

ESSENTIAL COMPONENTS OF THE SOLUTION SET FOR MULTICLASS MULTICRITERIA TRAFFIC EQUILIBRIUM PROBLEMS

Xiao-Jun YU¹, Hui YANG²

In this paper, we study multiclass multicriteria traffic equilibrium (MMTE) problem in the fixed demand case and investigate relations between vector variational inequality and weak vector equilibrium flows. We show that there exists at least one essential components of the solution set for each MMTE problem.

Keywords: Weak vector equilibrium principle, vector variational inequality, traffic, essential component

1. Introduction

Wardrop [1] introduced the famous user equilibrium principle for traffic network, which is a scalar equilibrium principle. Smith [2] investigated that a Wardrop's user equilibrium flow is equivalent to the solution of a class of variational inequalities when the travel cost function is a scalar function. Recently, many researchers have proposed equilibrium models based on multicriteria consideration or vector-valued cost functions. Chen and Yen [3] first proposed (weak) vector equilibrium principle for a vector traffic network without capacity constraints, which is a generalization of the classic Wardrop's user equilibrium principle. In [4], Yang and Goh investigated equivalent relations between vector variational inequalities and vector equilibrium flows based on vector equilibrium principle. Daniele et al. [5, 6] studied a traffic equilibrium problem with capacity constraints in dynamic case and obtained sufficient and necessary conditions for a traffic equilibrium flow. Lin [7] extended weak vector equilibrium principle to the case of capacity constraints of arcs and showed that there exists at least one essential components of the solution set for traffic equilibrium problems with capacity constraints of arcs. However, all the researches mentioned above assumed that the users in the traffic network are homogenous. In reality, we have to group users in different classes due to their differences in the income, age, gender, education, travel destination, and so on. Nagurney [8], Nagurney and Dong [9] discussed MMTE problem without capacity constraints with fixed demand and elastic demand, respectively, and

¹ School of Mathematics and Statistics, Guizhou College of Finance and Economics, China, e-mail: xjyu-my@163.com

² Department of Mathematics, Guizhou University, China

obtained the equivalent relations between equilibrium flows and variational inequalities. Raciti [10] derived the relations between vector variational inequality and MMTE problems with path constraints. For other results of MMTE problems, we refer to [11-15], and for other results of essential components relating to equilibrium problems, we refer to [16-19] and the references therein.

In this study, we investigate MMTE problems when demands are fixed. First, we derive a sufficient condition of weak vector equilibrium flows based on the weak vector equilibrium principle. Then, we obtain an existence result of MMTE problem. Finally, we show that there exists at least one essential components of the solution set for each MMTE problem.

2. MMTE problems models

Let $G = (N, E)$ denote a transportation network, with a finite set of nodes N and a finite set of directed links E ($|E|$ is the number of all directed links in the network). Let W be the set of all Origin-Destination (OD) pairs and P be the set of all paths in network ($n_p = |P|$ is the number of all paths in the network). Let P_w be the set of all paths between OD pair $w \in W$. Assume that there are M classes of users in the network with a typical class denoted by m . The other notations used throughout this paper are as follows: d_w^m is the demand of class m between OD pair $w \in W$, which is assumed to be constant; v_a^m is the flow of user class m on link $a \in E$; v_a is the aggregate flow on link $a \in E$; the vector of link flow is $\mathbf{v} = (v_{a_1}^1, \dots, v_{a_{|E|}}^1, \dots, v_{a_1}^M, \dots, v_{a_{|E|}}^M)$; f_p^m is the flow of class m on path $p \in P_w$; $\mathbf{f}^m = (f_{p_1}^m, \dots, f_{p_{n_p}}^m)^T \in R^{n_p}$ is the vector flow of the class m , where p_1, \dots, p_{n_p} denote n_p distinct paths in the network G ; $\mathbf{f} = ((\mathbf{f}^1)^T, (\mathbf{f}^2)^T, \dots, (\mathbf{f}^M)^T)^T \in R^{Mn_p}$ is the vector of path flow in the network G ; $\mathbf{C}_a^m(\mathbf{f}) = (C_{ja}^m(\mathbf{f}))^T \in R^l, j = 1, 2, \dots, l$ is the vector cost of class m on link a ; $\mathbf{C}_p^m(\mathbf{f}) = (C_{jp}^m(\mathbf{f}))^T \in R^l, j = 1, 2, \dots, l$ is the vector cost of class m on path p ; $\delta_{ap} = 1$ if path p traverses link $a \in E$, and $\delta_{ap} = 0$ otherwise.

Therefore, the following relationship must be satisfied, i.e.

$$C_{jp}^m(\mathbf{f}) = \sum_{a \in E} C_{ja}^m(\mathbf{f}) \delta_{ap}, \forall p \in P_w, w \in W, m = 1, \dots, M, j = 1, \dots, l.$$

The link flow and the path flow have relation as follows:

$$v_a^m = \sum_{w \in W} \sum_{p \in P_w} f_p^m \delta_{ap}, \forall a \in E, m = 1, 2, \dots, M.$$

We denote the feasible path flow set

$$\Lambda = \{ \mathbf{f} \mid \mathbf{f} \in R^{Mn_p} \mid \sum_{p \in P_w} f_p^m = d_w^m, \forall w \in W, m = 1, 2, \dots, M; \\ f_p^m \geq 0, \forall p \in P_w, w \in W, m = 1, 2, \dots, M \}$$

Clearly, Λ is convex and compact. We introduce matrix-valued functions $\mathbf{C}^m (m = 1, \dots, M)$ from Λ to $R^{l \times n_p}$ and a matrix-valued function \mathbf{C} from Λ to $R^{l \times Mn_p}$ as follows:

$$\mathbf{C}^m(\mathbf{f}) = \left(\mathbf{C}_{p_1}^m(\mathbf{f}), \dots, \mathbf{C}_{p_{n_p}}^m(\mathbf{f}) \right) = \begin{bmatrix} C_{1p_1}^m(\mathbf{f}) & \dots & C_{1p_{n_p}}^m(\mathbf{f}) \\ \dots & \dots & \dots \\ C_{lp_1}^m(\mathbf{f}) & \dots & C_{lp_{n_p}}^m(\mathbf{f}) \end{bmatrix}, m = 1, 2, \dots, M.$$

and $\mathbf{C}(\mathbf{f}) = (\mathbf{C}^1(\mathbf{f}), \dots, \mathbf{C}^M(\mathbf{f}))$.

Now, we introduce following definitions.

Definition 2.1. (Weak vector equilibrium principle) A flow $\mathbf{f} \in \Lambda$ is said to be in weak vector equilibrium if for each class m , for all OD pairs w and for any path $p, q \in P_w$ such that $\mathbf{C}_p^m(\mathbf{f}) - \mathbf{C}_q^m(\mathbf{f}) \in \text{int } R_+^l \Rightarrow f_p^m = 0$, where $\text{int } R_+^l$ is the interior of R_+^l .

\mathbf{f} is said to be a weak vector equilibrium flow. A MMTE problem is usually denoted by $\Gamma = \{G, \Lambda, \mathbf{C}\}$ (in brief, $\{\mathbf{C}\}$). \mathbf{f} is said to be a solution of Γ if \mathbf{f} is a weak vector equilibrium flow of Γ .

Definition 2.2. Let X, Y are two Hausdorff topological vector space and K is a nonempty subset of X , and $H: K \mapsto 2^Y$ is a set-valued mapping, where 2^Y denotes the family of all nonempty subset of Y , then

(1) H is said to be upper semicontinuous at $x \in K$, if for each open set U in Y with $U \supset H(x)$, there exists an open neighborhood $O(x)$ of x such that $U \supset H(x')$ for any $x' \in O(x)$; and upper semicontinuous on K if it is upper semicontinuous at each point of K .

(2) H is said to be lower semicontinuous at $x \in K$, if for each open set U in Y with $U \cap H(x) \neq \emptyset$, there exists an open neighborhood $O(x)$ of x such that $U \cap H(x') \neq \emptyset$ for any $x' \in O(x)$; and lower semicontinuous on K if it is lower semicontinuous at each point of K .

(3) H is said to be continuous at $x \in K$ if it is upper semicontinuous and lower semicontinuous at $x \in K$; and continuous on K if it is continuous at each point of K .

(4) H is an usco mapping, if H is upper semicontinuous on K , and for each $x \in K, H(x)$ is compact.

Definition 2.3. Let X, Y are two Hausdorff topological vector space and K is a nonempty subset of X and $g : K \mapsto Y$ is a vector-valued function, and C is a nonempty closed, convex and pointed cone in Y with $\text{int } C \neq \emptyset$. g is said to be C -continuous at $x_0 \in K$, if for any open set V of the zero element θ in Y , there exists an open neighborhood $O(x_0)$ of x_0 in K , for all $x \in O(x_0), g(x) \in g(x_0) + V + C$; and C -continuous on K if it is C -continuous at every point of K .

The following result is a particular form of a maximal element theorem for a family of set-valued mapping due to Deguire et al. (see [20], Theorem 1).

Lemma 2.1. Let K be a nonempty compact convex subset of a Hausdorff topological vector space X . Suppose that $H : K \mapsto 2^K \cup \{\emptyset\}$ is a set-valued mapping with following conditions:

- (i) for each $x \in K, x \notin H(x)$;
- (ii) for each $x \in K, H(x)$ is convex;
- (iii) for each $y \in K, H^{-1}(y) = \{x \in K : y \in H(x)\}$ is open in K .

Then there exists $\bar{x} \in K$ such that $H(\bar{x}) = \emptyset$.

3. Existence of weak vector equilibrium flows for MMTE problem

First, we establish a sufficient condition for a weak vector equilibrium flow as follows:

Theorem 3.1. The flow $\mathbf{f}^* \in \Lambda$ is in weak vector equilibrium if \mathbf{f}^* solves the following vector variational inequality problem: find $\mathbf{f}^* \in \Lambda$ such that

$$\langle C(\mathbf{f}^*), \mathbf{f} - \mathbf{f}^* \rangle \notin -\text{int } R_+^l, \forall \mathbf{f} \in \Lambda.$$

Proof: Suppose that $\mathbf{f}^* \in \Lambda$ satisfy above vector variational inequality but it is not a weak vector equilibrium flow. Then there exist $1 \leq m^* \leq M, w^* \in W$ and

$$q, r \in P_{w^*} \text{ such that } C_q^{m^*}(\mathbf{f}^*) - C_r^{m^*}(\mathbf{f}^*) \in \text{int } R_+^l, (f_q^*)^{m^*} > 0.$$

Construct a path flow vector \mathbf{f} to be as follows: If $m \neq m^*, \mathbf{f}^m = (\mathbf{f}^*)^m$, otherwise, the components of \mathbf{f}^m is

$$f_p^m = \begin{cases} (f_p^*)^{m^*}, & p \neq q, r; \\ 0, & p = q; \\ (f_q^*)^{m^*} + (f_r^*)^{m^*}, & p = r. \end{cases}$$

It is easy to verify that $\mathbf{f} \in \Lambda$. So, we have

$$\langle \mathbf{C}(\mathbf{f}^*), \mathbf{f} - \mathbf{f}^* \rangle = \sum_{m=1}^M \sum_{p \in P} \langle \mathbf{C}_p^m(\mathbf{f}^*), f_p^m - (f_p^*)^{m^*} \rangle = \langle \mathbf{C}_r^{m^*}(\mathbf{f}^*) - \mathbf{C}_q^{m^*}(\mathbf{f}^*), (f_q^*)^{m^*} \rangle \in -\text{int } R_+^l$$

which is a contradiction. The proof is complete.

The following theorem is our existence result.

Theorem 3.2. Consider a MMTE problem $\Gamma = \{\mathbf{C}\}$. Assume that for each $i (1 \leq i \leq n_p)$, $m (1 \leq m \leq M)$ and each $\mathbf{h} \in \Lambda$, $\mathbf{C}_{p_i}^m(\mathbf{f})(h_{p_i}^m - f_{p_i}^m)$ is $-R_+^l$ continuous on Λ , then Γ has a solution.

Proof: Define the set-valued mapping $S : \Lambda \rightarrow 2^\Lambda \cup \{\emptyset\}$ by

$$S(\mathbf{f}) = \{\mathbf{h} \in \Lambda : \mathbf{C}(\mathbf{f})(\mathbf{h} - \mathbf{f}) \in -\text{int } R_+^l\}.$$

(1) It is easy to verify that for each $\mathbf{f} \in \Lambda$, $\mathbf{f} \notin S(\mathbf{f})$.

(2) For each $\mathbf{f} \in \Lambda$, let $\mathbf{h}_1, \mathbf{h}_2 \in S(\mathbf{f})$, then $\mathbf{C}(\mathbf{f})(\mathbf{h}_1 - \mathbf{f}) \in -\text{int } R_+^l$ and $\mathbf{C}(\mathbf{f})(\mathbf{h}_2 - \mathbf{f}) \in -\text{int } R_+^l$. Since $-\text{int } R_+^l$ is convex, we obtain that for any $\lambda \in [0, 1]$ $\lambda \mathbf{C}(\mathbf{f})(\mathbf{h}_1 - \mathbf{f}) + (1 - \lambda) \mathbf{C}(\mathbf{f})(\mathbf{h}_2 - \mathbf{f}) \in -\text{int } R_+^l$. Thus,

$$\mathbf{C}(\mathbf{f})(\lambda \mathbf{h}_1 + (1 - \lambda) \mathbf{h}_2 - \mathbf{f}) = \lambda \mathbf{C}(\mathbf{f})(\mathbf{h}_1 - \mathbf{f}) + (1 - \lambda) \mathbf{C}(\mathbf{f})(\mathbf{h}_2 - \mathbf{f}) \in -\text{int } R_+^l.$$

Therefore, the set $S(\mathbf{f})$ is convex.

(3) If $\mathbf{h} \in S(\mathbf{f})$, then $\mathbf{C}(\mathbf{f})(\mathbf{h} - \mathbf{f}) \in -\text{int } R_+^l$, which implies that there is an open neighborhood V of the zero element θ such that $\mathbf{C}(\mathbf{f})(\mathbf{h} - \mathbf{f}) + V \subset -\text{int } R_+^l$. Thus, there is an open neighborhood $O(\mathbf{f})$ of \mathbf{f} such that, for each $\mathbf{f}' \in O(\mathbf{f})$,

$$\begin{aligned} \mathbf{C}(\mathbf{f}')(\mathbf{h} - \mathbf{f}') &= \sum_{m=1}^M \sum_{p_i \in P} \left[\mathbf{C}_{p_i}^m(\mathbf{f}') (h_{p_i}^m - (f_{p_i}')^m) \right] \\ &\in \sum_{m=1}^M \sum_{p_i \in P} \left[\mathbf{C}_{p_i}^m(\mathbf{f}') (h_{p_i}^m - (f_{p_i}')^m) + \frac{V}{Mn_p} - R_+^l \right] \\ &\subset \mathbf{C}(\mathbf{f})(\mathbf{h} - \mathbf{f}) + V - R_+^l \subset -\text{int } R_+^l - R_+^l \subset -\text{int } R_+^l \end{aligned}$$

which implies that $O(\mathbf{f}) \subset S^{-1}(\mathbf{h}) = \{\mathbf{f} \in \Lambda : \mathbf{C}(\mathbf{f})(\mathbf{h} - \mathbf{f}) \in -\text{int } R_+^l\}$, i.e., $S^{-1}(\mathbf{h})$ is open. By lemma 2.1, the result follows and our proof is finished.

4. Essential components of weak vector equilibrium flows

By theorem 3.2, it is easy to obtain the following corollary:

Corollary 4.1. Consider a MMTE problem $\Gamma = \{\mathbf{C}\}$. Suppose that for each $i(1 \leq i \leq n_p)$ $m(1 \leq m \leq M)$, $\mathbf{C}_{p_i}^m(\cdot)$ is continuous on Λ , then Γ has a solution.

Assume that Θ is the collection of all MMTE problems $\Gamma = \{\mathbf{C}\}$ satisfying the conditions of corollary 4.1.

For each $\bar{\Gamma} = \{\bar{\mathbf{C}}\}, \hat{\Gamma} = \{\hat{\mathbf{C}}\} \in \Theta$, define

$$\rho(\bar{\Gamma}, \hat{\Gamma}) = \max_{\substack{1 \leq m \leq M, 1 \leq j \leq l \\ 1 \leq i \leq n_p, \mathbf{f} \in \Lambda}} \left| \bar{\mathbf{C}}_{jp_i}^m(\mathbf{f}) - \hat{\mathbf{C}}_{jp_i}^m(\mathbf{f}) \right|$$

Clearly, (Θ, ρ) is a metric space. For each $\Gamma \in \Theta$, denote by $F(\Gamma)$ the solution set of Γ . Then F defines a set-valued mapping from Θ into Λ and, by corollary 4.1, $F(\Gamma) \neq \emptyset$ for any $\Gamma \in \Theta$.

Lemma 4.2 $F : \Theta \rightarrow 2^\Lambda$ is an usco mapping.

Proof: Since Λ is compact, by Theorem 7.1.16 of [21], it suffices to show that F is a closed mapping, i.e., the graph $\text{Graph}(F)$ of F is closed in $\Theta \times \Lambda$, where $\text{Graph}(F) = \{(\Gamma, \mathbf{f}) \in \Theta \times \Lambda : \mathbf{f} \in F(\Gamma)\}$.

Let $\{(\Gamma^n, \mathbf{f}^n)\}_{n \in \mathbb{Z}^+}$ be an arbitrary net in $\text{Graph}(F)$ with $(\Gamma^n, \mathbf{f}^n) \rightarrow (\Gamma^*, \mathbf{f}^*) \in \Theta \times \Lambda$, where $\Gamma^n = \{\mathbf{C}^n\}, \Gamma^* = \{\mathbf{C}^*\}$ and $\mathbf{f}^n \in F(\Gamma^n)$. Next we need to prove that $\mathbf{f}^* \in F(\Gamma^*)$. Suppose that $\mathbf{f}^* \notin F(\Gamma^*)$, then there exist $1 \leq m^* \leq M, w^* \in W, q, r \in P_{w^*}$ such that $(\mathbf{C}_q^*)^{m^*}(\mathbf{f}^*) - (\mathbf{C}_r^*)^{m^*}(\mathbf{f}^*) \in \text{int } R_+^l$ and $(f_q^*)^{m^*} > 0$, which implies that there is an open neighborhood V of the zero element θ in R^l such that $(\mathbf{C}_q^*)^{m^*}(\mathbf{f}^*) - (\mathbf{C}_r^*)^{m^*}(\mathbf{f}^*) + V \in \text{int } R_+^l$. Moreover, since $\mathbf{C}^n \rightarrow \mathbf{C}^*, \mathbf{f}^n \rightarrow \mathbf{f}^*$, there exist $N_0 \in \mathbb{Z}^+$ such that, for any $n \geq N_0$

$$\begin{aligned}
& (\mathbf{C}_q^n)^{m^*}(\mathbf{f}^n) - (\mathbf{C}_r^n)^{m^*}(\mathbf{f}^n) \\
&= \left[(\mathbf{C}_q^n)^{m^*}(\mathbf{f}^n) - (\mathbf{C}_q^n)^{m^*}(\mathbf{f}^*) \right] + \left[(\mathbf{C}_q^n)^{m^*}(\mathbf{f}^*) - (\mathbf{C}_q^*)^{m^*}(\mathbf{f}^*) \right] + (\mathbf{C}_q^*)^{m^*}(\mathbf{f}^*) \\
&\quad - \left[(\mathbf{C}_r^n)^{m^*}(\mathbf{f}^n) - (\mathbf{C}_r^n)^{m^*}(\mathbf{f}^*) \right] - \left[(\mathbf{C}_r^n)^{m^*}(\mathbf{f}^*) - (\mathbf{C}_r^*)^{m^*}(\mathbf{f}^*) \right] - (\mathbf{C}_r^*)^{m^*}(\mathbf{f}^*) \\
&\in (\mathbf{C}_q^*)^{m^*}(\mathbf{f}^*) - (\mathbf{C}_r^*)^{m^*}(\mathbf{f}^*) + V \in \text{int } R_+^l
\end{aligned}$$

and $(f_q^n)^{m^*} > 0$, which is a contradiction. Therefore, $(\Gamma^n, \mathbf{f}^n) \in \text{Graph}(F)$, and thus $\text{Graph}(F)$ is closed. The proof is complete.

For each $\Gamma \in \Theta$, the component of a point $\mathbf{f} \in F(\Gamma)$ is the union of all connected subsets of $F(\Gamma)$ containing \mathbf{f} . Note that each component of $F(\Gamma)$ is connected closed subset of $F(\Gamma)$ (see [22], p.356), thereby a connected compact subset as well. The connected components of two distinct points of $F(\Gamma)$ are either superposition or no-intersection. So $F(\Gamma)$ can be decomposed a family of each other non-intersection summation set, i.e., $F(\Gamma) = \bigcup_{\alpha \in I} F_\alpha(\Gamma)$, where I is index set, for each $\alpha \in I$, $F_\alpha(\Gamma)$ is a nonempty connected compact subset of $F(\Gamma)$ and for any $\alpha, \beta \in I, \alpha \neq \beta, F_\alpha(\Gamma) \cap F_\beta(\Gamma) = \emptyset$.

Definition 4.3. Let $\Gamma \in \Theta$ and Z is a nonempty closed subset of $F(\Gamma)$, Z is said to be an essential set of $F(\Gamma)$ if, for any open set $O \supset Z$, there exists $\delta > 0$ such that for any $\Gamma' \in \Theta$ with $\rho(\Gamma, \Gamma') < \delta, F(\Gamma') \cap O \neq \emptyset$. If a component $F_\alpha(\Gamma)$ of $F(\Gamma)$ is an essential set, then $F_\alpha(\Gamma)$ is said to be an essential component of $F(\Gamma)$. An essential set Z of $F(\Gamma)$ is said to be a minimal essential set of $F(\Gamma)$ if Z is a minimal element of the family sets in $F(\Gamma)$ ordered by set inclusion.

In order to prove the following theorem, we firstly present the following condition (c): Let $(X, d), (Y, \rho)$ are two metric spaces, $H : X \rightarrow 2^Y$ is a set-valued mapping. There exists $b > 0$ such that for any two nonempty closed sets K_1, K_2 in Y with $\rho(K_1, K_2) > 0$, there exists $a > 0$ such that for any $x_1, x_2 \in X$ with $d(x_1, x_2) < a, H(x_1) \cap K_1 = \emptyset, H(x_2) \cap K_2 = \emptyset$, there is $x' \in X$ satisfying $d(x', x_1) \leq bd(x_1, x_2), d(x', x_2) \leq bd(x_1, x_2)$ and $H(x') \cap [K_1 \cup K_2] = \emptyset$.

The following lemma is Theorem 2.1 in [23].

Lemma 4.4. Let $H : X \rightarrow 2^Y$ be an usco mapping, and condition (c) holds. Then ,

(1) for any $x \in X$, there is at least one minimal essential set of $H(x)$, and every minimal essential set must be connected;

(2) for any $x \in X$, there is at least one essential component of $H(x)$.

Theorem 4.5. For any $\Gamma \in \Theta$, there exists at least one essential component of $F(\Gamma)$.

Proof: Since F is an usco mapping, by lemma 4.4, we only need to verify the condition (c) holds. Let $b=1$, for any two nonempty closed subsets K_1, K_2 of Λ with $d(K_1, K_2) > 0$ and any $\bar{\Gamma}, \hat{\Gamma} \in \Theta$ with $\rho(\bar{\Gamma}, \hat{\Gamma}) < a=1$ such that $F(\bar{\Gamma}) \cap K_1 = \emptyset, F(\hat{\Gamma}) \cap K_2 = \emptyset$. We construct $\tilde{\Gamma} = \{\tilde{C}\}$: for each $1 \leq i \leq n_p, 1 \leq m \leq M$ and each $\mathbf{f} \in \Lambda$, $\tilde{C}_{p_i}^m(\mathbf{f}) = \lambda(\mathbf{f})\bar{C}_{p_i}^m(\mathbf{f}) + \mu(\mathbf{f})\hat{C}_{p_i}^m(\mathbf{f})$, Where

$$\lambda(\mathbf{f}) = \frac{d(\mathbf{f}, K_2)}{d(\mathbf{f}, K_1) + d(\mathbf{f}, K_2)}, \mu(\mathbf{f}) = \frac{d(\mathbf{f}, K_1)}{d(\mathbf{f}, K_1) + d(\mathbf{f}, K_2)}$$

Note that $\lambda(\mathbf{f}), \mu(\mathbf{f})$ are continuous and for any $\mathbf{f} \in \Lambda, \lambda(\mathbf{f}) \geq 0, \mu(\mathbf{f}) \geq 0, \lambda(\mathbf{f}) + \mu(\mathbf{f}) = 1$. It can be easily checked that $\tilde{\Gamma} \in \Theta$. We have

$$\begin{aligned} \rho(\tilde{\Gamma}, \bar{\Gamma}) &= \max_{\substack{1 \leq m \leq M, 1 \leq j \leq l \\ 1 \leq i \leq n_p, \mathbf{f} \in \Lambda}} |\tilde{C}_{jp_i}^m(\mathbf{f}) - \bar{C}_{jp_i}^m(\mathbf{f})| \\ &= \max_{\substack{1 \leq m \leq M, 1 \leq j \leq l \\ 1 \leq i \leq n_p, \mathbf{f} \in \Lambda}} \mu(\mathbf{f}) |\hat{C}_{jp_i}^m(\mathbf{f}) - \bar{C}_{jp_i}^m(\mathbf{f})| \\ &\leq \rho(\hat{\Gamma}, \bar{\Gamma}) \end{aligned}$$

Similarly, $\rho(\tilde{\Gamma}, \hat{\Gamma}) \leq \rho(\bar{\Gamma}, \hat{\Gamma})$.

If $\mathbf{f} \in K_1$, then $\lambda(\mathbf{f}) = 1, \mu(\mathbf{f}) = 0, \tilde{C}(\mathbf{f}) = \bar{C}(\mathbf{f})$. Since $\mathbf{f} \notin F(\bar{\Gamma})$, we have $\mathbf{f} \notin F(\tilde{\Gamma})$. If $\mathbf{f} \in K_2$, then $\lambda(\mathbf{f}) = 0, \mu(\mathbf{f}) = 1, \tilde{C}(\mathbf{f}) = \hat{C}(\mathbf{f})$. Since $\mathbf{f} \notin F(\hat{\Gamma})$, we have $\mathbf{f} \notin F(\tilde{\Gamma})$. Hence, $F(\tilde{\Gamma}) \cap [K_1 \cup K_2] = \emptyset$. Thus condition (c) holds. The proof is complete.

5. Conclusions

This paper studies the stability of MMTE problem with fixed demand. A sufficient condition of weak vector equilibrium flows of MMTE problem is obtained. Thus, an existence result of MMTE problem is derived and the stability of the solution set for MMTE problem is investigated.

Future work aims at the existence result and stability of MMTE problem with elastic demand.

Acknowledgment

This work was supported by the Science Technology Foundation of Guizhou (2010J2131) and Natural Science Foundation of Department of Education of Guizhou Province (2010029).

REFERENCES

- [1] *J.G. Wardrop*, Some theoretical aspects of road traffic research, in Proceedings of the Institute of Civil Engineers, part II, **vol. 1**, 1952, pp 325-378
- [2] *M.J. Smith*, Existence, uniqueness and stability of traffic equilibrium, Transportation Research Part B, **vol. 13**, no.4, 1979, pp 295-304
- [3] *G.Y. Chen and N.D. Yen*, On the variational inequality model for network equilibrium, Internal Report 3. 196 (724), Department of Mathematics, University of Pisa, 1993.
- [4] *X.Q. Yang, C.J. Goh*, On vector variational inequalities: Application to vector equilibria, Journal of Optimization Theory and Applications, **vol. 95**, no. 2, 1997, pp 431-443
- [5] *P. Daniele, A. Maugeri, W. Oettli*, Time-dependent traffic equilibria, Journal of Optimization Theory and Applications, **vol. 103**, no. 3, 1999, pp 543-555
- [6] *P. Daniele, A. Maugeri, W. Oettli*, Variational inequalities and time-dependent traffic equilibria, Comptes Rendus de l'Académie des Sciences Series I Mathematics, **vol. 326**, no. 9, 1998, pp 1059-102
- [7] *Z. Lin*, The study of traffic equilibrium problems with capacity constraints of arcs, Nonlinear Analysis: Real World Applications, **vol. 11**, no. 4, 2010, pp 2280-2284
- [8] *A. Nagurney*, A multiclass, multicriteria traffic network equilibrium model, Mathematical and Computer Modelling, **vol. 32**, no. 3-4, 2000, pp 393-411
- [9] *A. Nagurney, J. Dong*, A multiclass, multicriteria traffic network equilibrium model with elastic demand, Transportation Research Part B: Methodological, **vol. 36**, no. 5, 2002, pp 445-469
- [10] *F. Raciti*, Equilibrium conditions and vector variational inequalities: a complex relation, Journal of Global Optimization, **vol. 40**, no. 1-3, 2008, pp 353-360
- [11] *G.Y. Chen, C.J. Goh, X.Q. Yang*, Vector network equilibrium problems and nonlinear scalarization methods, Mathematical Methods of Operations Research, **vol. 49**, no. 2, 1999, pp 239-253
- [12] *C.J. Goh, X.Q. Yang*, Theory and methodology of vector equilibrium problem and vector optimization, European Journal of Operational Research, **vol. 116**, no. 3, 1999, pp 615-628

- [13] *H. Yang, H.J. Huang*, The multiclass, multicriteria traffic network equilibrium and systems optimum problem, *Transportation Research Part B: Methodological*, **vol. 38**, no. 1, 2004, pp 1-15
- [14] *H.J. Huang, Z.C. Li*, A multiclass, multicriteria logit-based traffic equilibrium assignment model under ATIS, *European Journal of Operational Research*, **vol. 176**, no. 3, 2007, pp 1464-1477
- [15] *S.J. Li, K.L. Teo, X.Q. Yang*, A remark on a standard and linear vector network equilibrium problem with capacity constraints, *European Journal of Operational Research*, **vol. 184**, no. 1, 2008, pp 13-23
- [16] *Z. Lin*, The study of the system of generalized vector quasi-equilibrium problems, *Journal of Global Optimization*, **vol. 36**, no. 4, 2006, pp 627-635
- [17] *H. Yang, J. Yu*, Essential component of the set of weakly Pareto-Nash equilibrium points, *Applied Mathematics Letters*, **vol. 15**, no. 5, 2002, pp 553-560
- [18] *J. Yu, H. Yang*, The essential components of the set of equilibrium points for set-valued maps, *Journal of Mathematical Analysis and Applications*, **vol. 300**, no. 2, 2004, pp 334-342
- [19] *J. Yu, S.W. Xiang*, On essential components of the Nash equilibrium points, *Nonlinear Analysis: Theory, Methods and Application*, **vol.38**, no. 2, 1999, pp 259-264
- [20] *P. Deguire, K.K. Tan, X.Z. Yuan*, The study of maximal elements, fixed points for L_s -majorized mapping and their applications to minimax and variational inequalities in the product topological spaces, *Nonlinear Analysis: Theory, Methods and Application*, **vol.37**, no. 7, 1999, pp 933-951
- [21] *E. Klein, A.C. Thompson*, *Theory of correspondences*, John Willey and Sons, 1984.
- [22] *R. Engelking*, *General Topology*, Heldermann Verlag, Berlin, 1989.
- [23] *J. Yu, H. Yang, S. W. Xiang*, Unified approach to existence and stability of essential components, *Nonlinear Analysis: Theory, Methods and Application*, **vol. 63**, no. 5-7, 2005, pp 2415-2425