

PLANE WAVES IN GENERALIZED MAGNETO-THERMO-MICRO-STRETCH ELASTIC SOLID FOR MODE-I CRACK PROBLEM

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The aim of this paper is to study the effect of magnetic field on wave propagation in generalized thermo micro-stretch for a homogeneous isotropic elastic half space solid whose surface is subjected to a mode-I crack. The normal mode analysis is used to obtain the exact expressions for the field variables i.e., displacement components, temperature distribution, force stress and microstress in the presence of magnetic field. It is seen that on the displacement, temperature and normal stress distribution magnetic field has decreasing and increasing effect in with and without energy dissipation respectively.

Keywords: Green and Naghdi theory. Energy dissipation. Thermoelasticity. Magnetic field. Microstretch. Mode-I crack.

1. Introduction

The effect of magnetic field on wave propagation in elastic solid was introduced by using Maxwell's equations. In the context of generalized thermoelasticity Nayfeh and Nemat-Nasser [1] studied the propagation of plane waves in solid under influence of electromagnetic field. Choudhuri [2] extended these results to rotating media.

Eringen [3] introduced the theory of microstretch elastic solids. That theory is a generalization of the theory of micropolar elasticity [4-6]. The material points of microstretch elastic solids can stretch and contract independent of their transformations. The microstretch is used to characterize composite materials and various porous media [7]. The basic results in the theory of microstretch elastic solids were obtained in the literature [8-10]. The theory of thermo-microstretch elastic solids was introduced by Eringen [11]. The asymptotic behavior of the solutions and an existence result were presented by Bofill and Quintanilla [12]. A

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reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa [13]. De Cicco and Nappa [14] extended the linear theory of thermo-microstretch elastic solids to permits the transmission of heat as thermal waves at finite speed. The theory is based on the entropy production inequality proposed by Green and Laws [15]. In [14], the uniqueness of the solution of the mixed initial-boundary-value problem is also investigated. The basic results and an extensive review on the theory of thermo-microstretch elastic solids can be found in [11].

The normal mode analysis was used to obtain the exact expression for the field variables that are displacement components, temperature, stresses and microstress distributions.

The purpose of the present paper is to check the effect of magnetic field on the field variables. The problem of generalized thermo-microstretch in an infinite space weakened by a finite linear opening mode-I crack is solved for the above field variables. The distributions of the considering field variables are represented graphically. A comparison is carried out for both with and without energy dissipation and these effect was known as Green Naghdi theories [16, 17] named GN-II [17] and GN-III [16] for the propagation of waves in semi-infinite microstretch elastic solids.

2. Formulation of the Problem:

The region $z \geq 0$ is occupied by linear isotropic microstretch elastic solid. We use a rectangular coordinate system (x, y, z) having origin on the surface $y = 0$ and z -axis pointing vertically into the medium. A magnetic field with intensity $\mathbf{H} = (0, H_0, 0)$, acting parallel to the boundary plane (taken as the direction of the y -axis). The surface of the half-space is subjected to a thermal shock which is a function of z and t . Thus, all the quantities considered will be functions of the time variable t , and of the coordinates x and z . We begin our consideration with linearized equations of electro-dynamics of slowly moving medium [18].

$$\left. \begin{aligned} \mathbf{J} &= \text{curl } \mathbf{h} - \varepsilon_0 \dot{\mathbf{E}}, \\ \text{curl } \mathbf{E} &= -\mu_0 \dot{\mathbf{h}}, \\ \mathbf{E} &= -\mu_0 (\dot{\mathbf{u}} \times \mathbf{H}), \\ \nabla \cdot \mathbf{h} &= 0. \end{aligned} \right\} \quad (1)$$

The equation of motion in the presence of Lorentz force is as follows.

$$\sigma_{im,m} + F_i = \rho \ddot{u}_i, \quad (2)$$

when

$$F_i = \mu_0 (J \times H)_i, \quad i = 1, 2, 3. \quad (3)$$

The basic governing equations of linear generalized thermo-elasticity in the absence of body forces are taken from [19]

$$\left. \begin{aligned} (\lambda + \mu) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + (\mu + k) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) - k \frac{\partial \varphi_2}{\partial z} + \lambda_0 \frac{\partial \varphi^*}{\partial x} - \hat{\gamma} \frac{\partial T}{\partial x} + F_1 &= \rho \frac{\partial^2 u}{\partial t^2}, \\ (\lambda + \mu) \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right) + (\mu + k) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + k \frac{\partial \varphi_2}{\partial x} + \lambda_0 \frac{\partial \varphi^*}{\partial z} - \hat{\gamma} \frac{\partial T}{\partial z} + F_3 &= \rho \frac{\partial^2 w}{\partial t^2}, \\ (\alpha + \beta + \gamma) \nabla (\nabla \cdot \vec{\varphi}) - \gamma \nabla \times (\nabla \times \vec{\varphi}) + k (\nabla \times \vec{u}) - 2k \vec{\varphi} &= j \rho \frac{\partial^2 \vec{\varphi}}{\partial t^2}, \\ \alpha_0 \nabla^2 \varphi^* - \frac{1}{3} \lambda_4 \varphi^* - \frac{1}{3} \lambda_0 (\nabla \cdot \vec{u}) + \frac{1}{3} \hat{\gamma}_1 T &= \frac{3}{2} \rho j \frac{\partial^2 \varphi^*}{\partial t^2}, \\ K^* \nabla^2 T + K \nabla^2 \dot{T} &= \rho C_E \ddot{T} + \hat{\gamma} T_0 \ddot{u}_{i,i} + \hat{\gamma}_1 T_0 \frac{\partial \varphi^*}{\partial t}. \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \sigma_{il} &= \left(\lambda_0 \varphi^* + \lambda u_{r,r} \right) \delta_{il} + (\mu + k) u_{l,i} + \mu u_{i,l} - k \varepsilon_{ilr} \varphi_r - \hat{\gamma} T \delta_{il}, \\ m_{il} &= \alpha \varphi_{r,r} \delta_{il} + \beta \varphi_{i,l} + \gamma \varphi_{l,i}, \\ \lambda_i &= \alpha_0 \varphi_{,i}^*, \\ e_{ij} &= \frac{1}{2} (u_{i,j} + u_{j,i}). \end{aligned} \right\} \quad (5)$$

where μ_0 is magnetic permeability; ε_0 is electric permeability; \mathbf{h} is the induced magnetic field vector; \mathbf{E} is the induced electric field vector, \mathbf{J} is the current density vector; T is the temperature above the reference temperature T_0 chosen so that $|(T - T_0)/T_0| < 1$, λ, μ are the counterparts of Lamé's parameters, the components of displacement vector \mathbf{u} are u_i , t is the time, σ_{ij} are the components of stress tensor, e is the dilatation, e_{ij} are the components of strain tensor, j is the micro inertia moment, k, α, β, γ are the micropolar constants, $\alpha_0, \lambda_0, \lambda_1$ are the microstretch elastic constants, φ^* is the scalar microstretch, $\vec{\varphi}$ is the rotation vector, m_{ij} is the couple stress tensor, δ_{ij} is the Kronecker delta, ε_{ijr} is the alternate tensor, the mass density is ρ , the specific heat at constant strain is C_E , the thermal conductivity is $K (\geq 0)$ and K^* is the material characteristic.

Where $\hat{\gamma} = (3\lambda + 2\mu + k)\alpha_{t_1}$, $\hat{\gamma}_1 = (3\lambda + 2\mu + k)\alpha_{t_2}$, $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$ and

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$. The state of plane strain parallel to the xz -plane is defined by.

$u_1 = u(x, z, t)$, $u_2 = 0$, $u_3 = w(x, z, t)$, $\phi_1 = \phi_3 = 0$, $\phi_2 = \phi_2(x, z, t)$, $\phi^* = \phi^*(x, z, t)$ and $\mathbf{h} = -H_0(0, e, 0)$. The constants $\hat{\gamma}$ and $\hat{\gamma}_1$ depend on the mechanical as well as the thermal properties of the body and the dot denote the partial derivative with respect to time, $\alpha_{t_1}, \alpha_{t_2}$ are the coefficients of linear thermal expansions.

Component of electric intensity and current density is represented as,

$$\mathbf{E} = -\mu_0 H_0(-\dot{w}, 0, \dot{u}), \quad \mathbf{J} = (H_0 \frac{\partial e}{\partial z}, 0, -H_0 \frac{\partial e}{\partial x}) - \varepsilon_0 \dot{\mathbf{E}},$$

External force can be represented as,

$$\mathbf{F} = \mu_0 H_0^2 \left(\frac{\partial e}{\partial x} - \varepsilon_0 \mu_0 \ddot{u}, 0, \frac{\partial e}{\partial z} - \varepsilon_0 \mu_0 \ddot{w} \right). \quad (6)$$

For convenience, the following non-dimensional variables are used:

$$\begin{aligned} \bar{x}_i &= \frac{\omega^*}{c_2} x_i, \quad \bar{u}_i = \frac{\rho c_2 \omega^*}{\hat{\gamma} T_0} u_i, \quad \bar{t} = \omega^* t, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma} T_0}, \quad \bar{m}_{ij} = \frac{\omega^*}{c_2 \hat{\gamma} T_0} m_{ij}, \\ \bar{\phi}_2 &= \frac{\rho c_2^2}{\hat{\gamma} T_0} \phi_2, \quad \bar{\lambda}_3 = \frac{\omega^*}{c_2 \hat{\gamma} T_0} \lambda_3, \quad \bar{\phi}^* = \frac{\rho c_2^2}{\hat{\gamma} T_0} \phi^*, \quad \bar{\omega}^* = \frac{\rho C_E c_2^2}{K^*}, \quad \bar{h} = \frac{h^*}{H_0}, \quad \bar{c}_2^2 = \frac{\mu}{\rho}. \end{aligned} \quad (7)$$

Using Eq. (7), Eqs. (4) become (dropping the bar for convenience)

$$\left. \begin{aligned} &\frac{(\mu+k)}{\rho c_2^2} \nabla^2 u + \left(\frac{(\mu+\lambda)}{\rho c_2^2} + R_H \right) \frac{\partial e}{\partial x} - \frac{k}{\rho c_2^2} \frac{\partial \phi_2}{\partial z} + \frac{\lambda_0}{\rho c_2^2} \frac{\partial \phi^*}{\partial x} - \frac{\partial T}{\partial x} = \beta^2 \frac{\partial^2 u}{\partial t^2}, \\ &\frac{(\mu+k)}{\rho c_2^2} \nabla^2 w + \left(\frac{(\mu+\lambda)}{\rho c_2^2} + R_H \right) \frac{\partial e}{\partial z} + \frac{k}{\rho c_2^2} \frac{\partial \phi_2}{\partial x} + \frac{\lambda_0}{\rho c_2^2} \frac{\partial \phi^*}{\partial z} - \frac{\partial T}{\partial z} = \beta^2 \frac{\partial^2 w}{\partial t^2}, \\ &\frac{j \rho c_2^2}{\gamma} \frac{\partial^2 \phi_2}{\partial t^2} = \nabla^2 \phi_2 - \frac{2k c_2^2}{\gamma \omega^*} \phi_2 + \frac{k c_2^2}{\gamma \omega^{*2}} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right), \\ &\left(\frac{c_3^2}{c_2^2} \nabla^2 - \frac{c_4^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2} \right) \phi^* - \frac{c_5^2}{\omega^{*2}} e + a_9 T = 0, \\ &\varepsilon_2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) + \varepsilon_3 \left(\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial z^2} \right) = \ddot{T} + \varepsilon_1 \ddot{e} + \varepsilon_4 \frac{\partial \phi^*}{\partial t}. \end{aligned} \right\} \quad (8)$$

Assuming the scalar potential functions $R(x, z, t)$ and $\psi(x, z, t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial R}{\partial x} + \frac{\partial \psi}{\partial z}, w = \frac{\partial R}{\partial z} - \frac{\partial \psi}{\partial x} \text{ this gives } e = \nabla^2 R. \quad (9)$$

Using Eq. (9) in Eqs. (8), we obtain.

$$\left. \begin{aligned} [\alpha^2 \nabla^2 - \beta^2 a_0 \frac{\partial^2}{\partial t^2}] R - a_0 T + a_1 \phi^* &= 0, \\ [\nabla^2 - \beta^2 a_2 \frac{\partial^2}{\partial t^2}] \psi - a_3 \phi_2 &= 0, \\ [\nabla^2 - 2a_4 - a_5 \frac{\partial^2}{\partial t^2}] \phi_2 + a_4 \nabla^2 \psi &= 0, \\ [a_6 \nabla^2 - a_7 - \frac{\partial^2}{\partial t^2}] \phi^* - a_8 \nabla^2 R + a_9 T &= 0, \\ \varepsilon_2 \nabla^2 T + \varepsilon_3 \nabla^2 \dot{T} = \ddot{T} + \varepsilon_1 \nabla^2 \ddot{R} + \varepsilon_4 \frac{\partial \phi^*}{\partial t} &. \end{aligned} \right\} \quad (10)$$

$$\begin{aligned} \text{where } c^2 &= \frac{1}{\mu_0 \varepsilon_0}, \beta^2 = \frac{V_A^2}{c^2} + 1, V_A^2 = \frac{\mu_0 H_0^2}{\rho}, R_H = \frac{V_A^2}{c_2^2}, c_3^2 = \frac{2\alpha_0}{3\rho j}, c_4^2 = \frac{2\lambda_1}{9\rho j}, \\ c_5^2 &= \frac{2\lambda_0}{9\rho j}, \varepsilon_1 = \frac{\hat{\gamma}^2 T_0}{\rho^2 C_E C_2^2}, \varepsilon_2 = \frac{K^*}{\rho C_E C_2^2}, \varepsilon_3 = \frac{K \omega^*}{\rho C_E C_2^2}, \varepsilon_4 = \frac{\hat{\gamma} \hat{\gamma}_1 T_0}{\rho^2 C_E \omega^* C_2^2}, \\ c_1^2 &= \frac{\lambda + 2\mu + k}{\rho}, \alpha^2 = 1 + R_H a_0, a_0 = \frac{c_2^2}{c_1^2}, a_1 = \frac{\lambda_0}{\lambda + 2\mu + k}, a_2 = \frac{\rho c_2^2}{\mu + k}, a_3 = \frac{k}{\mu + k}, \\ a_4 &= \frac{k c_2^2}{\gamma \omega^{*2}}, a_5 = \frac{\rho j c_2^2}{\gamma}, a_6 = \frac{c_3^2}{c_2^2}, a_7 = \frac{c_4^2}{\omega^{*2}}, a_8 = \frac{c_5^2}{\omega^{*2}} \text{ and } a_9 = \frac{2\hat{\gamma}_1 c_2^2}{9\hat{\gamma} j \omega^{*2}}. \end{aligned} \quad (11)$$

3. Solution of the Problem by Normal Mode Analysis:

Decompose the physical variables in terms of normal mode analysis method as follows,

$$[R, \psi, \phi^*, \phi_2, \sigma_{il}, m_{il}, T, \lambda_z](x, z, t) = [\bar{R}, \bar{\psi}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](z) \exp(\omega t + i b x) \quad (12)$$

Where $[\bar{R}, \bar{\psi}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](z)$ the amplitudes of the functions ω is a complex and b is the wave number, by using Eq. (12), Eqs. (10) become,

$$\left. \begin{aligned} (\alpha^2 D^2 - A_1)\bar{R} - a_0 \bar{T} + a_1 \bar{\varphi}^* &= 0, \\ (D^2 - A_2)\bar{\psi} - a_3 \bar{\varphi}_2 &= 0, \\ (D^2 - A_3)\bar{\varphi}_2 + a_4(D^2 - b^2)\bar{\psi} &= 0, \\ (a_6 D^2 - A_4)\bar{\varphi}^* - a_8(D^2 - b^2)\bar{R} + a_9 \bar{T} &= 0, \\ [\varepsilon(D^2 - b^2) - \omega^2]\bar{T} - \varepsilon_1 \omega^2(D^2 - b^2)\bar{R} - \varepsilon_4 \omega \bar{\varphi}^* &= 0. \end{aligned} \right\} \quad (13)$$

where $D = \frac{d}{dz}$, $A_1 = \alpha^2 b^2 + \beta^2 a_0 \omega^2$, $A_2 = b^2 + a_2 \beta^2 \omega^2$, $A_3 = b^2 + 2a_4 + a_5 \omega^2$,

$$A_4 = b^2 a_6 + a_7 + \omega^2, \quad \varepsilon = \varepsilon_2 + \varepsilon_3 \omega.$$

Eliminating $\bar{\varphi}_2, \bar{\psi}, \bar{R}, \bar{T}$ and $\bar{\varphi}^*$ from Eqs. (13), we get the following two ordinary differential equations.

$$\left. \begin{aligned} [D^6 - AD^4 + BD^2 - C] \left\{ \bar{R}, \bar{T}, \bar{\varphi}^* \right\}(z) &= 0, \\ [D^4 - g_4 D^2 + g_5] \left\{ \bar{\varphi}_2, \bar{\psi} \right\}(z) &= 0. \end{aligned} \right\} \quad (14)$$

This implies,

$$\left. \begin{aligned} (D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2) \left\{ \bar{R}, \bar{T}, \bar{\varphi}^* \right\}(z) &= 0, \\ (D^2 - k_4^2)(D^2 - k_5^2) \left\{ \bar{\varphi}_2, \bar{\psi} \right\}(z) &= 0. \end{aligned} \right\} \quad (15)$$

Where, $k_r^2 (r = 1, 2, 3, 4, 5)$ are the roots of the characteristic equation of Eqs. (14).

$$\begin{aligned} A &= g_{11} / \varepsilon g_{10}, & B &= g_{12} / \varepsilon g_{10}, & C &= g_{13} / \varepsilon g_{10}, & g_1 &= \varepsilon_4 \omega \alpha^2 - a_1 \varepsilon_1 \omega^2, \\ g_1 &= -\varepsilon_1 \omega A_1 + a_1 \varepsilon_1 \omega^1 b^1, & g_3 &= a_1 \varepsilon b^2 + a_1 \omega^2 + a_0 \varepsilon_4 \omega, & g_4 &= A_2 + A_3 - a_3 a_4, \\ g_5 &= A_2 A_3 - a_3 a_4 b^2, & g_6 &= a_6 (\varepsilon b^2 + \omega^2) + A_4 \varepsilon, & g_7 &= \varepsilon_4 a_9 \omega + A_4 (\varepsilon b^2 + \omega^2), \\ g_8 &= \varepsilon_1 a_6 \omega^2 b^2 + A_4 \varepsilon_1 \omega^2 - a_8 \varepsilon_4 \omega, & g_9 &= \varepsilon_4 a_8 \omega b^2 - \varepsilon_1 \omega^2 A_4 b^2, \\ g_{10} &= (a_6 g_1 + a_1 \varepsilon_1 \omega^2 a_6), & g_{11} &= -a_6 (\varepsilon g_2 - \varepsilon_1 \omega^2 g_3) + g_6 g_1 + a_1 \varepsilon g_8, \\ g_{12} &= -g_6 g_2 + g_7 g_1 - a_1 \varepsilon g_9 + g_3 g_8, & g_{13} &= -g_7 g_2 - g_3 g_9. \end{aligned}$$

By using Eq. (13) solution of Eq. (14), has the form

$$\left. \begin{aligned} \bar{R} &= \sum_{j=1}^3 M_j(b, \omega) e^{-k_j z}, \\ \bar{T} &= \sum_{j=1}^3 H_{1j} M_j(b, \omega) e^{-k_j z}, \\ \bar{\varphi}^* &= \sum_{j=1}^3 H_{2j} M_j(b, \omega) e^{-k_j z}, \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned} \text{and } \bar{\varphi}_2 &= \sum_{n=4}^5 M_n(b, \omega) e^{-k_n z}, \\ \bar{\psi} &= \sum_{n=4}^5 H_{3n} M_n(b, \omega) e^{-k_n z}. \end{aligned} \right\} \quad (17)$$

$$\left. \begin{aligned} \text{Where, } H_{1j} &= [k_j^4 a_6 \varepsilon_1 \omega^2 - k_j^2 g_8 - g_9] / [k_j^4 a_6 \varepsilon - k_j^2 g_6 + g_7], \\ H_{2j} &= [a_8 (k_j^2 - b^2) - a_9 H_{1j}] / [a_6 k_j^2 - A_4], \\ H_{3n} &= a_3 / [k_n^2 - A_2]. \quad \text{and } j=1, 2, 3 \text{ and } n=4, 5. \end{aligned} \right\} \quad (18)$$

4. Boundary Condition for Mode-I Crack:

The plane boundary subjects to an instantaneous normal point force and the boundary surface is isothermal, the boundary conditions at the vertical plan $y = 0$ and in the beginning of the crack, at $z = 0$ are as follows:

- (1) Mechanical boundary condition is that the surface of the half-space obeys,

$$\left. \begin{aligned} \sigma_{xx} &= -p(z), & |z| < a, \\ \sigma_{xz} &= 0, & -\infty < z < \infty, \\ \sigma_{zz} &= 0, & -\infty < z < \infty, \\ \lambda_z &= 0 & -\infty < z < \infty. \end{aligned} \right\} \quad (19a)$$

- (2) Thermal boundary condition is that the surface of the half-space subjects to a thermal shock,

$$T = f(z), \quad |z| < a \quad (19b)$$

We obtain the non-dimensional expressions for the displacement components, force stress, coupled stress and temperature distribution of the microstretch generalized thermoelastic medium as follows,

$$\left. \begin{aligned} \bar{u} &= \sum_{j=1}^3 ibM_j(b, \omega) e^{-k_j z} - \sum_{n=4}^5 k_n H_{3n} M_n(b, \omega) e^{-k_n z}, \\ \bar{w} &= - \sum_{j=1}^3 k_n M_j(b, \omega) e^{-k_j z} - \sum_{n=4}^5 ibH_{3n} M_n(b, \omega) e^{-k_n z}, \end{aligned} \right\} \quad (20)$$

$$\left. \begin{aligned} \bar{\sigma}_{xx} &= \sum_{r=1}^5 S_{1r} M_r(b, \omega) e^{-k_r z}, \\ \bar{\sigma}_{zz} &= \sum_{r=1}^5 S_{2r} M_r(b, \omega) e^{-k_r z}, \\ \bar{\sigma}_{xz} &= \sum_{r=1}^5 S_{3r} M_r(b, \omega) e^{-k_r z}, \\ \bar{m}_{xy} &= \sum_{n=4}^5 -a_{15} k_n M_n(b, \omega) e^{-k_n z}, \\ \bar{\lambda}_z &= \sum_{j=1}^3 -k_j a_{16} H_{2j} M_j(b, \omega) e^{-k_j z}. \end{aligned} \right\} \quad (21)$$

where

$$a_{10} = \frac{\lambda_0}{\rho C_2^2}, a_{11} = \frac{C_1^2}{C_2^2}, a_{12} = \frac{\lambda}{\rho C_2^2}, a_{13} = \frac{\mu + k}{\rho C_2^2}, a_{14} = \frac{k}{\rho C_2^2}, a_{15} = \frac{\gamma \omega^{*2}}{\rho C_2^4}, a_{16} = \frac{\alpha_0 \omega^*}{\rho C_2^3}$$

$$S_{1j} = a_{10} H_{2j} - b^2 a_{11} + k_n^2 a_{12} - H_{1j}, \quad S_{1n} = ib(-a_{11} + a_{12}) k_n H_{3n},$$

$$S_{2j} = a_{10} H_{2j} + k_j^2 a_{11} - b^2 a_{12} - H_{1j}, \quad S_{2n} = ib(a_{11} - a_{12}) k_n H_{3n},$$

$$S_{3j} = -ib(1 + a_{13}) k_j, \quad S_{3n} = (k_n^2 + a_{13} b^2) H_{3n} + a_{14}, \quad j = 1, 2, 3 \text{ and } n = 4, 5.$$

Applying the boundary conditions (19) at the surface $z = 0$ of the plane, we obtain a system of five equations. These equations can be represented in matrix form as under,

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ -k_1 H_{21} & -k_2 H_{22} & -k_3 H_{23} & 0 & 0 \\ H_{11} & H_{12} & H_{13} & 0 & 0 \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} -\bar{p} \\ 0 \\ 0 \\ 0 \\ \bar{f} \end{pmatrix}. \quad (22)$$

By applying inverse matrix operation one can find the values of unknown constants M_n , $n = 1, 2, 3, 4, 5$ and hence, obtain the expressions for the field variables.

5. Particular Cases:

Case-1: Micropolar Effect without Stretch:

The corresponding equations for the medium can be obtained by putting,

$$\alpha_0 = \lambda_0 = \lambda_1 = \phi^* = 0. \quad (23)$$

Case 2: Without Micropolar Effect:

To find field variables for without micropolar thermoelastic medium adjust the constants as,

$$k = \alpha = \beta = \gamma = 0. \quad (30)$$

Proceeding on the same way as we did one can find the field.

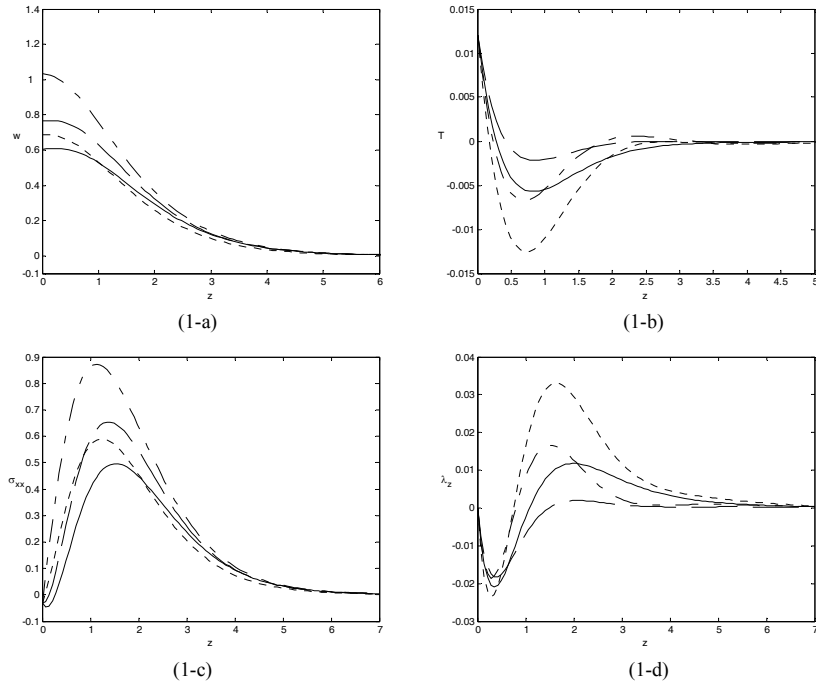
6. Application and Discussions

We take a magnesium crystal [4] as the model material. Since, ω is a complex constant, we take $\omega = \omega_0 + i\zeta$ and set $\omega_0 = -2.5$ and $\zeta = 1$. The physical constants used are:

$$\begin{aligned} \rho &= 1.74 \times 10^3 \text{ kg m}^{-3}, \quad j = 0.2 \times 10^{-21} \text{ m}^3, \quad \lambda = 9.4 \times 10^{11} \text{ kg m}^{-1} \text{ s}^{-2}, \quad T_0 = 293 \text{ K}, \\ \mu &= 4.0 \times 10^{11} \text{ kg m}^{-1} \text{ s}^{-2}, \quad k = 1 \times 10^{11} \text{ kg m}^{-1} \text{ s}^{-2}, \quad \gamma = 0.779 \times 10^{-8} \text{ kg m s}^{-2}, \quad \varepsilon_1 = 1.78, \\ \varepsilon_2 &= 1.1, \quad \varepsilon_3 = 0.4, \quad K^* = 0.1 \times 10^{-3} \text{ W m}^{-1} \text{ K}^{-1}, \quad K = 1.3 \times 10^{-4} \text{ W m}^{-1} \text{ K}^{-1}, \quad z = 1.3, \\ \lambda_0 &= 0.5 \times 10^{11} \text{ kg m}^{-1} \text{ s}^{-2}, \quad \lambda_1 = 0.1 \times 10^9 \text{ kg m}^{-1} \text{ s}^{-2}, \quad \alpha_0 = 0.779 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-2}, \\ \bar{p} &= -1.2, \quad t = 0.1 \text{ s}, \quad \bar{f} = 0.08, \quad b = 1.0. \end{aligned}$$

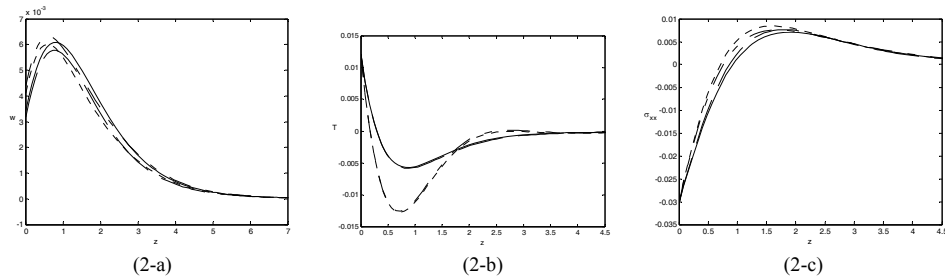
To check the effect of magnetic field on considered variables we chose at random two different intensities of magnetic field *i.e.*, $H_0 = 0$ and 1000. Also with and without energy dissipation *i.e.*, under Green Naghdi theories II and III are

considered and showed their behavior in the graphs. The following graphs shows the behavior of displacement component w , temperature T , normal stress σ_{xx} and microstress λ_z in the presence and absence of magnetic field i.e., $H_0 = 0$ and 1000 in the context of GN-II and GN-III. Solid line and dashed line for GN-II at $H_0 = 0$ and 1000, dashed with dot and dotted line are for GN-III at $H_0 = 0$ and 1000 respectively



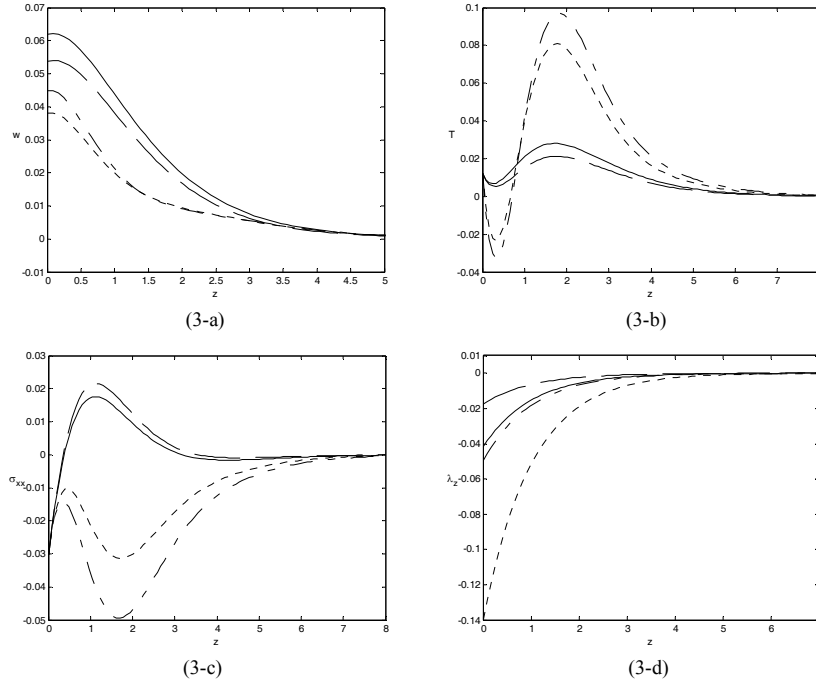
It is observed that magnetic field has increasing effect on w , T and σ_{xx} for GN-II and decreasing effect for the case of GN-III. For the case of microstress distribution magnetic field has dual effect.

Case-1: Micropolar Effect without Stretch



These curves are representing the case of without stretching parameters, magnetic field has decreasing effect on displacement component and temperature distribution. Amplitude of normal stress distribution increased by increasing the influence of magnetic field. The microstress distribution does not exist.

Case-2: Without Micropolar affect



Magnetic field has decreasing effect on displacement component and temperature distribution for both the theories of GN. Where as normal stress σ_{xx} and microstress λ_z has increasing effect in GN-II and decreasing effect in GN-III.

6. Conclusion

In this paper the effect of magnetic field on plane waves in a generalized thermo-microstretch elastic media is studied. In this paper effect of H_0 is also been investigated for material i) without microstretch and ii) without micropolar effect. The importance of this paper is to consider effect of magnetic field for mode-I crack, taken as particular example for each case. We can obtain the following conclusions according to the analysis above. The problem considering effect of magnetic field in generalized thermo-microstretch elastic media can be described by two characteristic equations of order six and four. Distributions of all physical quantities throughout the medium depend on the nature of material.

Variable quantities have a dual nature for magnetic field in majority of cases its presence is having an increasing effect. All the curves obtained converge to zero representing decaying of each and every field variable as distance from edge of crack increases.

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