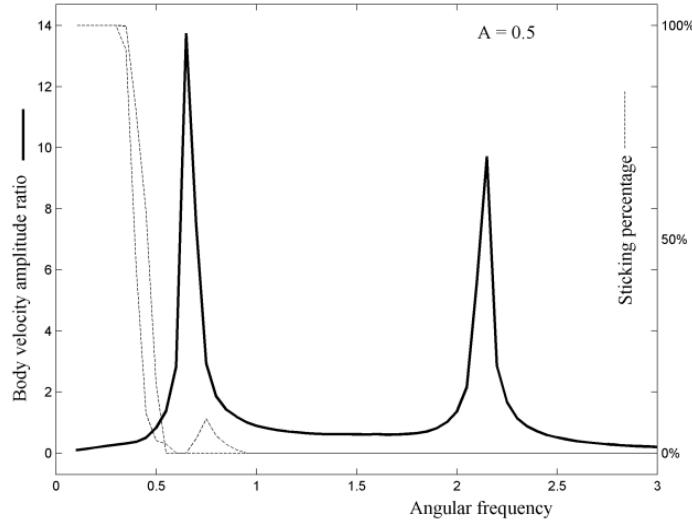


Fig. 6. Car body velocity transfer functions and sticking time percentage of each suspension

The system sensitivity to its parameter variation proves the difficulty of the suspension optimization as the external perturbations exerted over vehicles may vary in a wide range. Besides, simulations revealed large transient times for the steady-state solutions and, more than that, forced dry friction oscillators may exhibit chaotic dynamics [2].

It should be pointed out that solution uniqueness is uncertain for pure Coulomb friction or in the case of a decreasing friction characteristic [4].

4.2. Transient solutions

Common vehicle operation records significant parameter changes in time; hence, the prevailing regime is described mainly by the transient solutions. The following figures illustrate transient stick-slip behavior of the vehicle suspension by means of the friction forces and oscillation velocities time histories.

Solutions corresponding to the particular suspensions configurations, modeled by Equations 3 are plotted in Figures 7 and 8, respectively. First mass time histories were plotted using thick line and thin line was used for the car body mass.

The most striking features about friction forces are the step discontinuities which point out the slip to stick transitions or the inversion of the relative motion in the contact point. In the latter case the gap of the friction force is two times the traction value ($2F_{stick}$), see Figure 7.

Before the relative movement switches from slip, the friction attains its limit value and, when the stick begins it sharply changes to equilibrate the other forces resultant which should be smaller than the traction to allow stiction; therefore, the friction gap will be smaller than $2F_{stick}$, see Figure 7. Forces discontinuities correspond to shock occurrence in the vehicle suspension.

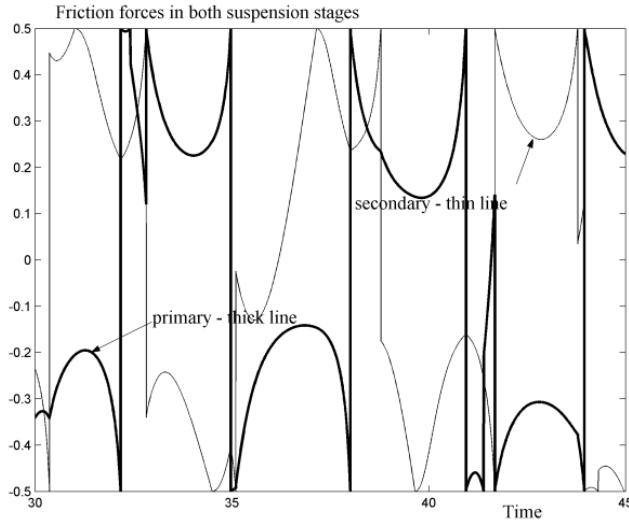


Fig. 7. Friction forces time histories for a suspension with damping in both suspension stages

During slip modes, friction forces are also smaller than F_{stick} because their values are given by Equation 6.

Regarding the oscillation velocities the most outstanding aspects are, on one hand, the singularities which mark the beginning of the stick phases or change in the relative velocity sign and, on the other hand, the graphs splits which mark the slip commencement.

Hence, the plot intervals where the velocities are overlapping highlight the stick phases and the collapse of a DOF. The technical consequence is that a suspension stage will not function.

Figure 7 illustrates the non periodic motion aspect of the transient solutions and Figure 8 the correlation of the oscillation phases with the friction forces variation.

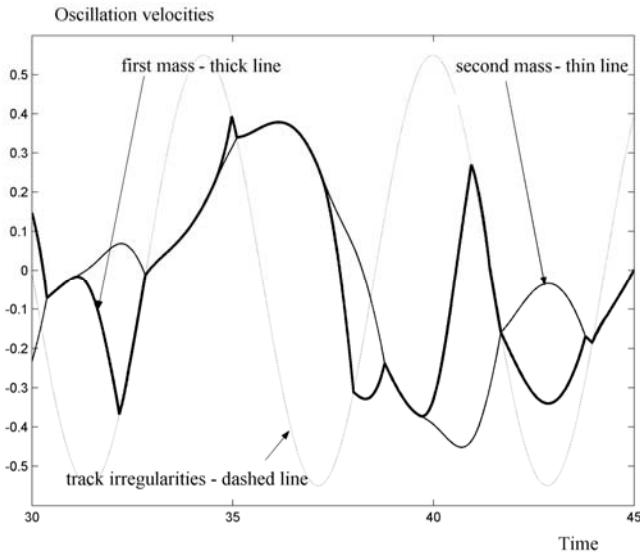


Fig. 8. Oscillations velocities time histories for a suspension with damping in both suspension stages

Noticeable results are also obtained regarding the equilibrium points which are not singular positions; instead, they lie in certain intervals. This effect is due to the alteration of the equilibrium sets induced by dry friction [14]. However, this phenomenon may not occur in practice due to the natural dither, as it has been shown that dither significantly influences ride dynamics of freight wagons [3].

5. Conclusions

In the present work, a switch model for the study of the dry friction suspensions is described. Coulomb's friction law is generally formulated as a set-valued force function. The contact problem of rigid multi-body systems with set-valued contact laws may be formulated as a nonlinear algebraic inclusion [13]. This choice proves hard to model but it has the advantage to be an accurate model. Therefore, the set-valued nature of the problem should be assumed and the nonlinear algebraic inclusion may be straightly approached.

From a technical perspective, it may be concluded that the stick slip phenomenon leads to poor performance of the vehicle suspension. Friction forces discontinuities generate shocks which affect comfort, vehicle structure and running safety [16]. The strong dependence of the system behavior to the parameter fluctuations indicate the numerical study as the most appropriate for such suspensions design.

Optimal values of the friction forces should provide an efficient damping without blocking the suspension (stick phases). However, vehicles with dry

friction damping may exhibit extremely dissimilar behaviors for slight variations of the external perturbations. Common vehicle operation records significant changes, mainly in running speed and track irregularities and, consequently, the suspension regime is extremely varied. More than that, friction forces values and characteristics are dispersed.

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