

## KINEMATICS OF THE 3-RRR PLANAR PARALLEL ROBOT

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*Lucrarea stabileşte relaţii matriceale recurente pentru cinematica robotului plan paralel cunoscut 3-RRR cu acţionare revolută. Cele trei picioare identice ale robotului, care sunt legate la platforma mobilă, sunt localizate în acelaşi plan. Cunoscând mişcarea platformei, problema de cinematică inversă oferă expresii şi grafice pentru unghiurile de rotaţie, vitezele unghiulare şi accelerările unghiulare ale celor trei sisteme active.*

*Recursive matrix relations for kinematics of the commonly known 3-RRR planar parallel robot with revolute actuators are established in this paper. The three identical legs of the robot, connecting to the moving platform, are located in the same plane. Knowing the motion of the platform, the inverse kinematical problem offers expressions and graphs for the rotation angles, the angular velocities and the angular accelerations of the three actuators.*

**Key-words:** kinematics, parallel manipulator, platform

### List of symbols

- $a_{k,k-1}$  : orthogonal transformation matrix
- $R$  : rotation matrix of the moving platform
- $\vec{u}_1, \vec{u}_2, \vec{u}_3$  : three orthogonal unit vectors
- $\varphi_{k,k-1}$  : relative rotation angle of  $T_k$  rigid body
- $\vec{\omega}_{k,k-1}$  : relative angular velocity of  $T_k$
- $\vec{\omega}_{k0}$  : absolute angular velocity of  $T_k$
- $\tilde{\omega}_{k,k-1}$  : skew-symmetric matrix associated to the angular velocity  $\vec{\omega}_{k,k-1}$
- $\vec{\varepsilon}_{k,k-1}$  : relative angular acceleration of  $T_k$
- $\vec{\varepsilon}_{k0}$  : absolute angular acceleration of  $T_k$
- $\tilde{\varepsilon}_{k,k-1}$  : skew-symmetric matrix associated to the angular acceleration  $\vec{\varepsilon}_{k,k-1}$
- $\vec{r}_{k,k-1}^A$  : relative position vector of the centre  $A_k$  of joint
- $\vec{v}_{k,k-1}^A$  : relative velocity of the centre  $A_k$
- $\vec{\gamma}_{k,k-1}^A$  : relative acceleration of the centre  $A_k$

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## 1. Introduction

Parallel manipulators are closed-loop mechanisms equipped with revolute or prismatic actuators that consist of separate serial chains connecting the fixed base to the moving platform. They have a robust construction and can move bodies of large dimensions with high velocities and accelerations. This is the reason why the devices, which produce translations or spherical motion to a platform, technologically are based on the concept of parallel manipulators [1].

Compared with serial manipulators, the followings are the potential advantages of parallel architectures: higher kinematical precision, lighter weight and better stiffness, greater load bearing, stable capacity and suitable position of arrangement of actuators. But, from application point of view, a limited workspace and complicated singularities are two major drawbacks of parallel mechanisms.

Over the past decades, parallel manipulators have received more and more attention from researchers and industries. Accuracy and precision in the direction of the tasks are essential since the robot is intended to operate on fragile objects, where positioning errors of the tool could end in costly damages.

Considerable efforts have been devoted to the kinematics and dynamics analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2], Merlet [3], Parenti Castelli and Di Gregorio [4]). The prototype of Delta parallel robot (Clavel [5], Tsai and Stamper [6], Staicu and Carp-Ciocardia [7]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [8]) are equipped with three motors, which train on the mobile platform in a three-degrees of freedom general translation motion. Angeles, Gosselin, Gagné and Wang [9], [10], [11] analysed the kinematics, dynamics and singularities loci of Agile Wrist spherical robot with three actuators.

A mechanism is said to be a *planar robot* if all the moving links of the mechanism perform planar motions that are situated in parallel planes. For a planar mechanism, the loci of all points in all links can be drawn conveniently on a plane. In a planar linkage, the axes of all revolute joints must be normal to the plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion.

Aradyfio and Qiao [12] examined the inverse kinematic solution for the three different 3-DOF planar parallel robots. Gosselin and Angeles [13] and Pennock and Kassner [14] each presents a kinematical study of a planar parallel robot where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints. Sefrioui and Gosselin [15] give an interesting numerical solution in the inverse and direct

kinematics of this kind of planar robot. Recently, more general approaches have been presented. Daniali *et al.* [16] present a study of velocity relationships and singular conditions for general planar parallel robots. Merlet [17] solved the forward pose kinematics problem for a broad class of planar parallel manipulators. Williams *et al.* [18] analysed the dynamics and the control of a planar three-degrees-of-freedom parallel manipulator at Ohio University while Yang *et al.* [19] concentrate on the singularity analysis of a class of 3-RRR planar parallel robots developed in its laboratory. Bonev, Zlatanov and Gosselin [20] describe several types of singular configurations by studying the direct kinematic model of a 3-RRR planar parallel robot with actuated base joints.

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for kinematics of the 3-RRR planar parallel robots.

## 2. Kinematics analysis

Having a closed-loop structure, the planar parallel robot 3-RRR is a special symmetrical mechanism composed of three planar kinematical chains with identical topology, all connecting the fixed base to the moving platform (Fig. 1).

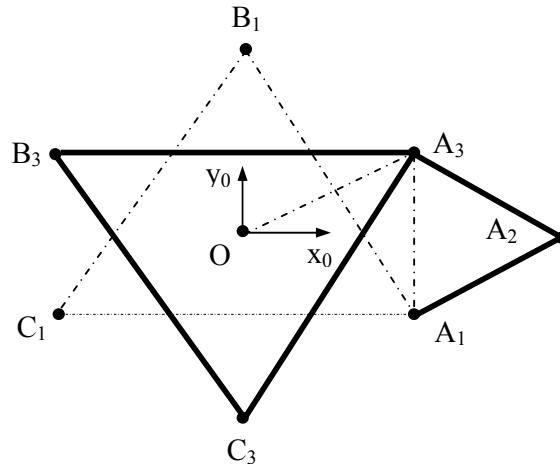


Fig. 1 The 3-RRR planar parallel robot

The centres  $A_1, B_1, C_1$  of three fixed pivots define the geometry of a fixed base and the three moving revolute joints  $A_3, B_3, C_3$  define the geometry of the moving platform. Each leg consists of two binary links with three parallel revolute joints. Together, the mechanism consists of seven moving links and nine revolute

joints. Grübler mobility equation predicts certainly three degrees of freedom of the robot.

In the present kind of the robot (RRR) we consider the moving platform as the output link and the links  $A_1A_2$ ,  $B_1B_2$ ,  $C_1C_2$  as the input links. Thus, all actuators can be installed on the fixed base.

For the purpose of analysis, we attach a Cartesian frame  $x_0y_0z_0(T_0)$  to the fixed base with its origin located at triangle centre  $O$ , the  $z_0$  axis perpendicular to the base and the  $x_0$  axis pointing along the direction  $C_1A_1$ . Another mobile reference frame  $x_Gy_Gz_G$  is attached to the moving platform. The origin of this coordinate central system is located just at the centre  $G$  of the moving triangle (Fig. 2).

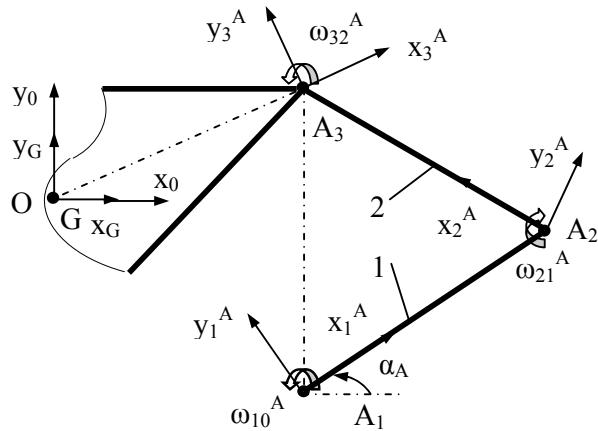


Fig. 2 Kinematical scheme of first leg  $A$  of the mechanism

To simplify the graphical image of the kinematical scheme of the mechanism, in the followings we will represent the intermediate reference systems by only two axes, so as one proceeds in most of robotics papers [1], [3], [9]. It is noted that the relative rotation of  $T_k$  body with  $\varphi_{k,k-1}$  angle must be always pointing about the direction of  $z_k$  axis.

In what follows we consider that the moving platform is initially located at a *central configuration*, where the platform is not rotated with respect to the fixed base while the mass centre  $G$  is at the origin  $O$  of the fixed frame.

One of three active legs (for example leg  $A$ ) consists of a fixed revolute joint, a moving crank **1** of length  $l_1$ , which has a rotation about  $z_1^A$  axis with the angle  $\varphi_{10}^A$ , the angular velocity  $\omega_{10}^A = \dot{\varphi}_{10}^A$  and the angular acceleration  $\varepsilon_{10}^A = \ddot{\varphi}_{10}^A$ . A new element of the leg is a rigid rod **2** of length  $l_2$ , linked at the  $x_2^A y_2^A z_2^A$  frame, having a relative rotation with the angle  $\varphi_{21}^A$ , velocity  $\omega_{21}^A = \dot{\varphi}_{21}^A$  and acceleration

$\varepsilon_{21}^A = \ddot{\phi}_{21}^A$ . Finally, a revolute joint is introduced at the moving platform, which is schematised as an equilateral triangle congruent to the base having the edge  $l = r\sqrt{3}$ .

At the central configuration, we also consider that all legs are symmetrically extended and that the angles of orientation of fixed pivots are given by

$$\alpha_A = \frac{\pi}{6}, \alpha_B = \frac{5\pi}{6}, \alpha_C = -\frac{\pi}{2}. \quad (1)$$

Pursuing the first leg  $A$  in the  $OA_1A_2A_3$  way, for example, we obtain the following matrices of transformation [21]:

$$a_{10} = a_{10}^\varphi a_\alpha^A, \quad a_{21} = a_{21}^\varphi \theta, \quad a_{32} = a_{32}^\varphi \theta, \quad (2)$$

where

$$a_\alpha^A = \begin{bmatrix} \cos \alpha_A & \sin \alpha_A & 0 \\ -\sin \alpha_A & \cos \alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \theta = \frac{1}{2} \begin{bmatrix} -1 & \sqrt{3} & 0 \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$a_{k,k-1}^\varphi = \begin{bmatrix} \cos \varphi_{k,k-1}^A & \sin \varphi_{k,k-1}^A & 0 \\ -\sin \varphi_{k,k-1}^A & \cos \varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{k0} = \prod_{j=1}^k a_{k-j+1, k-j}, \quad (k=1, 2, 3). \quad (3)$$

Analogous relations can be written for the other two legs of the mechanism. Three rotations angles  $\varphi_{10}^A, \varphi_{10}^B, \varphi_{10}^C$  of the active links are the joint variables that give the input vector  $\vec{\varphi}_{10} = [\varphi_{10}^A \ \varphi_{10}^B \ \varphi_{10}^C]^T$  of the instantaneous position of the mechanism in the present study configuration. But, in the inverse geometric problem, it can be considered that the position of the mechanism is completely given through the coordinates  $x_0^G, y_0^G$  of the mass centre  $G$  of the moving platform and the orientation angle  $\phi$  of the movable frame  $x_Gy_Gz_G$ . The orthogonal rotation matrix of the moving platform from  $x_0y_0z_0$  to  $x_Gy_Gz_G$  reference system is

$$R = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

Further, we suppose that the position vector of the centre  $G$ , respectively  $\vec{r}_0^G = [x_0^G \ y_0^G \ 0]^T$  and the orientation angle  $\phi$ , which are expressed by the following analytical functions

$$\begin{aligned}
x_0^G &= x_0^{G*} (1 - \cos \frac{\pi}{3} t) \\
y_0^G &= y_0^{G*} (1 - \cos \frac{\pi}{3} t) \\
\phi &= \phi^* (1 - \cos \frac{\pi}{3} t)
\end{aligned} \tag{5}$$

can describe the general absolute motion of the moving platform. From the rotation conditions of the moving platform

$$a_{30}^{\circ T} a_{30} = b_{30}^{\circ T} b_{30} = c_{30}^{\circ T} c_{30} = R, \tag{6}$$

taking, for example,

$$a_{30}^{\circ} = a_{\alpha}^A, b_{30}^{\circ} = a_{\alpha}^B, c_{30}^{\circ} = a_{\alpha}^C, \tag{7}$$

we obtain the following relations between angles:

$$\varphi_{10}^A - \varphi_{21}^A + \varphi_{32}^A = \varphi_{10}^B - \varphi_{21}^B + \varphi_{32}^B = \varphi_{10}^C - \varphi_{21}^C + \varphi_{32}^C = \phi. \tag{8}$$

The six variables  $\varphi_{10}^A, \varphi_{21}^A, \varphi_{10}^B, \varphi_{21}^B, \varphi_{10}^C, \varphi_{21}^C$  will be determined by several vector-loop equations, as follows

$$\begin{aligned}
\vec{r}_{10}^A + \sum_{k=1}^2 a_{k0}^T \vec{r}_{k+1,k}^A + a_{30}^T \vec{r}_3^{GA} &= \\
= \vec{r}_{10}^B + \sum_{k=1}^2 b_{k0}^T \vec{r}_{k+1,k}^B + b_{30}^T \vec{r}_3^{GB} &= \\
= \vec{r}_{10}^C + \sum_{k=1}^2 c_{k0}^T \vec{r}_{k+1,k}^C + c_{30}^T \vec{r}_3^{GC} &= \vec{r}_0^G,
\end{aligned} \tag{9}$$

where one denoted

$$\begin{aligned}
\vec{u}_1 &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, & \vec{u}_2 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, & \vec{u}_3 &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & \tilde{u}_3 &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\vec{r}_{10}^A &= 0.5r[\sqrt{3} & -1 & 0]^T \\
\vec{r}_{10}^B &= r[0 & 1 & 0]^T \\
\vec{r}_{10}^C &= 0.5r[-\sqrt{3} & -1 & 0]^T \\
\vec{r}_{21}^i &= r\vec{u}_1, \quad \vec{r}_{32}^i = r\vec{u}_1, \quad \vec{r}_3^{Gi} = -r\vec{u}_1 \quad (i = A, B, C).
\end{aligned} \tag{10}$$

Actually, these vector equations means that there is only one inverse geometrical solution for the manipulator, namely:

$$r \sin(\varphi_{10}^i + \alpha_i) - r \sin(\varphi_{10}^i + \alpha_i - \varphi_{21}^i - \pi/3) = y_0^G - y_{10}^i + r \sin(\phi + \alpha_i)$$

$$r \cos(\varphi_{10}^i + \alpha_i) - r \cos(\varphi_{10}^i + \alpha_i - \varphi_{21}^i - \pi/3) = x_0^G - x_{10}^i + r \cos(\phi + \alpha_i) \quad (11)$$

$$(i = A, B, C)$$

We develop the inverse kinematics problem and determine the velocities and accelerations of the manipulator, supposing that the planar motion of the moving platform is known. Firstly, we compute the linear and angular velocities of each leg in terms of the angular velocity  $\omega_0^G = \dot{\phi} \vec{u}_3$  and the centre's velocity  $\vec{v}_0^G = \dot{\vec{r}}_0^G$  of the moving platform.

The motions of the component elements of each leg (for example the leg *A*) is characterized by the following skew symmetrical matrices:

$$\tilde{\omega}_{k0}^A = a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T + \omega_{k,k-1}^A \tilde{u}_3, \quad \omega_{k,k-1}^A = \dot{\phi}_{k,k-1}^A \quad (12)$$

which are *associated* to the absolute angular velocities given by the following recursive relations

$$\bar{\omega}_{k0}^A = a_{k,k-1} \bar{\omega}_{k-1,0}^A + \omega_{k,k-1}^A \vec{u}_3 \quad (k = 1, 2, 3). \quad (13)$$

The following relations give the velocities  $\vec{v}_{k0}^A$  of the joints *A*<sub>*k*</sub>

$$\vec{v}_{k0}^A = a_{k,k-1} \vec{v}_{k-1,0}^A + a_{k,k-1} \tilde{\omega}_{k-1,0}^A \vec{r}_{k,k-1}^A. \quad (14)$$

The equations of geometrical constraints (8) and (9) can be derived with respect to the time to obtain the following *matrix conditions of connectivity* [22]

$$\begin{aligned} & \omega_{10}^A \vec{u}_i^T a_{10}^T \tilde{u}_3 \{ \vec{r}_{21}^A + a_{21}^T \vec{r}_{32}^A + a_{21}^T a_{32}^T \vec{r}_3^{GA} \} + \omega_{21}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 \{ \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} + \\ & + \omega_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \vec{r}_3^{GA} = \vec{u}_i^T \dot{\vec{r}}_0^G, \quad (i = 1, 2) \\ & \omega_{10}^A - \omega_{21}^A + \omega_{32}^A = \dot{\phi} \end{aligned} \quad (15)$$

where  $\tilde{u}_3$  is a skew-symmetrical matrix associated to the unit vector  $\vec{u}_3$  pointing in the positive sense of the axis  $z_k$ . From these equations, we obtain the relative velocities  $\omega_{10}^A, \omega_{21}^A, \omega_{32}^A$  as functions of the angular velocity of the platform and of the velocity of the mass centre *G*. But, the conditions (15) give the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and for the particular configurations of the singularities where the manipulator becomes uncontrollable.

Rearranging, the above six constraint equations (11) of the planar robot can immediately be written as follows.

$$\begin{aligned} & [x_0^G - x_{10}^i + r \cos(\phi + \alpha_i) - r \cos(\varphi_{10}^i + \alpha_i)]^2 + \\ & + [y_0^G - y_{10}^i + r \sin(\phi + \alpha_i) - r \sin(\varphi_{10}^i + \alpha_i)]^2 = r^2 \quad (16) \\ & (i = A, B, C), \end{aligned}$$

where the “zero” position  $x_0^{0G} = 0$ ,  $y_0^{0G} = 0$ ,  $\phi^0 = 0$  corresponds to the joints variables  $\vec{\phi}_{10}^0 = [0 \ 0 \ 0]^T$ . The derivative with respect to the time of the conditions (16) leads to the matrix equation

$$J_{1f} \dot{\vec{\phi}}_{10} = J_{2f} [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{\phi}]^T \quad (17)$$

for the planar robot with *fixed revolute actuators*.

Matrices  $J_{1f}$  and  $J_{2f}$  are, respectively, the inverse and forward Jacobian of the manipulator and can be expressed as:

$$J_{1f} = \text{diag} \{ \delta_{Af} \quad \delta_{Bf} \quad \delta_{Cf} \}, \quad J_{2f} = \begin{bmatrix} \beta_{1f}^A & \beta_{2f}^A & \beta_{3f}^A \\ \beta_{1f}^B & \beta_{2f}^B & \beta_{3f}^B \\ \beta_{1f}^C & \beta_{2f}^C & \beta_{3f}^C \end{bmatrix}, \quad (18)$$

with

$$\begin{aligned} \delta_{if} &= -(x_0^G - x_{10}^i) \sin(\phi_{10}^i + \alpha_i) + (y_0^G - y_{10}^i) \cos(\phi_{10}^i + \alpha_i) + r \sin(\phi - \phi_{10}^i) \\ &\quad (i = A, B, C) \\ \beta_{1f}^i &= \frac{1}{r} [x_0^G - x_{10}^i - r \cos(\phi_{10}^i + \alpha_i) + r \cos(\phi + \alpha_i)] \\ \beta_{2f}^i &= \frac{1}{r} [y_0^G - y_{10}^i - r \sin(\phi_{10}^i + \alpha_i) + r \sin(\phi + \alpha_i)] \\ \beta_{3f}^i &= -(x_0^G - x_{10}^i) \sin(\phi + \alpha_i) + (y_0^G - y_{10}^i) \cos(\phi + \alpha_i) + r \sin(\phi - \phi_{10}^i) \end{aligned} \quad (19)$$

The three kinds of singularities of the three closed-loop kinematical chains can be determined by the analysis of two Jacobian matrices  $J_{1f}$  and  $J_{2f}$  [23], [24].

As for the relative accelerations  $\varepsilon_{10}^A$ ,  $\varepsilon_{21}^A$ ,  $\varepsilon_{32}^A$  of the robot, the derivatives with respect to the time of the equations (15) give other following conditions of connectivity [25], [26]

$$\begin{aligned} &\varepsilon_{10}^A \vec{u}_i^T a_{10}^T \tilde{u}_3 \{ \vec{r}_{21}^A + a_{21}^T \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} + \varepsilon_{21}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 \{ \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} + \\ &+ \varepsilon_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \hat{r}_3^{GA} = \vec{u}_i^T \vec{r}_0^{GA} - \omega_{10}^A \omega_{10}^A \vec{u}_i^T a_{10}^T \tilde{u}_3 \{ \vec{r}_{21}^A + a_{21}^T \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} - \\ &- \omega_{21}^A \omega_{21}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 \{ \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} - \omega_{32}^A \omega_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \hat{r}_3^{GA} - \\ &- 2\omega_{10}^A \omega_{21}^A \vec{u}_i^T a_{10}^T \tilde{u}_3 a_{21}^T \tilde{u}_3 \{ \vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA} \} - \\ &- 2\omega_{10}^A \omega_{32}^A \vec{u}_i^T a_{10}^T \tilde{u}_3 a_{21}^T a_{32}^T \tilde{u}_3 \hat{r}_3^{GA} - \\ &- 2\omega_{21}^A \omega_{32}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 a_{32}^T \tilde{u}_3 \hat{r}_3^{GA}, \quad (i=1,2) \\ &\varepsilon_{10}^A - \varepsilon_{21}^A + \varepsilon_{32}^A = \ddot{\phi}. \end{aligned} \quad (20)$$

The relationships (15) and (20) represent the *inverse kinematics model* of the planar parallel robot.

The following recursive relations give the angular accelerations  $\vec{\varepsilon}_{k0}^A$  and the accelerations  $\vec{\gamma}_{k0}^A$  of the joints  $A_k$

$$\begin{aligned}\vec{\varepsilon}_{k0}^A &= a_{k,k-1} \vec{\varepsilon}_{k-1,0}^A + \varepsilon_{k,k-1}^A \vec{u}_3 + \omega_{k,k-1}^A a_{k,k-1} \vec{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3 \\ \vec{\omega}_{k0}^A \vec{\omega}_{k0}^A + \vec{\varepsilon}_{k0}^A &= a_{k,k-1} \left( \vec{\omega}_{k-1,0}^A \vec{\omega}_{k-1,0}^A + \vec{\varepsilon}_{k-1,0}^A \right) a_{k,k-1}^T + \\ &+ \omega_{k,k-1}^A \omega_{k,k-1}^A \vec{u}_3 \vec{u}_3 + \varepsilon_{k,k-1}^A \vec{u}_3 + \\ &+ 2\omega_{k,k-1}^A a_{k,k-1} \vec{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3 \\ \vec{\gamma}_{k0}^A &= a_{k,k-1} \vec{\gamma}_{k-1,0}^A + a_{k,k-1} \{ \vec{\omega}_{k-1,0}^A \vec{\omega}_{k-1,0}^A + \vec{\varepsilon}_{k-1,0}^A \} \vec{r}_{k,k-1}^A \quad (k = 1, 2, 3).\end{aligned}\tag{21}$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.

As application let us consider a 3-RRR planar robot, which has the following characteristics:

$$\begin{aligned}x_0^{G*} &= -0.025 \text{ m}, y_0^{G*} = 0.025 \text{ m}, \phi^* = \frac{\pi}{12} \\ r &= 0.3 \text{ m}, l = r\sqrt{3}, l_1 = l_2 = 0.3 \text{ m}, \Delta t = 3 \text{ s}.\end{aligned}\tag{22}$$

Using the MATLAB software, a computer program was developed to solve the inverse kinematics of the robot. Finally, the angles of rotation (Fig. 3), the angular velocities (Fig. 4) and the angular accelerations (Fig. 5) of the three revolute actuators were plotted versus time, using this program.

### 3. Conclusions

Within the inverse kinematical analysis, some exact relations that give the time-history evolution of the angles of rotation, angular velocities and angular accelerations of each element of the parallel robot have been established in the present paper.

The simulation by the presented program certifies that one of the major advantages of the current matrix recursive formulation is a reduced number of additions or multiplications and consequently a smaller processing time of numerical computation. Also, the proposed method can be applied to various types of complex robots when the number of the components of the mechanism is increased.

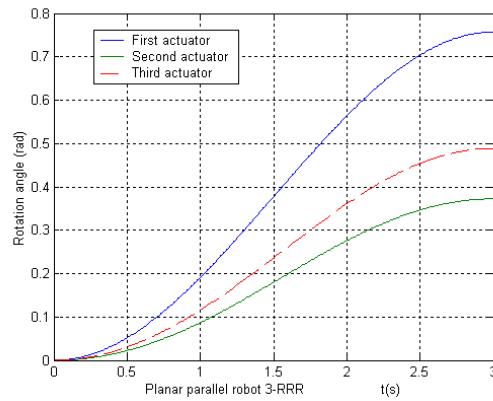


Fig. 3. Rotation angles of the three actuators

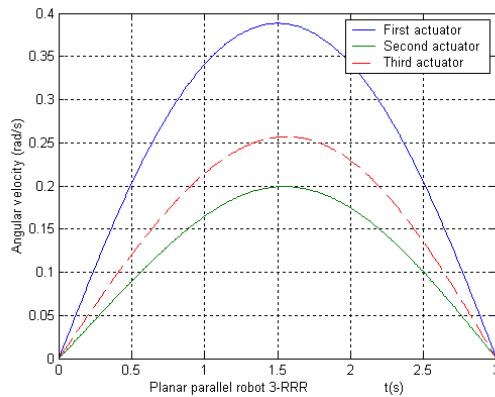


Fig. 4. Angular velocities of the three actuators

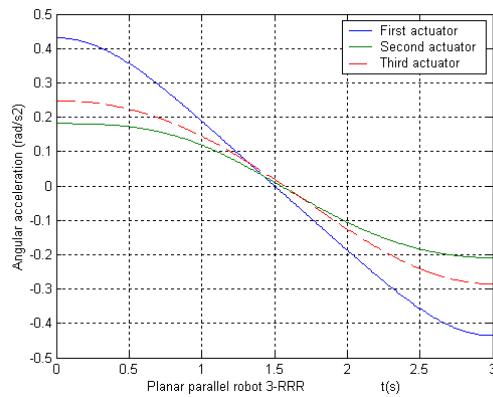


Fig. 5. Angular accelerations of the three actuators

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