

NUMERICAL VALIDATION OF A CONSTITUTIVE MODEL FOR UHMWPE-BASED COMPOSITES AT HIGH STRAIN RATES

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In order to design and build ballistic protection structures containing composite materials, a multitude of experiments performed in dedicated ranges are required. In the frame of limiting the costs demanded by the final result, numerical simulation is used as a tool to estimate the outcome of a certain experiment without performing it first. Using the laboratory results from a more accessible type of mechanical test such as Hopkinson bars compression, and a simple material model implemented within the LS-Dyna program, we have iteratively obtained results that accurately approximate the dynamic behavior of the Ultra High Molecular Weight Polyethylene composite material.

Keywords: Hopkinson bars, UHMWPE, constitutive model, numerical simulation

1. General considerations on the method

The interest in the materials response at dynamical loading has led over time to the development of several types of standardized tests capable of inducing a wide range of strain rates in the samples [1]. Among the high strain rates dedicated tests, the Split Hopkinson Bars (SHPB) and the Taylor gun tests are the most accessible in laboratory conditions. Although the Taylor gun test allows for higher strain rates, SHPB has proven to be extremely versatile, both in the wide range of materials that can be tested (metal alloys, ceramic materials, composites) and in the loading variations (compression, tension, shearing, or twisting), as well as in the special shapes that can be used for the samples [2].

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In order to avoid damage to the gauges, the measurements within such tests are performed in remote locations from the samples. It allows obtaining the overall response of the samples rather than the local stress, strain and strain rate values. Since in multilayered composite materials or specimens the stress and strain conditions are uneven within its volume, the overall performance does not uniquely determine the material characteristics on the ply level. In order to get a better understanding of the phenomena occurring in samples undergoing uneven deformations, and to allow for a correct determination of the material characteristics, it is possible to resort to numerical models based on Finite Element Method (FEM) [3] in which the degree of detail can descend to modeling each layer of the material [4]. Thus, the calibration of the material coefficients used in these numerical models is based on obtaining the most accurate reproduction of the experimental measurements [5].

2. Purpose of the article. Materials and models considered

The materials of interest for the present paper are the Ultra High Molecular Weight Polyethylene (UHMWPE) composite materials, represented by the Tensylon® brand, a type of oriented UHMWPE tape. These types of materials are already used worldwide in the defense industry for the production of multilayered protection products and even as a single constituent for the manufacture of various types of plates or panels. Since cross-plied, UHMWPE-based products are not as performant in the „soft” (unpressed) variant [6], they are most often used in the form of blocks or composite panels obtained by high temperature pressing [7]. At the base of obtaining hard UHMWPE panels stands the pre-preg, which is a two (in the case of Tensylon® brand) or several plies thin tape, lightly stuck to each other in order to facilitate handling (Fig. 1). This form of use is the one studied in this paper.

The targeted application is the protection (on different levels [8]) against ballistic threats – such as bullets, fragments and even shock absorption in the event of an explosion.

The primary objective of the present paper is to establish, through FEM, a relatively simple material model that allows the prediction of its behaviour under ballistic impact. Since the ballistic data on UHMWPE is relatively limited [9], [10], and is confined to general post-mortem observations, residual velocity and V_{50} (the velocity at which there are 50% chances of panel full penetration) determination, there is not enough quantitative data to be used for the constitutive model validation. The SHPB experimental data are chosen as the base for validation, as long as these data are obtained in close strain rates domains (10^2 s^{-1} for Hopkinson bars testing compared to an estimated 10^4 - 10^5 s^{-1} for small calibers ballistic impact).

The material parameters in the constitutive model are obtained by means of an iterative procedure. The parameter values are chosen (using a trial-and-error method) to superpose the totality of the available experimental data to the highest accuracy achievable.

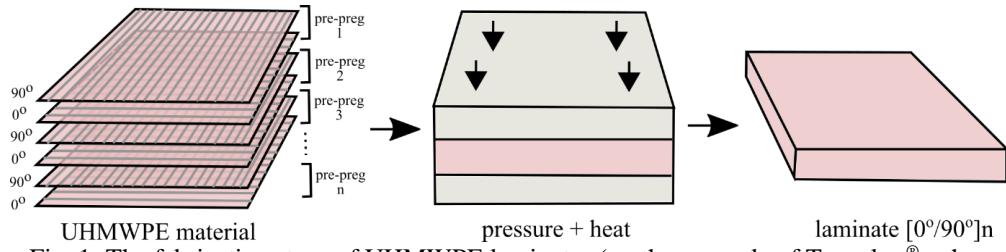


Fig. 1: The fabrication steps of UHMWPE laminates (on the example of Tensylon[®], whose pre-preg is made of two plies) by hot pressing

In numerical simulation approaches, a composite material based on oriented polyethylene can be assessed as it follows (Fig. 2) [11]:

- At micro-scale: the individual fibers, the matrix, and sometimes a third media, representing the fiber-matrix interface, are explicitly modeled;
- At meso-scale: the properties of the individual layers are homogenized in the main directions, and subsequently are modeled and superposed to form a composite;
- At macro-scale: the composite is modeled as a continuum, and the properties of the laminate are homogenized in the main directions.

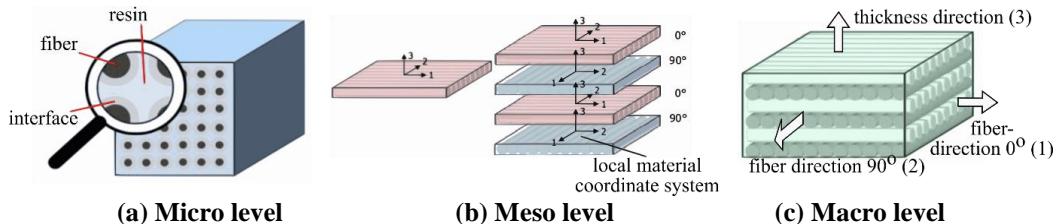


Fig. 2: Micro, meso and macro-scale for mechanical modelling of fibre and filament-based composites; Fig. borrowed from [12]

In the present paper, in order to model the material specimen, we realized two axisymmetric physical models, which fall into the macro and meso category, starting from the pre-preg thickness size (equivalent to two unidirectional tapes), as it follows:

- ✓ Model 1 of the sample is an isotropic solid for every layer, with joint nodes on all the interfaces. Such a representation is, on one hand, identical to a bulk isotropic solid, and so it constitutes a "macro" model of the sample. On the other hand, it allows to test the multi-layer approach within

LS-DYNA, and develop this model by adding some properties/media on the layer interfaces.

- ✓ Model 2 makes a use of this possibility by introducing an "adhesive" layer (consisting of a different body) with specific parameters, instead of an inter-ply interface. This model is an example of a "meso" representation of the sample.

3. Experimental results from Hopkinson bars testing

The experiments performed on Hopkinson bars made of Maraging steel were organized with the purpose to determine the response of Tensylon® composite material to high strain rates. The strain rates were determined by time derivation of the obtained deformations, thus the strain rates turned out to be of the order 10^2 s^{-1} (the highest recorded value being $3.6 \cdot 10^2 \text{ s}^{-1}$). A sketch containing all the dimensions and the location of the strain gauges on the SHPB setup used to perform the experiments is presented in Fig. 3. The bars assembly consists of three bodies: the striker, the incident bar and the transmission bar.

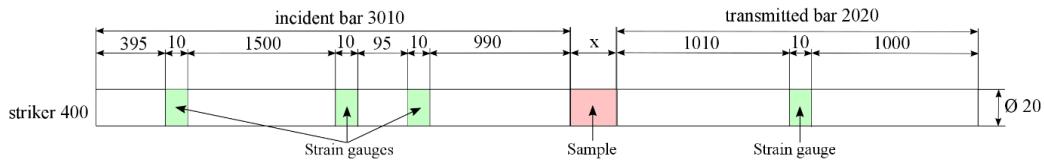


Fig. 3: The Hopkinson bars assembly sketch (dimensions in mm)

The parameters of the bars and transducers installed on them are listed in Table 1. These were used in calculating the dimensions of interest by transforming the exported signal (in volts), but also in the numerical simulations.

The 20 mm diameter Tensylon® cylindrical samples used in the tests, which were obtained by water jet cutting from a 25x30 cm hot pressed plate, are illustrated in Fig. 4 (a). These samples were individually fixed, during the tests, between the incident and the transmission bars, without any glue (due to the lightweight, the sample was self-supported between the bars), as shown in Fig. 4 (b).

By processing the signals from the four transducers, it was possible to extract the following time-dependencies:

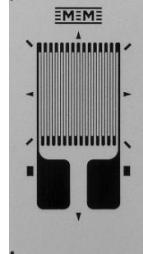
- Displacement of the sample ends, the period of interest being comprised between the arrival of the compression wave through the incident bar and release of the compressive stress on the sample (as predicted by the bar gauges) after the wave passed;

- Velocity of the sample ends;
- Average sample stress diagram;

- Average sample strain diagram;

Table 1

The parameters of the Hopkinson bars set

Hopkinson Bars	Strain gauges measuring circuit
Material: Maraging steel	Strain gauges type: resistive
Density: 7819 kg/m ³	Producer: Micro measurements (Vishay, Inc.)
Sound speed: 5015 m/s	Measuring circuit type: full-bridge
Young Modulus: 183.9 GPa	External voltage: 12.148 V
Poisson Coefficient: 0.32	Aquisition system: Genesis
Lamé Coefficients: 69.6 și 123.8 GPa	Input type: differential
	Acquisition speed: 1MSamples/s (maximum available on the equipment)
	

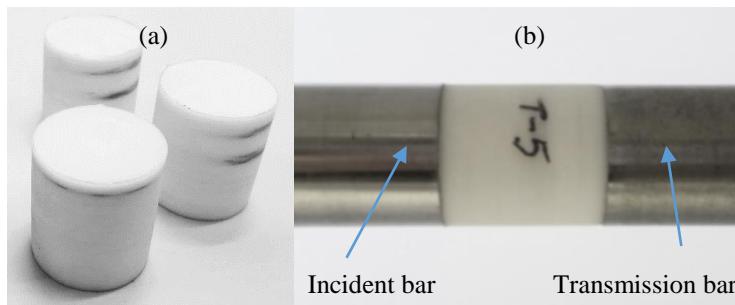


Fig. 4: (a) Cylindrical Tensylon samples for SHPB tests;
(b) The SHPB testing configuration in the sample securing zone.

For the simulations we considered only one case, the particular experimental conditions being shown in Table 2. The graphs used for the validation of the simulated model were those recorded by the transducer located on the incident bar at 990 mm away from the sample.

Table 2

The SHPB experimental conditions

No.	Condition	Value
1	Sample diameter	20 mm
2	Initial thickness of the sample	22.06 mm
3	Final thickness of the sample	22.04 mm
4	Pressure in gas chamber	1 bar

No.	Condition	Value
5	Striker velocity	9.1 ± 0.06 m/s
6	Sample code	T-5

4. The numerical simulation of the SHPB impact phenomenon

The modeling of the tests was done using LS-DYNA R7.0.0, a specialized software for the modeling of non-linear transient phenomena. In our approach, axisymmetric volume weighted solid elements, type SHELL 15, were used. In this case the y axis is the axis of symmetry and the x axis corresponds to the radial direction. The integral difference method defines the components of the gradient of a function F in terms of the line integral about the contour S which encloses the area A :

$$\frac{\partial F}{\partial x} = \frac{\int_C F(n * i) dS}{\lim_{A \rightarrow 0} A}; \quad (1)$$

$$\frac{\partial F}{\partial y} = \frac{\int_C F(n * j) dS}{\lim_{A \rightarrow 0} A}, \quad (2)$$

where n is the normal vector to S and i and j are unit vectors in the x and y directions, respectively.

In this approach the velocity gradients which define the strain rates are element centered and the velocities and nodal forces are node centered [13].

4.1. Numerical model of the Hopkinson bars

In the modeling of the experiments carried out using the Hopkinson bars, we took into account the following aspects:

- The experimental conditions allow simplification of the modeling approach, thus the problem can be expressed in an axisymmetric non-linear statement;
- Due to the length of the bars, in order to realize the meshing, a compromise had to be done between the size of the elements, the precision of the solution and the time to solve the problem. The elements dimensions were 1x1mm;
- The paraboloid of revolution shape of the striker's tip was determined by actual measurements of the striker.

A discretization detail in the bars can be observed in Fig. 5.

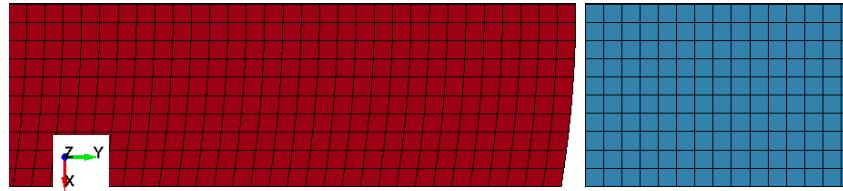


Fig. 5: The meshing of Hopkinson bars – detail.
Red – Striker; Blue – Incident bar.

The interaction between bodies was achieved using the *CONTACT_2D_AUTOMATIC_SURFACE_TO_SURFACE_ID card. The initial striker speed was defined using the *INITIAL_VELOCITY_GENERATION card.

The characteristics of every component of the Hopkinson bars set are gathered in table 3 and its physical properties in table 4. The bar material (maraging steel) is described by the plastic-kinematic model.

Table 3
The characteristics of the Hopkinson bars used as input for modeling in the LS-Dyna pre-processor

Part	Number of elements	Number of nodes	Initial conditions
Striker	5210	5742	Velocity is 9.1 m/s / along OY
Incident bar	30100	33121	At rest
Transmitted bar	20100	22121	At rest
Total	55410	60984	

Table 4
The plastic kinematic material model properties used as input for modeling the maraging steel and the „adhesive” material

Material considered	The plastic kinematic model parameters							
	Ro [tons/mm ³]	E [MPa]	Pr	Sigy [MPa]	Etan [MPa]	Beta	Src [s ⁻¹]	srp
Maraging steel	7.82E-9	183900	0.32	800	800	1	0	0
Adhesive	1.2E-9	4000	0.3	500	2000	1	1E09	1

Symbols: Ro = mass density, E = Young's Modulus; Pr = Poisson ratio; $Sigy$ = Yield stress; $Etan$ = tangent modulus, $Beta$ = hardening parameter, Src = strain rate parameter, C , for Cowper Symonds strain rate model (if zero, rate effects are not considered)

4.2. The macro-scale modeling of the sample

As previously mentioned, the numerical simulation was realized for two different representations of the sample – with and without „adhesive” layers, that corresponds to modelling on two different scales (macro and meso, respectively).

In the "macro" model, we considered a cylindrical sample having the thickness of 22.0 mm (closest to the real model). The sample was modelled as two bodies, each consisting of 110 layers of the thickness 0.1 mm each. The two bodies were shifted 0.1 mm with respect to each other, thus covering the whole space. For each of them, the same material model was used. The discretization was achieved using quadrilateral elements, having the dimensions of 0.1x0.5mm (Fig. 6). Since experimentaly no sign of delamination was observed, the contact between the two components of the composite material was achieved by joining the common nodes. The construction characteristics of the component parts of the specimen are shown in Table 5.

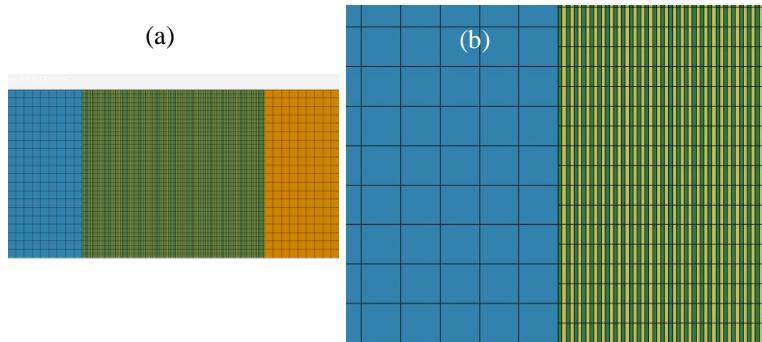


Fig. 6: The meshing (discretization) of the sample. (a) The assembly: blue – incident bar, orange – transmitted bar, green and yellow – the two parts representing the UHMWPE sample; (b) detail from the contact zone between the sample and the incident bar.

Table 5

The construction characteristics of the composite material

Part	Number of elements	Number of nodes	Observations
Part 1	2200	4620	110 layers
Part 2	2200	4620	110 layers
Total	4400	4641	

The density of the sample was determined by direct calculation, and for the first iteration, the values of the other parameters were taken from specialized works. [14]

Table 6 shows the values of material properties for UHMWPE used in numerical simulation. A number of 14 calculations were performed, by implementing gradual modifications in both Young's Modulus and Poisson's coefficient, as well as in all the other parameters presented in the table.

After each trial, the time variation graphs were plotted for several parameters of interest: displacement of sample ends, sample ends velocities, stresses and strain recorded. In order to compare the obtained results, we

considered the graphical representation of only a few of the obtained solutions, highlighted in the table. These are presented in Figs. 7-8.

In the experimental trials, the information on the behavior of the material is determined by processing the data obtained from the strain gauges placed on the bars. Thus, it is considered that the displacement of the bar ends coincides with the displacement of the sample ends while the sample remains in contact with the bars. Therefore, in order to maintain the level of fidelity from simulations to reality, we chose to determine the sample ends displacement in two nodes outside the sample: one on the incident bar end and the other one on the transmission bar end.

Table 6
**The evolution of the essential parameters during the Hopkinson bars simulation,
the macro-scale version**

Trial	E	pr	Sigy	Etan
	Young's Modulus [MPa]	Poisson Coefficient	Stress limit [Mpa]	Tangential modulus [MPa]
1.	3500	0.45	75	500
2.	2500	0.45	75	500
3.	1500	0.45	75	500
4.	1500	0.47	75	500
5.	1500	0.47	75	700
6.	1500	0.47	35	300
7.	1500	0.47	45	450
8.	1750	0.47	45	450
9.	2500	0.47	45	450
10.	2500	0.47	40	400
11.	2500	0.47	50	500
12.	2500	0.47	50	600
13.	2500	0.47	70	500
14.	2500	0.47	60	500

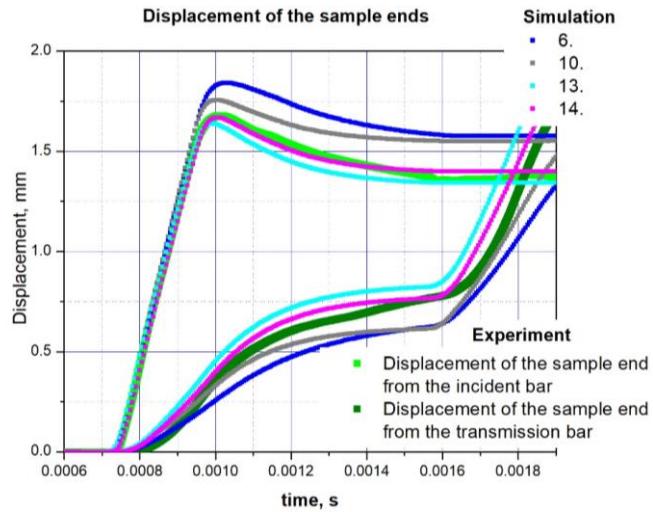


Fig. 7: Displacement of the sample ends in the macro model

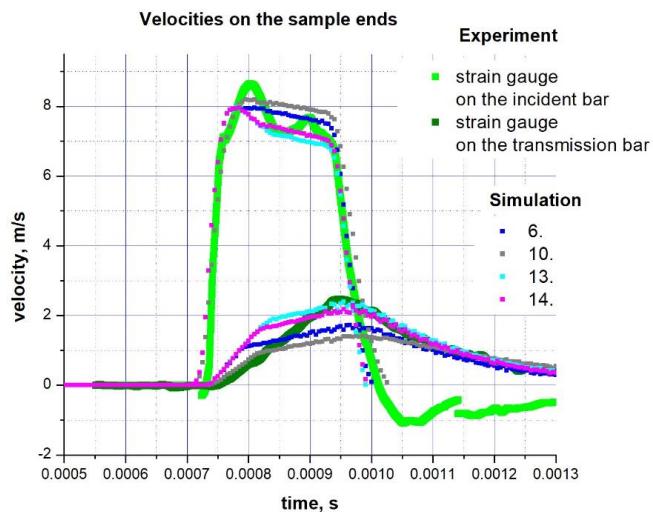


Fig. 8: Velocities of the sample ends in the macro model

Between the four curves represented, obtained from the simulations, curve no. 14 (magenta) approximates best the experiment, as it can be observed in Figs. 7 – 8.

For a better validation of the calculation model by numerical simulation, comparison between the experimental and theoretical values of stresses and deformations in time was performed (Figs. 9 – 10). A simple analysis of the curves below shows that the simulation results provide a very good approximation of the material behavior, version no. 14 being once again the closest to the experimental results.

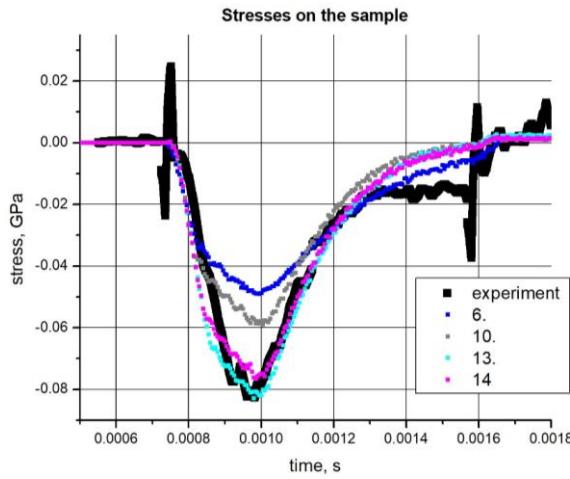


Fig. 9: Stresses in the sample versus time graphs for the macro model

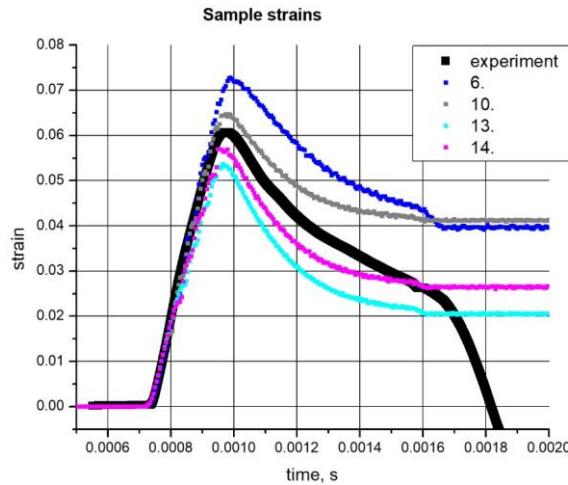


Fig. 10: Strains in the sample versus time for the macro model

4.3. The meso-scale modeling of the sample

The modeling of the specimen was done in this case on three individual layers, two representing the pre-pregs and a third representing a „linking” layers between the two - the "adhesive". Three parts corresponding to two material models were considered. The discretization was done using quadrilateral elements, with the dimensions of 0.1x0.5mm for the Tensylon® layers, and 0.015x0.5mm for the adhesive layers. The overall thickness of the sample was 22.065 mm. Because of the absence of delamination in the experimental trials, the contact between the three components of the composite material was made also by joining the common nodes. The construction characteristics of the sample component parts are shown in Table 7.

Table 7
The construction characteristics of the composite material – version with „adhesive”

Part	Number of elements	Number of nodes	Observations
Part 1	1920	4032	96 layers
Part 2	1920	4032	96 layers
Adhesive	3820	8022	191 layers
Total	7760	16086	

The adhesive parameters were considered constant: it was described by a plastic-kinematic equation of state with parameters given in table 4. Several sets of values of the parameters for the sample material were tested. For the first iteration, they were determined in the light of past experience (version „without adhesive”). In Fig. 11 the new configuration of the sample is presented, corresponding to a detail in the contact zone with the incident bar.

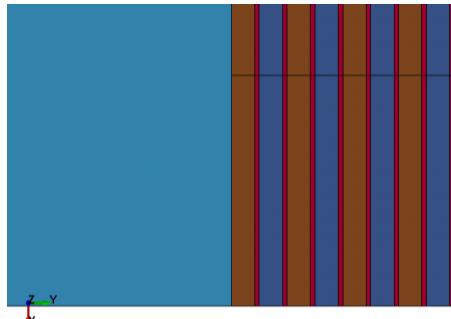


Fig. 11: The meso-scale model configuration of the Hopkinson bars experiment, in numerical representation; light blue – incident bar (brown and dark blue – UHMWPE, red – adhesive layers)

In this case we applied as well the previously presented algorithm, proceeding in the same iterative manner in order to determine the constituent materials models. Table 8 presents the values of the material properties used in numerical simulation. Considering the fact that the number of model elements increased and taking into account the previous experience regarding the material parameters, only 6 iterations were performed, in which both Young's and Poisson's coefficient and the other parameters presented in the table were modified.

Next, comparative graphs are presented (Figs. 12-13), considering the experimental results and the simulation trials no. 5 and 6.

Analyzing Figs. 12-13, we notice that from the four represented curves, obtained from the simulation, the curve no. 6 (magenta) approximates best the experiment.

We have also compared the experimental and theoretical values of stresses and strains versus time (Figs. 14-15), highlighting the fact that the simulation

results provide a very good approximation of the behavior of the material, trial no. 6 (magenta) being the closest to the experimental results.

Table 8
The evolution of the essential parameters in the simulation of the SHPB tests, meso-scale version

Trial	E	pr	Sigy	Etan
	Young Modulus [MPa]	Poisson Coefficient	Stress limit [Mpa]	Tangential modulus [MPa]
1.	2500	0.47	60	500
2.	2500	0.47	60	1000
3.	2500	0.47	60	700
4.	2500	0.47	100	1000
5.	2000	0.47	70	700
6.	3000	0.47	60	500

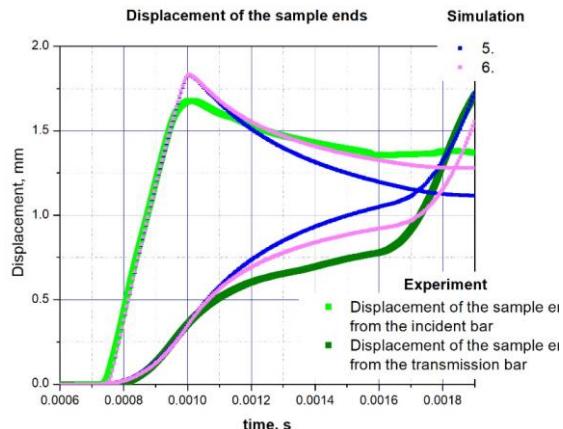


Fig. 12: Displacement of the sample ends for the meso-scale model

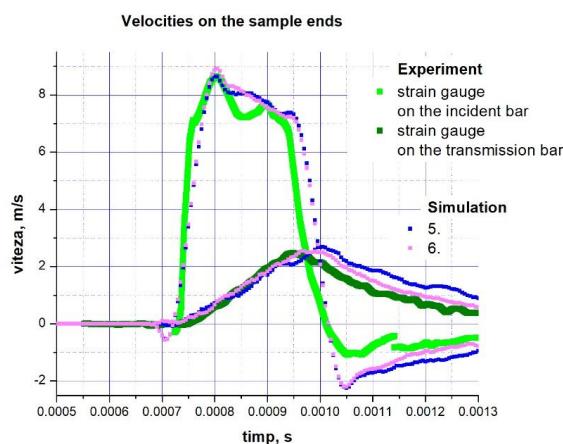


Fig. 13: Velocities on sample ends for the meso-scale model

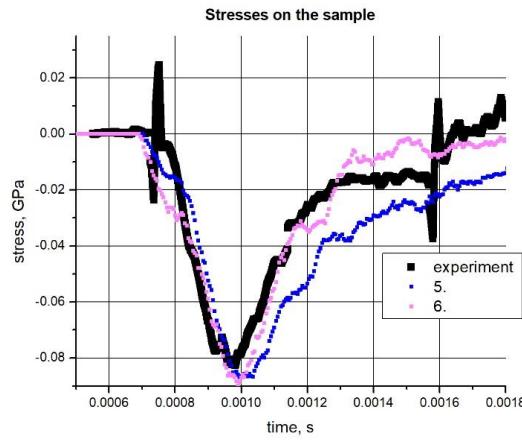


Fig. 14: Stresses inside the sample, meso-scale model

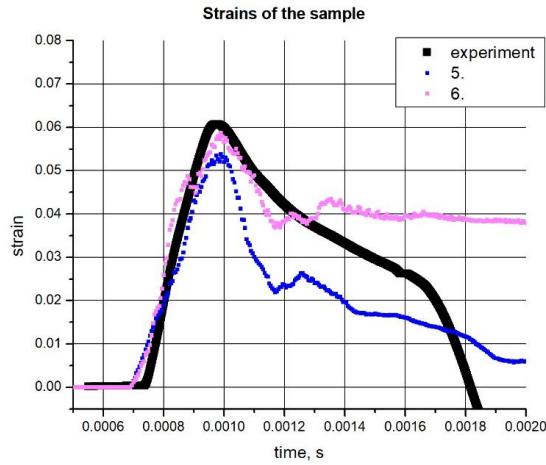


Fig. 15: Strains of the sample, meso-scale model

5. Comparative results and conclusions

As shown in the previous section, the solution No 14 for the macro model and the solution No 6 for the meso model give the best correlation with the experimental data in the limits of their respective constitutive models. In this section, these two solutions are compared with each other (Figs. 16-19). It should be noted that the macro model better fits the sample interfaces displacements, whereas the meso model better represents their velocities.

The numerical simulations indicate a residual deformation of about 2.5% in the case of the macro optimum model and 4% in the case of the micro optimum model (Fig. 19). These values correspond to a deformation of 0.55 mm and 0.88 mm respectively, compared to 0.02 mm in the experiment. Therefore, the values obtained in both simulation cases overestimate the actual values, experiments

showing that the sample, even after several consecutive compressive SHPB tests, does not exhibit cumulative deformations greater than 0.15 mm.

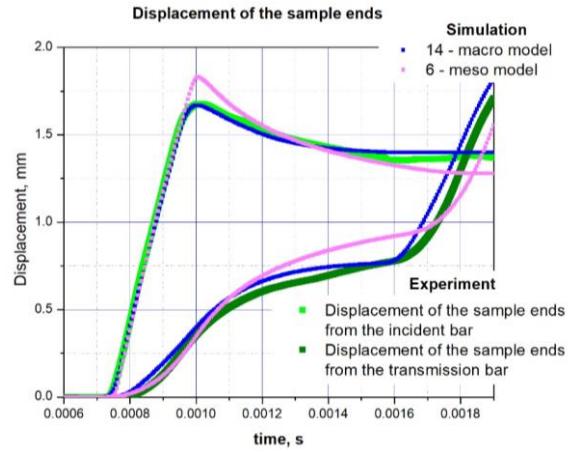


Fig. 16: Comparative graphs of the sample ends displacements

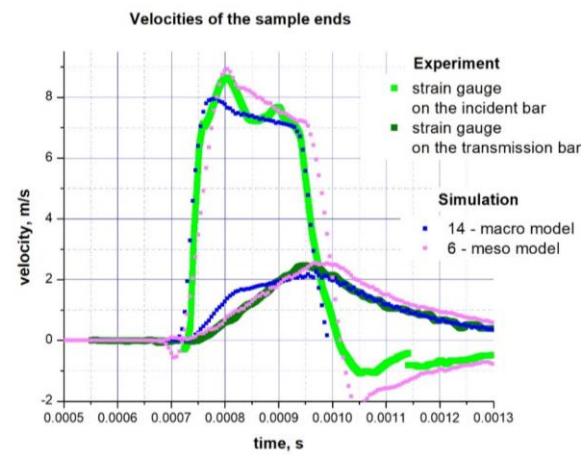


Fig. 17: Comparative graphs of the sample ends velocities

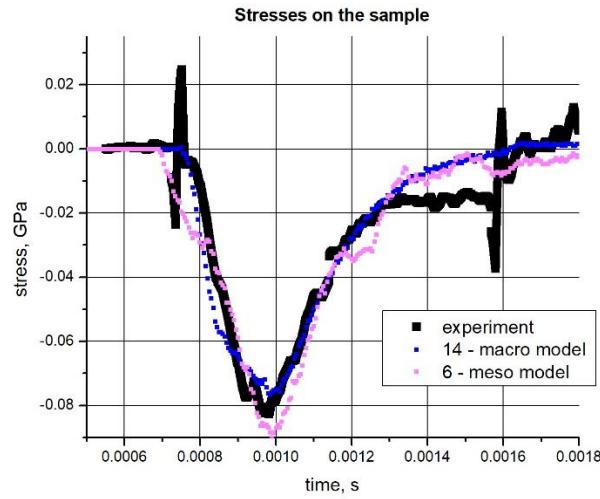


Fig. 18: Comparative graph of the stresses on the sample

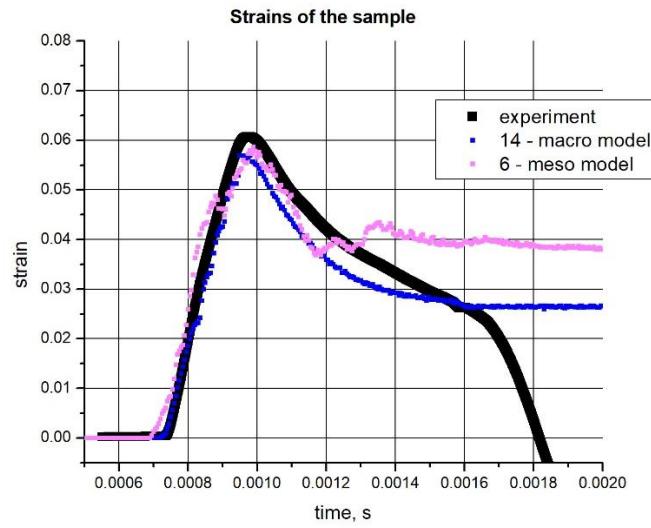


Fig. 19: Comparative graphs on strains

In conclusion, it can be admitted that, using an inverse approach by performing numerical simulation in order to fit experimental results, a relatively precise parameter calibration for a simple plastic-kinematic material model was determined. This model can be used further for modeling the criss-cross composite materials obtained from oriented polyethylene tape, involved in ballistic impact events, considering the fact that these materials represent a common choice nowadays for obtaining individual ballistic protection solutions.

In order to improve the accuracy, other material models such as Johnson-Cook, orthotropic, hypelastic, etc., as well as a finer mesh or a three-dimensional

model can be used. The disadvantage of these versions, however, is a higher computing time or the need for more powerful computers.

The obtained "macro" and "meso" models will, in the following, be used for estimative axisymmetric numerical simulations of the impact between bullet penetrators and ballistic protection structures which contain a UHMWPE composite material component. The numerical example presented here shows that the post-mortem deformations are not likely to be predicted, but the stress state under dynamic loading is expected to be numerically estimated with sufficient (10% at least) accuracy. Considering the achieved results by the inverse approach, we can continue to perform numerical simulations of the impact between bullet penetrators and ballistic protection structures which are composed of UHMWPE composite materials, without relying on more expensive and difficult experiments.

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