GEOMETRICAL AND OPERATIONAL CONSTRAINTS OF AN ACKERMANN STEERING LINKAGE

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In lucrare se analizează restricţiile geometrice şi funcţionale ale trapezului de direcţie al automobilelor, folosindu-se sistematic lungimile raportate ale elementelor acestuia. Se analizează în ce măsură funcţia de transmitere necesară a mecanismului poate fi realizată folosind la sinteza mecanismului metoda dezvoltării în serie Taylor a acestei funcţii.

In the paper the geometrical and the operational constraints of an Ackermann steering linkage are analyzed by systematic using of the normalized lengths of the elements of the mechanism. One analyzes to what extent the necessary transmission function of the mechanism can be carried out when the synthesis of the mechanism is performed by method of the Taylor's series expansion.

Keywords: automobile**,** Ackermann steering linkage, constraints, steering, steering angle, vehicle

1. Introduction

The steering system of an automobile with the rigid steering axle (beam and steering knuckles hinge-connected to the beam with king pins) comprises the steering mechanism which is represented by the steering trapezium otherwise known as Ackermann steering linkage. For the kinematical synthesis of an Ackermann steering linkage analytical and graphical methods are used. In the beginning of the automobile development graphical methods have been employed. For the facility of design, on the basis of these methods, tables and diagrams have been achieved as those by Lutz [1].

The analysis and synthesis of the Ackermann linkage have been tackled in numerous papers. A comprehensive list of these works together with the essential ideas and their concise conclusions written before 1977 are presented in monograph [2]. In [3] the conditions in which the graphs of the transmission function of the Ackermann steering linkage and the theoretical transmission function corresponding to the correct steering condition have the same curvature at origin has been investigated in connection with the synthesis of this

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mechanism. The approximated synthesis of an Ackermann steering linkage using Taylor's expansion has been tackled in [2] and again in [4].

In the present paper the geometrical and operational constraints of an Ackermann steering linkage are minutely investigated using systematically normalized lengths of its elements. In connection with this fact one analyzes to what extent the transmission function of the Ackermann steering linkage may be approximated by the method of Taylor's series expansion in view of the synthesis of this mechanism. In the paper we consider the planar Ackermann steering linkage and the classical condition of correct steering of a vehicle given by the Ackermann's relation [2, 5]. In the most works, the mentioned relation is referred to as Ackermann's relation, but, curiously, in [1] it does not have a denomination.

2. Elements of geometry and kinematics of the Ackermann linkage

In the sketch of a four-bar mechanism shown in Fig. 1 the points A and D are fixed on the motor vehicle**.**

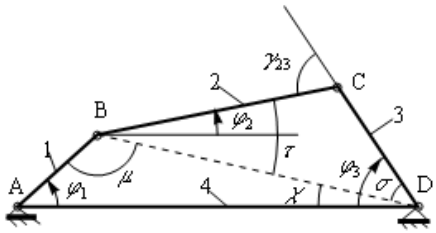

Fig. 1 Geometrical elements of a four-bar mechanism

They represent the intersections of the king pin axes with the mechanism plane, this being parallel to the ground (the king pin axes are perpendicular to the ground plane). The elements 1 and 3 are the steering knuckle arms, and element 2 is the tie-rod. For the present condition it is adequately to define the positioning elements of the four-bar mechanism in the way shown in Fig.1. The lengths of the elements 1, 2 and 3 are l_1, l_2 and l_3, respectively. The distance between A and D is l_4 : it represents the king pin track denoted by E_p also.

From triangle ABD we can write the relations

$$tg\left(\frac{\chi-\mu}{2}\right)=\frac{l_1-l_4}{l_1+l_4}\cdot\cot g\frac{\varphi_1}{2},\quad \chi+\mu=\pi-\varphi_1, \tag{1}$$

from which it follows that

$$\chi=\frac{\pi-\varphi_1}{2}+arctg\left(\frac{l_1-l_4}{l_1+l_4}\cdot\cot g\frac{\varphi_1}{2}\right). \tag{2}$$

Applying the cosine theorem in triangles BAD and DBC one can determine the angle τ. In asimilar way from triangles BAD and DBC one can calculate the angle σ. Further, one can determine the angles φ_2 and φ_3 using the relations

$$\varphi_2 = \tau - \chi, \quad \varphi_3 = \sigma + \chi. \tag{3}$$

Putting the normalized lengths

$$\lambda_1 = l_1 / l_4, \lambda_2 = l_2 / l_4, \lambda_3 = l_3 / l_4, \tag{4}$$

taking into account that in the case of the steering trapezium $\lambda_1=\lambda_3$, after performing above described operations we obtain

$$\varphi_2(\varphi_1) = \arccos\frac{1+\lambda_2^2-2\lambda_1\cos\varphi_1}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos\varphi_1}} - arctg\left(\frac{\lambda_1-1}{\lambda_1+1}\cdot\cot g\frac{\varphi_1}{2}\right) - \frac{\pi-\varphi_1}{2}, \tag{5}$$

$$\varphi_3(\varphi_1) = \arccos\frac{1+2\lambda_1^2-\lambda_2^2-2\lambda_1\cos\varphi_1}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos\varphi_1}} + arctg\left(\frac{\lambda_1-1}{\lambda_1+1}\cdot\cot g\frac{\varphi_1}{2}\right) + \frac{\pi-\varphi_1}{2}. \tag{6}$$

By projecting of the contour ABCD on the direction of AD after that on a perpendicular direction on same direction one arrives the relationship:

$$\lambda_1\cos\varphi_1 + \lambda_2\cos\varphi_2 + \lambda_1\cos\varphi_3 = 1, \tag{7}$$

$$\lambda_1\sin\varphi_1 + \lambda_2\sin\varphi_2 - \lambda_1\sin\varphi_3 = 0. \tag{8}$$

If the relations (7) and (8) are differentiated with respect to φ_1, then one obtains a linear system of two equations with two unknowns (the transmission ratios)

$$i_{21} = d\varphi_2 / d\varphi_1, \quad i_{31} = d\varphi_3 / d\varphi_1.$$

Solving this system and performing some trigonometrically transformations one arrives at the relations

$$i_{21} = -\frac{\lambda_1}{\lambda_2}\cdot\frac{\sin(\varphi_1+\varphi_3)}{\sin(\varphi_2+\varphi_3)}, \quad i_{31} = \frac{\sin(\varphi_2-\varphi_1)}{\sin(\varphi_2+\varphi_3)}. \tag{9}$$

3. Geometrical and operational constraints for the Ackermann linkage

As is known, the Ackermann linkage may be situated either before or at back of the front axle. Corresponding to the two cases one uses the following denominations: leading Ackermann linkage and trailing Ackermann linkage.

3.1. Trailing Ackermann steering linkage

Schematically, the disposal of the trailing Ackermann steering linkage is indicated in Fig.2.

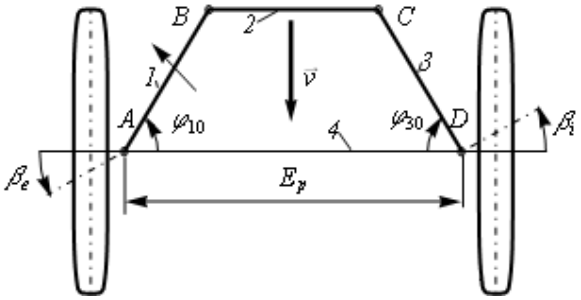

Fig. 3. Trailing Ackermann steering linkage-geometrical elements

Supposing that the turning is negotiated to the left ($\vec{v}$ -motor vehicle velocity), the outer wheel is turned by angle β_e and the inner wheel is turned by angle β_i. Inspecting the Fig. 2 one can write the relations:

$$\beta_e = \varphi_1 - \varphi_{10},\ \beta_i = \varphi_{10} - \varphi_3, \tag{10}$$

where $\varphi_{10}=\varphi_{30}$ represent the values of φ_1 and φ_3 when the motor vehicle is in a straight motion. Elementary geometrical considerations yield

$$\cos\varphi_{10} = \cos\varphi_{30} = (1-\lambda_2)/(2\lambda_1). \tag{11}$$

In the case of the trailing Ackermann linkage $l_2 \le l_4 = E_p$, so that $\lambda_2 \le 1$. The existence condition of the Ackermann linkage is

$$0 \le 1-\lambda_2 \le 2\lambda_1,\ (\lambda_1 \ne 0). \tag{12}$$

Applying the Grashof's theorem or the variant expounded in [4] one readily establishes that the trailing Ackermann linkage is double-rocker.

For a large turning angle of the outer wheel the Ackermann linkage may arrive in critical position when the points B, C and D lie in a straight line. In this situation the angle φ_1 has the value φ_{1l}. Applying law cosines for the triangle formed at mentioned position we obtain

$$\cos\varphi_{1l} = \frac{1+\lambda_1^2-(\lambda_1+\lambda_2)^2}{2\lambda_1}. \tag{13}$$

Obviously, the following inequalities must be satisfied

$$-2\lambda_1 \le 1+\lambda_1^2-(\lambda_1+\lambda_2)^2 \le 2\lambda_1, (\lambda_1 > 0). \tag{14}$$

The right inequality of (14) becomes after transformations

$$(\lambda_2+1)(2\lambda_1+\lambda_2-1) \ge 0.$$

Because $\lambda_2+1>0$, it results that $2\lambda_1+\lambda_2-1\ge 0$, which coincides with the right inequality of (12). The left inequality of (14) can be written as

$$(\lambda_2-1)(2\lambda_1+\lambda_2+1) \le 0.$$

Because $\lambda_2-1\le 0$, one deduces that $2\lambda_1+\lambda_2+1>0$, which is allways satisfied. Therefore, if the existence conditions (12) for the Ackermann linkage are satisfied then the inequalities (14) are satisfied.

Obviously, it is necessary that allways $\varphi_1 \le \varphi_{1l}$. Taking into account (10), it follows that

$$\varphi_{1l} - \varphi_{10} \le \beta_{e\max}, \tag{15}$$

where β_{emax} represents the maximum required turning angle of the outer wheel (its value corresponds to the minimum turning radius). Taking into account (11) and (13), the relation (15) becomes

$$\arccos\frac{1+\lambda_1^2-(\lambda_1+\lambda_2)^2}{2\lambda_1} - \arccos\frac{1-\lambda_2}{2\lambda_1} \ge \beta_{e\max}. \tag{16}$$

Obviously it is necessary that

$$\frac{1+\lambda_1^2-(\lambda_1+\lambda_2)^2}{2\lambda_1} < \frac{1-\lambda_2}{2\lambda_1},$$

which, after transformations, becomes $\lambda_2(2\lambda_1+\lambda_2-1)>0 \Rightarrow 2\lambda_1+\lambda_2-1>0$. The preceding inequality coincides with the inequality associated with the existence condition of the Ackermann linkage (without the equal sign because otherwise it would mean that $\beta_{emax}=0$).

For the bar mechanisms the motion transmitting can be readily achieved if the pressure angle does not exceed certain admissible value [4]. The transmission angle, which is complement of the pressure angle, can not be smaller than 20°-30° in the case of an Ackermann linkage [3]. One can prove that the transmission angle between the element 2 and the element 3 is given as

$$\gamma_{23} = \varphi_2 + \varphi_3. \tag{17}$$

If γ_a is the admissible value of the transmission angle and if we take into account (5), (6), the first relation in (10) and the relation (17) then the condition of the motion transmitting is written as

$$\arccos\frac{1+\lambda_2^2-2\lambda_1\cos(\beta_{e\max}+\varphi_{10})}{2\lambda_2\sqrt{1+\lambda_1^2-2\lambda_1\cos(\beta_{e\max}+\varphi_{10})}} + \arccos\frac{1+2\lambda_1^2-\lambda_2^2-2\lambda_1\cos(\beta_{e\max}+\varphi_{10})}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos(\beta_{e\max}+\varphi_{10})}}$$

$$\ge \gamma_a \tag{18}$$

The expression under root sign in (18) is allways positive. Indeed, from evident inequality $1+\lambda_1^2>2\lambda_1$ it follows that $1+\lambda_1^2>2\lambda_1\cos(\beta_{emax}+\varphi_{10})$ since $-1<\cos(\beta_{emax}+\varphi_{10})<1$. The numerators of the fractions from (18) are allways positive. Indeed, from evident inequality $\cos(\beta_{emax}+\varphi_{10})<\cos\varphi_{10}=(1-\lambda_2)/(2\lambda_1)$ it follows that : $1+\lambda_2^2-2\lambda_1\cos(\beta_{emax}+\varphi_{10})>\lambda_2+\lambda_2^2>0$, $1+2\lambda_1^2-\lambda_2^2-2\lambda_1\cos(\beta_{emax}+\varphi_{10})>2\lambda_1^2+\lambda_2(1-\lambda_2)>0$, ($\lambda_2 \le 1$).

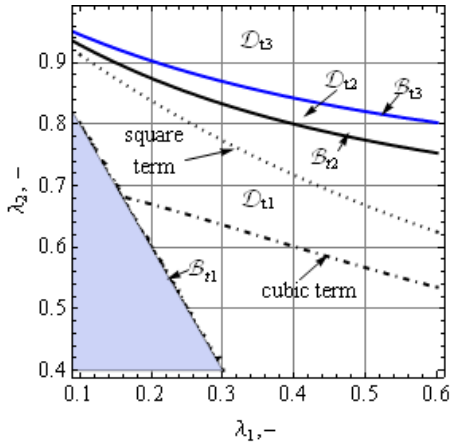

Fig. 3. The graphical representation of the conditions (12), (16) and (18) for the trailing Ackermann steering linkage : β_{emax}=35°, γ_a=25°

For a given value of β_{emax}, the condition (18) is satisfied when λ_1 and λ_2 are chosen so that the condition (16) without the equal sign is fulfilld. We can prove that in this conditions the arguments of the functions arccos from (16) are less than 1 (we readily verify that if in (16) the case of equality is considered the mentioned arguments are equal to 1).

For given values of β_{emax} and γ_a, the conditions (12), (16) and (18) are dependent on the normalized lengths λ_1 and λ_2. On the plotting plane (λ_1,λ_2) with λ_1>0, λ_2>0, the above mentioned conditions define the domains $\mathcal{D}_{t1}$, $\mathcal{D}_{t2}$ and $\mathcal{D}_{t3}$, respectively. Of course, the domain $\mathcal{D}_{t1}$ has boundary defined by a straight line. Using a proper program in *Mathematica*®, the boundaries of the domains $\mathcal{D}_{t2}$ and $\mathcal{D}_{t3}$ have been constructed. They are shown in Fig. 3. The boundaries are depicted by $\mathcal{B}_{t1}$, $\mathcal{B}_{t2}$ and $\mathcal{B}_{t3}$, respectively. It is found that $\mathcal{D}_{t3}\subset\mathcal{D}_{t2}\subset\mathcal{D}_{t1}$. One can prove that the boundary of the domain defined in the first quadrant of the coordinate system has a horizontal asymptote given by the equation $\lambda_2 = \sin\beta_{e\max}$. Therefore, the normalized lengths of the trailing Ackermann linkage should theoretically fulfil the conditions: $\lambda_1\in[\ \lambda_{1\varepsilon},\ \infty)$, $\lambda_2\in[\sin\beta_{emax},\ 1]$, where $\lambda_{1\varepsilon}$ is a positive number however small but finite. Also, one can prove that the boundary of the domain defined by (18) in the first quadrant of the coordinate system has a horizontal asymptote given by equation $\lambda_2=\sin\beta_{emax}/\cos\gamma_a$, so that $\lambda_1\in[\lambda_{1\varepsilon},\ \infty)$, $\lambda_2\in[\sin\beta_{emax}/\cos\gamma_a,\ 1]$. Of course, the values of λ_1 and λ_2 should be chosen according to design constraints.

3.2. Leading Ackermann steering linkage

Inspecting the Fig. 4 one can write

$$\beta_e = \varphi_{10} - \varphi_1, \quad \beta_i = \varphi_3 - \varphi_{10}. \tag{19}$$

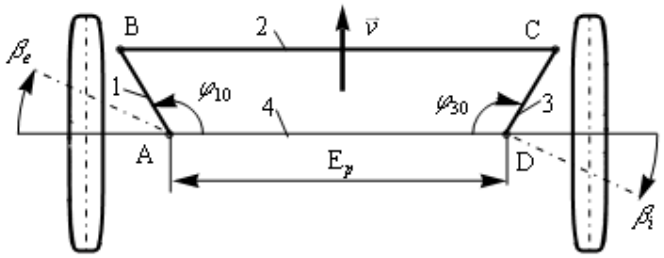

Fig. 4. Leading Ackermann steering linkage : geometrical elements

In this case $\lambda_2>1$ and the existence condition of the trapezium is

$$1-\lambda_2 \geq -2\lambda_1, (\lambda_1 \neq 0, \lambda_2 \geq 1) \tag{20}$$

In time, the Ackermann linkage is a double crank mechanism. In critical position the points B, C and D are collinear (the point C lies on the right side of the point D). Alike as in the preceding case the expression of the limit angle is obtained as:

$$\cos\varphi_{1l} = \frac{1+\lambda_1^2-(\lambda_2-\lambda_1)^2}{2\lambda_1}. \tag{21}$$

Taking into account the first relation (19) we obtain

$$\arccos\frac{1-\lambda_2}{2\lambda_1} - \arccos\frac{1+\lambda_1^2-(\lambda_2-\lambda_1)^2}{2\lambda_1} \geq \beta_{e\max}. \tag{22}$$

One can prove that the transmission angle is given by relation

$$\gamma_{23} = \pi - (\varphi_2 + \varphi_3). \tag{23}$$

In view of the preceding relation the condition of motion transmitting is written as

$$\arccos\frac{1+\lambda_2^2-2\lambda_1\cos(\varphi_{10}-\beta_{e\max})}{2\lambda_2\sqrt{1+\lambda_1^2-2\lambda_1\cos(\varphi_{10}-\beta_{e\max})}} + \arccos\frac{1+2\lambda_1^2-\lambda_2^2-2\lambda_1\cos(\varphi_{10}-\beta_{e\max})}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos(\varphi_{10}-\beta_{e\max})}}$$

$$\leq \pi - \gamma_a \tag{24}$$

As in the case of the trailing Ackermann linkage the imposed requirements on the parameters λ_1 and λ_2 relating to the relations (21) and (22) are associated with the existence condition of the trapezium (the proofs is similar). In the same way one can verify that the expression under sign roots in (24) is positive. To be fulfilled the inequality (24), λ_1 and λ_2 should satisfy the strict inequality (22). One can prove that the moduli of the arguments of the functions arcos in (24) are less 1. If one considers the equal sign in (22) then the mentioned arguments are 1 and -1 (the order may be inverse).

Following the same considerations as in the case of the trailing Ackermann linkage, the graphs shown in Fig.5 are obtained. At the same time, there are the domains $\mathcal{D}_{l1}$, $\mathcal{D}_{l2}$ and $\mathcal{D}_{l3}$. Also, the boundaries of these domains are

depicted by $\mathcal{B}_{l1}$, $\mathcal{B}_{l2}$ and $\mathcal{B}_{l3}$, respectively. We can prove that the boundary of the domain defined by (22) considering that the region marked by $\lambda_2 \geq 1$ has an oblique asymptote given by the equation $\lambda_2 = 2\lambda_1 + \cos\beta_{e\max}$. Also, the boundary defined by (24) has an oblique asymptote of which slope m satisfies the equation $\arccos(0.5m) + \arccos(1-0.5m^2) = \pi - \gamma_a$. It is found that λ_2 should be less than a certain specified value for a given λ_1. This condition is opposite to that of the trailing Ackermann linkage.

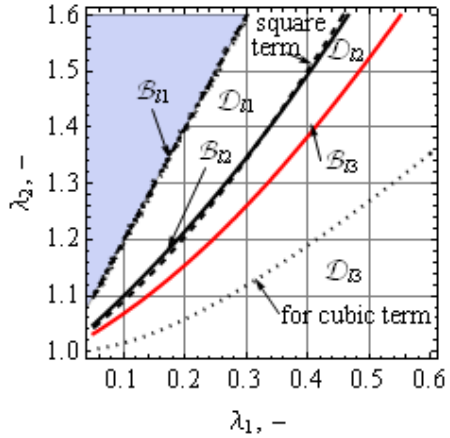

Fig. 5. The graphical representation of the conditions (20), (22) and (24) for the trailing Ackermann steering linkage : β_{emax}=35°, γ_a=25°

4. Analysis of the capability of an Ackermann steering linkage to carry out the condition of correct steering

The condition of correct steering is written as [2, 5]:

$$\cot g\beta_e - \cot g\beta_i = E_p / L, \tag{25}$$

where L is the wheel base of the automobile. If it is supposed that β_e is the input quantity and β_l is output quantity then the relation (25) leads to the relation

$$\beta_i = arctg\left(\frac{tg\beta_e}{1-(E_p/L)\cdot tg\beta_e}\right). \tag{26}$$

Taylor's series expansion around the origin of the second part of the relation (26) yields (obviously, β_e=0⇒β_i=0)

$$\beta_i = \beta_e + (E_p/L).\beta_e^2 + (E_p/L)^2.\beta_e^3 + [-(E_p/L)/3 + (E_p/L)^3].\beta_e^4 + [-(E_p/L)^2 + (E_p/L)^4].\beta_e^5 + [(E_p/L)(2/45) - 2(E_p/L)^3 + (E_p/L)^5].\beta_e^6 + O[\beta_e^7]. \quad (27)$$

If β_{it} is the value of β_1 determined by (26) and β_{in} is the value of the same angle determined by relation (27) when n terms are retained (n=2, 3, 4, 5, 6), then the error committed by the relation (27) is defined so

$$e_t = \frac{\beta_{in} - \beta_{it}}{\beta_{it}} \cdot 100\,[\%]. \quad (28)$$

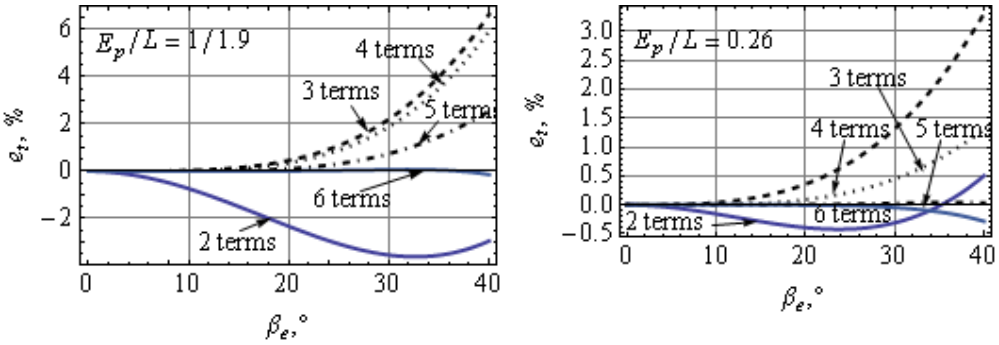

Fig. 6. The influence of the term number of Taylor's series on the error

From the inspection of the numerical results highlighted by Fig.6 it is found that the approximation with two terms is better than that with 3 and 4 terms for range angles greater than 20°-35°(obviously, this angle is dependent on the ratio E_p/L). The retaining of 6 terms provides a high accuracy for the large values of the angle.

The series expansion of the function $\varphi_3(\varphi_1)$ is (obviously, $\varphi_{30}=\varphi_{10}$):

$$\varphi_3(\varphi_1) = \varphi_{10} + \frac{1}{1!}\frac{d\varphi_3}{d\varphi_{10}}(\varphi_1 - \varphi_{10}) + \frac{1}{2!}\frac{d^2\varphi_3}{d\varphi_{10}^2}(\varphi_1 - \varphi_{10})^2 + \frac{1}{3!}\frac{d^3\varphi_3}{d\varphi_{10}^3}(\varphi_1 - \varphi_{10})^3 + O[(\varphi_1 - \varphi_{10})^4]. \quad (29)$$

The first derivative of the function φ_3 is given by relation (9). Taking into account that φ_{10}□φ_{20}=0, from (9) it results

$$\frac{d\varphi_3}{d\varphi_{10}} = -1, \quad \frac{d\varphi_2}{d\varphi_{10}} = -\frac{\lambda_1}{\lambda_2} \cdot \frac{\sin 2\varphi_{10}}{\sin \varphi_{10}}. \quad (30)$$

Using the expression of the first derivative we get

$$\frac{d^2\varphi_3}{d\varphi_1^2}=\frac{d}{d\varphi_1}[\frac{\sin(\varphi_2-\varphi_1)}{\sin(\varphi_2+\varphi_3)}]=\frac{1}{\sin^2(\varphi_2+\varphi_3)}[\cos(\varphi_2-\varphi_1)(\frac{d\varphi_2}{d\varphi_1}-1)\sin(\varphi_2+\varphi_3)-$$

$$-\cos(\varphi_2+\varphi_3)\sin(\varphi_2-\varphi_1)(\frac{d\varphi_2}{d\varphi_1}+\frac{d\varphi_3}{d\varphi_1})] \tag{31}$$

In considerations on (30), after a succession of transformations, we arrive at the relation

$$\frac{d^2\varphi_3}{d\varphi_{10}^2}=-2\frac{\cos\varphi_{10}}{\sin\varphi_{10}}(2\frac{\lambda_1}{\lambda_2}\cos\varphi_{10}+1). \tag{32}$$

In view of (11) the preceding relation becomes

$$\frac{d^2\varphi_3}{d\varphi_{10}^2}=-2\frac{1-\lambda_2}{\lambda_2\sqrt{4\lambda_1^2+2\lambda_2-\lambda_2^2-1}}. \tag{33}$$

In a similar way one proceeds for the third order derivative and the result is written as

$$\frac{d^3\varphi_3}{d\varphi_1^3}=-2\left[\frac{4}{\lambda_2}-\frac{\lambda_2}{\lambda_1^2}+2\left(\frac{3\lambda_1}{\lambda_2^2}+\frac{1}{\lambda_1}\right)\cos\varphi_{10}+\frac{4}{\lambda_2}\cos 2\varphi_{10}+2\frac{\lambda_1}{\lambda_2^2}\right]\frac{\cos\varphi_{10}}{\sin^2\varphi_{10}}, \tag{34}$$

Finally, in consideration on (11) we get

$$\frac{d^3\varphi_3}{d\varphi_{10}^3}=\frac{4(-1+\lambda_2)(-1+\lambda_2+\lambda_2^3)}{\lambda_2^2[-4\lambda_1^2+(-1+\lambda_2)^2]}. \tag{35}$$

Substituting $\varphi_3(\varphi_1)$ given by (29) into the second relation (10) and taking into account the first relation (10), and (30), (32), (33), we arrive at the expression

$$\beta_{itt}=\beta_e+\frac{1-\lambda_2}{\lambda_2\sqrt{4\lambda_1^2+\lambda_2^2+2\lambda_2-1}}\cdot\beta_e^2-\frac{2}{3}\cdot\frac{(-1+\lambda_2)(-1+\lambda_2+\lambda_2^3}{\lambda_2^2[-4\lambda_1^2+(-1+\lambda_2)^2]}.\beta_e^3+O[\beta_e^4], \tag{36}$$

where β_{itt} is the angle achieved by the trailing Ackermann linkage for the inner wheel. Comparing the relations (27) and (36) it is found that the linear terms are the same. Therefore, the Ackermann linkage provides the condition of correct steering to the squared terms (exclusively) of the turning angle β_e regardless the design characteristics. There is the ideal case when the expansions (27) and (36) would be identical, which is impossible because only two independent parameters are available (λ_1 and λ_2). Therefore we may set the condition that the terms of β_e^2 and β_e^3 be identical, namely :

$$\frac{1-\lambda_2}{\lambda_2\sqrt{4\lambda_1^2+\lambda_2^2+2\lambda_2-1}}=\frac{E_p}{L},\ -\frac{2}{3}\cdot\frac{(-1+\lambda_2)(-1+\lambda_2+\lambda_2^3}{\lambda_2^2[-4\lambda_1^2+(-1+\lambda_2)^2]}=\left(\frac{E_p}{L}\right)^2. \quad (37)$$

From system (37), λ_1 and λ_2 might be determined, but, generally, this system has no real positive solutions. To illustrate this, in Fig.3 the curves corresponding to the two equations (37) are also shown. It is found that the curves do not intersect into the domain which is of practical interest. Moreover, the curve for 3 degree term which is located within the domain does not yield $\beta_e \geq 35°$. The condition for the two degree term can be fulfilled but it does not assure the condition of the motion transmitting.

In the case of the leading Ackermann linkage the relations (27)-(35) remain valid. In consideration of (19) we get finally the expression

$$\beta_{itl}=\beta_e-\frac{1-\lambda_2}{\lambda_2\sqrt{4\lambda_1^2+\lambda_2^2+2\lambda_2-1}}\cdot\beta_e^2-\frac{2}{3}\cdot\frac{(-1+\lambda_2)(-1+\lambda_2+\lambda_2^3}{\lambda_2^2[-4\lambda_1^2+(-1+\lambda_2)^2]}.\beta_e^3+O[\beta_e^4], \quad (38)$$

where β_{ilt} is the angle achieved by the leading Ackermann steering linkage for the inner wheel. So we arrive at a system similar to (31) and the representation of the associated curves is shown in Fig. 5. And in this case the system does not have real positive solutions. The curve for two degree term is enough near to the limit curve for the motion transmitting. The curve corresponding to the 3 degree term is located into the admissible domain.

From the above results it follows that the graph of the transmission function of the Ackermann steering linkage has at origin the same tangent as the graph of the theoretical transmission function corresponding to the correct steering condition. In [3] the conditions in which the two mentioned graphs have the same curvature at origin has been investigated. The determination of the second derivative of the transmission function has been carried out by the acceleration method, which is more intricate than that presented here. The obtained result has a form which differs from that given in this paper, but one can prove that the proper result is not different (one can directly prove starting from (32)). The above mentioned method cannot be applied for determination of the third derivative, what explain probably the reason for which the question has not been tackled.

The applied method for the approximate synthesis in [2, 4] is different than that used in the present paper. In the implicit expression of the transmission function the angle β_e is substituted for the terms to third degree term of β_l from the Taylor's series expansion, after that a succession of approximations are made. Further, l_2 and λ_1 being chosen one determines φ_{10}. For example [4], for

E_p/L=0.40 and λ_1=0.30 the result is λ_2=0.80. This result leads to β_{emax}=31°, which is enough small.

Considering different values of E_p/L we can conclude that, generally, the approximate synthesis of the Ackermann linkage by the series expansion method cannot carried out only to at the most the square outer turning angle.

5. Conclusions

The relations established in the paper allow defining the admissible domains of the normalized lengths of the Ackermann steering linkage elements corresponding to the critical position of the mechanism and the admissible transmission angle. These domains are not dependent on the automobile characteristics. The mentioned relations render evident the fact that the constraints of the Ackermann steering linkage are hard enough. Also, these relations can be straightway used to optimize the Ackermann steering linkage, which we shall do in a next paper.

The synthesis of the Ackermann linkage using Taylor's series expansion has an important limitation because only the linear terms of the transmission function expansion can be exactly reproduced by the mechanism.

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