

## ON THE VIBRATION OF SYSTEMS WITH DEGRADING HYSTERETIC CHARACTERISTICS

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*Lucrarea analizează comportarea sistemelor cu caracteristici histeretice degradabile. Este studiat răspunsul unui sistem cu un singur grad de libertate la o excitație de tip armonic precum și cel la o excitație de tip seismic, considerând modificarea caracteristicilor structural dinamice determinată de efectele distructive generate de intrare. Comportarea dinamică a sistemelor histeretice cu degradare este analizată utilizând modelul Bouc-Wen. Studiul este axat pe punerea în evidență a modificărilor induse în comportamentul sistemului degradabil în funcție de modul în care frecvența proprie a structurii este situată relativ la componenta de frecvență dominantă a intrării: sub rezonanță sau peste rezonanță.*

*The paper is an analysis of structures with degrading hysteretic characteristics. The response of a SDOF degrading system to harmonic excitation and to a strong seismic input is studied by considering the modifications of the structural dynamic characteristics due to input-generated damages. A model based on Bouc-Wen representation of hysteretic behavior is employed to describe the dynamic behavior of degrading structures. The study is focused on the modifications of the degrading system behavior as a function of the mode in which the eigenfrequency of the structure is situated relative to the dominant frequency component of the input: below resonance or above resonance.*

**Keywords:** vibrations, stiffness degradation, hysteresis

### 1. Introduction

By stiffness degradation a system can be either “dragged” into the resonance regime or “pulled” away from it. The effect of structural degradation on the fatigue life of composite laminates was studied in [1] by spectral methods showing the modification of system resonance frequency as function of the number of testing cycles. Vibration amplification in oscillating systems with degrading characteristics is investigated in [2], [3].

The modeling of deteriorating hysteretic behavior is becoming increasingly important especially in the context of seismic analysis and design. Thus, hysteretic models for deteriorating inelastic structures are developed in [4].

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In [5], a nonlinear dynamic hysteresis model, that can take into account the strength degradation and the effect of the duration time of earthquake motion is evaluated through seismic response analysis using the proposed model is presented.

The dynamic behavior of degrading structures has been also studied by employing viscoelastic models [6]. The numerical results outlined the output amplification or attenuation effect of degradation. In this paper, similar aspects are addressed by employing a more realistic model of structural degradation, based on Bouc-Wen representation of the hysteretic behavior [7].

## 2. Analytical model

The model adopted involves one mass, the aim being to approximate the vibration of a building in the range of its lowest eigenmode (see Fig.1).

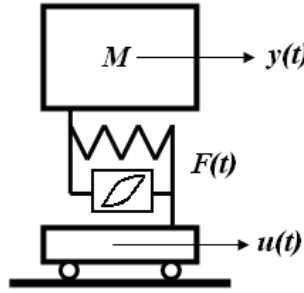


Fig. 1. Model of the mechanical system

Only the lateral motion is considered, the building being treated as a shear structure. The sprung mass  $M$ , is connected to the system base by a structural element generating a hysteretic force

$$F(t) = \alpha (F_y / x_y) x(t) + (1 - \alpha) F_y z(t),$$

$$\dot{z} = \left[ A - |z|^n (\beta + \gamma \text{sign}(\dot{x}z)) \right] \frac{\dot{x}}{x_y}, \quad (1)$$

where  $x(t) = y(t) - u(t)$  is the relative displacement between the top level and the building base,  $F_y$  the yielding force,  $x_y$  the yield displacement,  $\alpha$  the ratio of post-yield to pre-yield stiffness (rigidity ratio) and  $z(t)$  is the hysteretic parameter that obeys the Bouc-Wen differential equation [8], [9]. The dimensionless quantities  $A, \beta, \gamma$  and  $n$  are “loop parameters”, which control the shape and magnitude of the hysteresis loop  $z(x)$  for the imposed cyclic

displacement  $x(t)$ . Through appropriate choices of parameters  $\alpha$  and  $\beta$  in the model, it can represent the strain softening or strain hardening hysteretic behavior [10], [11]. For

$$\gamma \geq \beta \quad (2)$$

the hysteretic loop  $z(x)$  assumes a bulge shape as opposed to a slim one. The transition from elastic to post-elastic branch of the hysteresis loop is controlled by the exponential parameter  $n$ : smooth for small values and abrupt for large values. To reduce the hysteresis model to a formulation with well defined properties, the following constraint was suggested [11]:

$$\frac{A}{\beta + \gamma} = 1. \quad (3)$$

Moreover, it has been shown that fixing parameter  $A$  to unity is the best way to remove the mathematical redundancy of the model parameters [12]. These constraints and condition (2) are adopted herein by taking

$$A = 1, \quad n = 1, \quad \beta = 0.95, \quad \gamma = 0.05. \quad (4)$$

Degradation of the restoring force gradually increases as the structure experiences repeated stress reversals. The degradation mechanism can be modeled by allowing the parameters of the hysteretic restoring force to vary as a function of the response duration and severity [13], [7]. A convenient measure of the combined effect of duration and severity is the total energy dissipated through hysteresis over the time interval  $[0, t]$ .

We considered that only parameter  $\alpha$  is allowed to vary in order to describe both the stiffness degradation and damping capacity appreciation. The following simple linear functional relationship was considered to describe the time variation of  $\alpha$ , starting from its initial value  $\alpha_0$ , taken for the undamaged structure

$$\alpha(t) = \alpha_0 \left[ 1 - \frac{\varepsilon}{F_y x_y} \int_0^t F(s) \dot{x}(s) ds \right], \quad (5)$$

where  $\varepsilon$  is a non-negative parameter, measuring the structural degradation intensity. By denoting  $\omega_0^2 = F_y / M x_y$  and introducing the dimensionless parameters

$$\begin{aligned} \tau &= \omega_0 t, \quad \xi(\tau) = x(\tau/\omega_0) / x_y, \quad \xi'(\tau) = \dot{x}(\tau/\omega_0) / (\omega_0 x_y), \\ \xi''(\tau) &= \ddot{x}(\tau/\omega_0) / (\omega_0^2 x_y), \quad \eta''(\tau) = \ddot{u}(\tau/\omega_0) / (\omega_0^2 x_y), \\ \Phi(\tau) &= F(\tau/\omega_0) / F_y, \quad \upsilon = \omega / \omega_0, \end{aligned} \quad (6)$$

the dimensionless form of the equation of motion system and of the degradation relationship (5) can be written as

$$\begin{cases} \xi'' + \alpha\xi + (1-\alpha)\Phi = -\eta''(\tau) \\ \alpha(\tau) = \alpha_0 \left[ 1 - \varepsilon \int_0^\tau \Phi(\sigma) \xi'(\sigma) d\sigma \right] \end{cases} \quad (7)$$

For the sake of simplicity, the dimensionless quantities are given the same names as the corresponding physical ones:  $\tau$  - time,  $\xi(\tau)$  - relative displacement (building base-top drift),  $\nu$  - frequency,  $\Phi(\tau)$  - hysteretic restoring force,  $\eta''(\tau)$  - ground motion acceleration, etc. Equation (7) represents an integral-differential model, describing the dynamic behavior of a degrading oscillating system perturbed by the base acceleration  $\eta''(\tau)$ . In order to solve this problem one must assess the value of the degradation parameter  $\varepsilon$ . By assuming that the initial rigidity ratio  $\alpha_0 = 0.9$  decreases 2 times after completion of 25 loading cycles with an imposed harmonic relative displacement

$$\xi(\tau) = \xi_0 \cos \nu \tau, \quad \xi_0 = 4, \quad \nu = 1, \quad (8)$$

the value  $\varepsilon = 0.008$  is obtained.

Fig. 2 shows the evolution of the degrading hysteresis loops with the progress of cyclic loading.

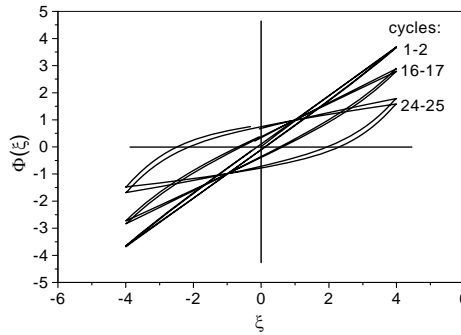


Fig. 2. Evolution of the degrading hysteresis loops with the number of loading cycles.

As one can see from these plots, the stiffness degradation is associated with an important increase of the dissipated energy per cycle. Therefore, the relative damping of the system becomes even considerably larger. The mathematical relation assuring the integrity of the structural element, which develops the hysteretic restoring force, is  $\alpha(\tau) > 0$  over the whole time interval on which the excitation is applied.

### 3. Harmonic input

The response of the degrading system to a harmonic excitation is described by the integral-differential system defined by (5)-(8), in which the base acceleration is:

$$\eta''(\tau) = \eta_0'' \sin \upsilon \tau. \quad (9)$$

The system output amplification or attenuation due to structural degradation can be outlined by taking two different values for the perturbation frequency  $\upsilon$ , slightly higher and respectively, lower, than the initial undamped natural frequency of the system  $\upsilon_0 = \sqrt{\alpha_0}$ .

Fig.3 and Fig.4 show comparatively the time histories of the relative displacement  $\xi(\tau)$  and of the rigidity ratio  $\alpha(\tau)$ , obtained for two close values of the perturbation frequency:  $\upsilon_1 = 1.05$ ,  $\upsilon_2 = 0.75$ .

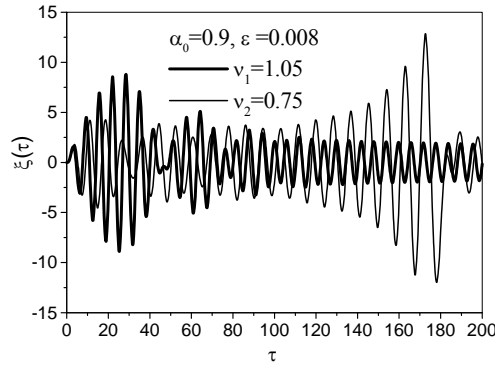


Fig. 3. Time history of the relative displacement

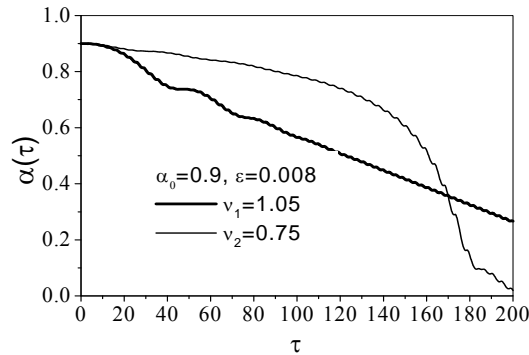


Fig. 4. Variation of the rigidity ratio with time

As shown in Fig.3, when the excitation frequency is higher than the initial value of the system eigenfrequency, by stiffness degradation the system is rapidly “pulled” away from resonance, resulting in and a very abrupt attenuation of the vibration level. Therefore, the structural strength is not affected very much. On the contrary, when the perturbation frequency is lower than the initial value of system eigenfrequency, the system is “dragged” gradually toward the resonance regime, resulting in an important output amplification. As the maximum output amplification is encountered after the system experienced already many stress reversals with increasing intensity, its structural integrity could be seriously affected. These assertions are also sustained by the plots in Fig. 6, showing the difference between the stiffness degradation patterns in the two considered case studies. Thus, in the second case, the system strength is almost entirely consumed by degradation, i.e.  $\alpha(\tau_{\max}) \cong 0$ .

#### 4. Seismic input

A real ground motion acceleration was chosen as input [3]. In Fig.5 the dimensionless input time history is shown.

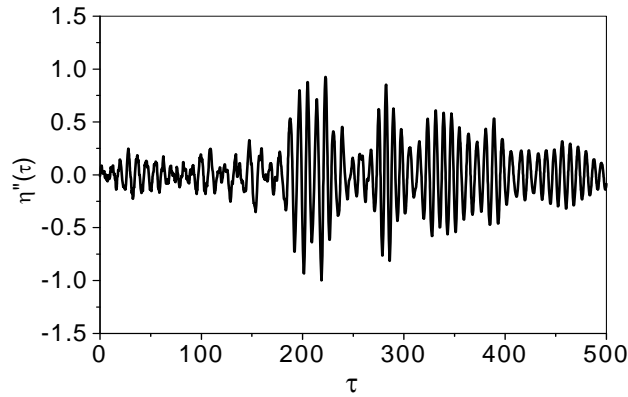


Fig. 5. Time history of the ground motion acceleration

The time scale of the recorded ground motion acceleration was slightly modified in order to analyze the building behavior when the initial value of natural frequency of the undamaged system  $\nu_0$  is below and above the dominant frequency component of the input amplitude spectrum, as shown in Fig.6.

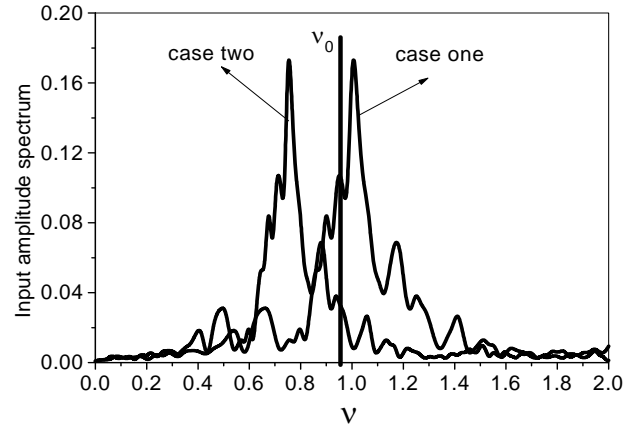


Fig. 6. Amplitude spectra of the ground motion acceleration for two case studies

The modifications of the system dynamic behavior due to structural damage are illustrated in Fig.7 and 8, showing the time history of the total inter storey drift  $\xi(\tau)$ . In both case studies, the dynamic behavior of the degrading system is compared to that of a non-degrading one, with the same initial rigidity ratio and seismic input.

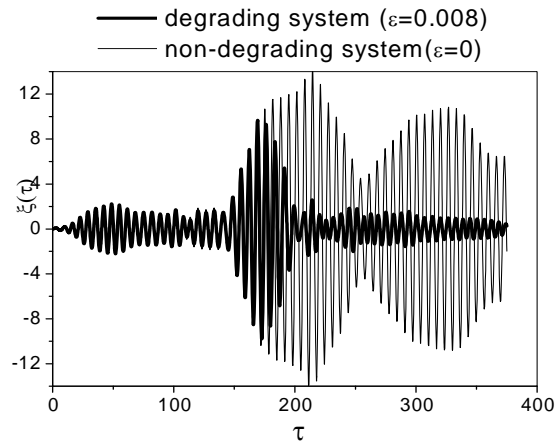


Fig. 7. Attenuation of the system seismic output due to structural degradation

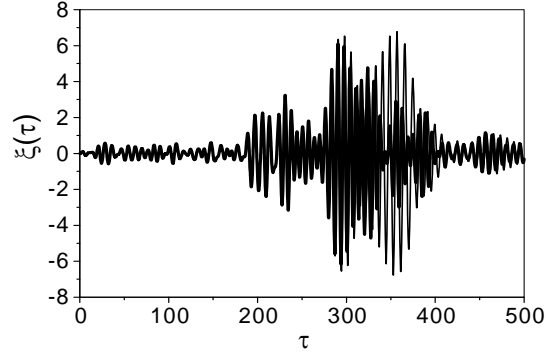


Fig. 8. Amplification of the system seismic output due to structural degradation

The first case corresponds to the situation when the natural frequency of the undamaged building is slightly lower than the frequency of the dominant spectral component of the seismic motion. In this case, the structural degradation “pulls” the system away from the vibration resonance regime, with a beneficial effect on the building seismic response level. In the second case study, the natural frequency of the undamaged system is higher than the frequency of the dominant spectral component of ground motion. In this case, by structural stiffness degradation, the building is “dragged” into the resonance range with an important amplification of stresses (especially at the end of the seismic motion) that can lead to severe structural damage or even to building collapse. The evolution of the degrading hysteretic force throughout the duration of the seismic action is presented in Fig. 9 and Fig.10 for the two considered case studies. It is worth mentioning that the modification of the building dynamic behavior during strong earthquakes is mostly influenced by the structural stiffness degradation and only to a less extent by the associated increase of the internal damping.

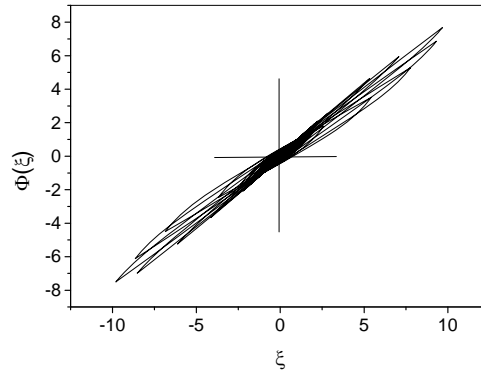


Fig. 9. Evolution of the degrading hysteretic force in the first case study

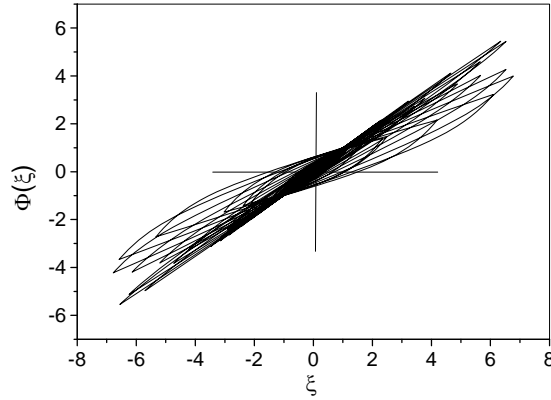


Fig. 10. Evolution of the degrading hysteretic force in the second case study

## 5. Conclusions

The results reported in this paper are based on simple models and possible scenarios describing the dynamic behavior of buildings with degrading hysteretic characteristics, during strong earthquakes. These results lead to several important qualitative conclusions regarding the effects of structural degradation on building dynamic response to strong earthquakes:

1. By overall stiffness degradation, a building can enter or exit the resonance range of the seismically induced vibration, depending on the relative position between its initial eigenfrequency and the frequencies of the dominant spectral components of seismic motion. It should be mentioned that stiffness degradation can occur only for sufficiently large building deflections, which are unlikely to develop unless the building motion is excited within or very close to the resonance range. Otherwise, the building mechanical filtering properties do not allow a significant amplification of the base motion or even they can lead to attenuation. Therefore, in this case the building protects itself against the seismic action.

2. The increase of the building self-damping capacity by internal energy dissipation due to structural degradation is not so important as to significantly reduce the building seismic response. The increase of the building damping capacity should be achieved by special devices that are capable to control and limit the structural deflections, rather than by accepting the local failure of the structural elements (beams or base columns).

3. By stiffness degradation, a building has always the tendency to vibrate above the resonance range, since a pure resonance could be possible only for

extremely short time intervals. As shown in [6], for this vibration regime, the maximum values of shear forces (stresses) occur in columns at a certain height from the foundation, generating in many cases column failure at about one third of their height from the foundation level. This damaging effect is mainly due to the fact that in the above discussed vibration pattern the building base and top level absolute displacements are in opposite phase.

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