

## A DMSACO approach to economic heat exchanger design

George Anescu<sup>1</sup>

*The problem of economic optimization of Shell-and-Tube Heat Exchangers (STHE) is well known in the literature. Since traditional design approaches do not guarantee the reach of the optimal solution, some heuristic approaches were developed and their results are published in the literature. Here is proposed a new method inspired from the multistart methods and Ant Colony Optimization (ACO) methods for continuous problems, the Distributed MultiStart Ant Colony Optimization (DMSACO) method. As a local optimization method used by DMSACO a novel Swarm Intelligence (SI) method is proposed, the Particle Swarm Local Optimization (PSLO) method. The results for two case studies are finally compared to those obtained by other approaches from literature, proving that the DMSACO global optimization method combined with PSLO local optimization method can be successfully applied to the STHE economic design problem.*

**Keywords:** Optimization, Continuous Global Optimization Problem (CGOP), Swarm Intelligence (SI), Ant Colony Optimization (ACO), Distributed MultiStart Ant Colony Optimization (DMSACO), Particle Swarm Local Optimization (PSLO), Shell-and-Tube Heat Exchanger (STHE)

**MSC2010:** 68T 20, 68W 10, 90C 26, 90C 56, 90C 59.

### 1. Introduction

Heat exchangers are devices used to transfer heat between two or more fluids that are at different temperatures and which in most cases are separated by a solid wall. Shell and tube heat exchangers, *STHEs*, are probably the most common type of heat exchangers applicable for a wide range of operating temperatures and pressures. Their widespread use can be justified by their versatility, robustness and reliability. The design of *STHE* involves a large number of geometric and operating variables as a part of the search for a heat exchanger geometry that meets the heat duty requirement and a given set of design constraints. A variety of techniques have been proposed to the design optimization problem such as, numerical resolution of the stationary point equations of a nonlinear objective function, mixed integer nonlinear programming, but also techniques pertaining to the larger field of Artificial Intelligence (*AI*), such as the optimization techniques from the Swarm Intelligence (*SI*)

---

<sup>1</sup>PhD Student, Power Plant Engineering Faculty, Polytechnic University of Bucharest, Romania, e-mail: [george.anescu@gmail.com](mailto:george.anescu@gmail.com)

class. In the present paper we will approach the problem of economic design of *STHE* by applying the novel Distributed MultiStart Ant Colony Optimization (*DMSACO*) method combined with the novel Particle Swarm Local Optimization (*PSLO*) method.

## 2. Continuous Global Optimization Problem (*CGOP*)

The general Continuous Global Optimization Problem (*CGOP*) is formulated as:

$$\text{minimize} \quad f(\mathbf{x}) \quad (1)$$

$$\text{subject to} \quad \mathbf{x} \in D$$

with

$$D = \{\mathbf{x} : \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}; g_i(\mathbf{x}) \leq 0, i = 1, \dots, G; \\ h_j(\mathbf{x}) = 0, j = 1, \dots, H\} \quad (2)$$

where  $\mathbf{x} \in \mathbb{R}^n$  is a real  $n$ -vector of decision variables,  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is the continuous objective function,  $D \subset \mathbb{R}^n$  is the non-empty set of feasible decisions (a proper subset of  $\mathbb{R}^n$ ),  $\mathbf{l}$  and  $\mathbf{u}$  are explicit, finite (component-wise) lower and upper bounds on  $\mathbf{x}$ ,  $g_i : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $i = 1, \dots, G$  is a finite collection of continuous inequality constraint functions, and  $h_j : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $j = 1, \dots, H$  is a finite collection of continuous equality constraint functions. We don't make any other additional assumptions on the *CGOP* problem and presume that we cannot get any additional knowledge about the collections of real continuous functions defined, in this way treating the *CGOP* problem as a black box, i.e. for any point  $\mathbf{x}$  in the boxed domain  $\{\mathbf{x} : \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}\}$  we presume that we are able calculate the values of the functions  $f(\mathbf{x})$ ,  $g_i(\mathbf{x})$ ,  $i = 1, \dots, G$ ,  $h_j(\mathbf{x})$ ,  $j = 1, \dots, H$ , but nothing more.

## 3. *PSLO* local optimization method

Conceptually the Particle Swarm Local Optimization (*PSLO*) method is similar to *PSO* methods ([?]). It is searching for the optimum of a problem by maintaining a population of candidate solutions called particles and moving these particles around in the search domain according to simple interaction rules. The movements of the particles are guided by the best found position in the search-space, which is continually updated as better positions are found by the particles. We denote by  $n$  the dimension of the search space and by  $f(\mathbf{x})$  our objective function to be minimized. If  $N$  is the number of particles in the swarm, then the position of a particle is given by the vector  $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \dots, x_{i,n})$ ,  $i = 1, 2, \dots, N$ . We will give interaction rules for iteratively calculating the next position of a particle based on its current position in each of the  $n$  dimensions. At each iterative step the particle determined to

be in the best position (which gives the best known optimal value for function  $f$ ) is immobile, while the other particles in the swarm will be attracted to it. Also there will be a random fluctuation component in the movement of the particles. The steps of the algorithm are as follows:

*Step 1: Initialization.* The method's parameters are initialized:  $N$  the number of particles,  $p$  the power parameter in Minkovski distance, and  $\epsilon$  the required precision. Also the bounds of the limiting box are defined. The particles are positioned at random positions in the search space (the limiting box).

*Steps 2-7* are performed in a loop:

*Step 2:* The diameters on each dimension are calculated as the difference between the maximum value on that dimension over all the particles in the swarm and the minimum value on that dimension over all the particles in the swarm:

$$d_j(k+1) = \max_{1 \leq i_1, i_2 \leq N, i_1 \neq i_2} |x_{i_1, j}(k) - x_{i_2, j}(k)|, \quad j = 1, 2, \dots, n \quad (3)$$

*Step 3:* The values of the objective function  $f(\mathbf{x})$  for the positions of the particles are updated:

$$v_i(k+1) = f(\mathbf{x}_i(k)), \quad i = 1, 2, \dots, N \quad (4)$$

and the minimum and maximum values over the particles in the swarm are determined:

$$v_{min}(k+1) = \min_{1 \leq i \leq N} v_i(k+1) \quad (5)$$

and

$$v_{max}(k+1) = \max_{1 \leq i \leq N} v_i(k+1) \quad (6)$$

*Step 4:* The satisfaction of the termination condition is checked:

$$d(k+1) = \left( \sum_{j=1}^n (d_j(k+1))^p \right)^{\frac{1}{p}} < \epsilon \quad (7)$$

where the overall swarm diameter  $d(k)$  is calculated according to the Minkovski distance with power parameter  $p$ . If the termination condition is satisfied then the iterative process is stopped (the loop is broken) and the particle which gives  $v_{min}(k+1)$ , positioned in  $\mathbf{x}_{min}(k)$  is taken as the solution to the global optimization problem.

*Step 5:* The normalized distances for each particle to the best particle are calculated according to:

$$r_i(k+1) = \frac{\|\mathbf{x}_{min}(k) - \mathbf{x}_i(k)\|_p}{d(k+1)}, \quad (8)$$

$$i = 1, 2, \dots, N$$

*Step 6:* The weights of the particles in the swarm are calculated according to:

$$w_i(k+1) = \frac{(v_i(k+1) - v_{min}(k+1))}{(v_{max}(k+1) - v_{min}(k+1))}, \quad (9)$$

$$i = 1, 2, \dots, N$$

*Step 7:* The new positions of the particles are calculated according to the formula:

$$x_{i,j}(k+1) = x_i(k) + attr_{i,j}(k+1) + fluct_{i,j}(k+1), \quad (10)$$

$$i = 1, 2, \dots, N, \quad j = 1, 2, \dots, n$$

where:

$$attr_{i,j}(k+1) = \min\{1, 4r_i(k+1)w_i(k+1)\}(x_{min,j}(k) - x_{i,j}(k))$$

$$fluct_{i,j}(k+1) = (x_{min,j}(k) - x_{i,j}(k))(rnd_{i,j,1}(k+1) - 0.5) +$$

$$+ d_j(k+1)w_i(k+1)(rnd_{i,j,2}(k+1) - 0.5) \quad (11)$$

$$i = 1, 2, \dots, N, \quad j = 1, 2, \dots, n$$

the attraction factor having an upper limit to the value 1 in order to not allow the movement beyond the center of attraction.  $rnd_{i,j,1}(k+1)$ ,  $rnd_{i,j,2}(k+1)$  are pseudo-randomly generated numbers uniformly distributed in the interval  $[0, 1)$ .

As can be seen from the above algorithm, the particles are interacting directly with the best particle in the swarm and indirectly among themselves through the set of diameters on directions and the set of weights.

#### 4. *DMSACO* global optimization method

The *DMSACO* global optimization method is combining ideas from multistart optimization methods and *ACO* optimization methods for continuous domains. The main idea is to maintain a Solution Archive (*SA*) of ants and to update the *SA* at each iteration by running a local optimization method (the *PSLO* method in our implementation) with initialization around the current ants in *SA*. The initialization boxes around the ants in *SA* are reduced in volume with a constant volumetric rate at each iteration step. After a finite number of iterations the initialization boxes around the ants attain in size the

required precision of the method, in this way providing a intrinsic stop condition. The steps of the algorithm are as follows:

*Step 1: Initialization.* The method's parameters are initialized:  $N$  the initial number of ants,  $\epsilon_g$  the precision of the global strategy,  $\epsilon_l$  the precision of the local optimization method (*PSLO*),  $\alpha_v$  the volumetric decrease rate. Also the limits of the constraining box are defined. The precision of the local optimization method  $\epsilon_l$  is the precision required for the last iteration of the method, at each iteration being used a current precision value  $\epsilon_{lc}$  which is initialized to  $\frac{\epsilon_l}{\epsilon_g}$ . The unidimensional decrease rate  $\alpha_u$  is calculated from the volumetric decrease rate as  $\alpha_u = (\alpha_v)^{\frac{1}{n}}$ , where  $n$  is the dimension of the search domain. The number of iterations can be pre-calculated and is given by

$$iter_{max} = \left\lceil \frac{\log\left(\frac{\epsilon_g}{2}\right)}{\log \alpha_u} \right\rceil + 1 \quad (12)$$

Another parameter is the length of the side of the initialization boxes around the ants,  $l_b$ . To simplify the notation we will assume that the limiting box is normalized to the unit hypercube and in this case all the sides of the initialization boxes centred in the ants are initialized to 2.

*Step 2: First run.* The local optimization method is run  $N$  times with precision  $\epsilon_{lc}$  and the particles of *PSLO* method are initialized randomly in the initialization boxes. The solutions of the local runs are used to populate the *SA* ordered by the objective function optimization values obtained from the local optimization run, the first ant being the best one (with the smallest optimization value). If two ants are in space at an Euclidian distance smaller than  $\epsilon_{lc}$  than the worst one is not saved in the *SA*. In this way the number of ants  $N$  is dynamically adapted to the structure of the objective function.

The next steps are run in a loop  $iter_{max}$  times with current loop index  $k$  starting at 1:

*Step 3:* The parameters  $\epsilon_{lc}$  and  $l_b$  are reduced in by the unidimensional decrease rate  $\alpha_u$ :

$$\begin{aligned} \epsilon_{lc}(k+1) &= \alpha_u \epsilon_{lc}(k) \\ l_b(k+1) &= \alpha_u l_b(k) \end{aligned} \quad (13)$$

*Step 4:* The local optimization method is run for each ant in *SA* with precision  $\epsilon_{lc}$  and the particles of *PSLO* method are initialized randomly in boxes obtained by intersecting the large limiting box with the current initialization boxes centred in the ants. The solutions of the local runs of method *PSLO* are saved in *SA*, added to or inserted among the existent solutions (ants) so that to maintain the sorting rule used for *SA*, i.e. increasingly by the value of the objective function. The dimension of *SA* has to be maintained to at most the current value of  $N(k)$  and therefore some ants have to be eliminated

from  $SA$ . If two ants are in space at an Euclidian distance smaller than  $\epsilon_{lc}$  than the worst one is eliminated from  $SA$ . Then a selection strategy based on elitism is applied, by which a percentage (usually 10%) from the best solutions from  $SA$  are maintained. The other solutions maintained in  $SA$  up to at most  $N(k)$  are obtained by applying a probabilistic selection algorithm based on rank (*Rank Selection*). If we denote by  $N'$  the number of solutions remained in  $SA$  after eliminating the ones selected through elitism, the probability of the best remained solution is  $N'/(N'(N'+1))/2 = \frac{2}{N'+1}$ , then decreasingly the solution in position  $l$  will have the probability  $l/(N'(N'+1))/2 = \frac{2l}{N'(N'+1)}$ , and finally the weakest solution will have the probability  $\frac{2}{N'(N'+1)}$ .

*Step 5:* At the end, after  $iter_{max}$  iterations the best solution is located in the first position in  $SA$  and it is calculated with the required local precision  $\epsilon_l$ .

It is important to note that each local optimization step can be run independently and the method presents a great opportunity for implementation on parallel and distributed computers, especially considering that we expect a very small overhead from communication between the execution threads.

For constraints treatment there are many methods proposed in the literature, one of the most popular methods used in  $SI$  optimization methods, and also used in the present study, being the exterior penalty function method.

## 5. Mathematical model

### Nomenclature:

$A$	heat exchanger surface area ( $m^2$ )
$a_1$	numerical constant (€)
$a_2$	numerical constant (€/m <sup>2</sup> )
$a_3$	numerical constant
$A_s$	shell side pass area ( $m^2$ )
$B$	baffles spacing ( $m$ )
$C$	numerical constant
$C_e$	energy cost (€/kWh)
$C_i$	capital investment (€)
$C_o$	annual operating cost (€/an)
$C_{od}$	total discounted operating cost (€)
$C_p$	specific heat ( $J/kgK$ )
$C_{tot}$	total annual cost (€)
$d_e$	equivalent shell diameter ( $m$ )
$d_i$	tube inside diameter ( $m$ )
$d_o$	tube outside diameter ( $m$ )
$D_s$	shell inside diameter ( $m$ )
$F$	temperature difference correction factor
$f_s$	shell side friction coefficient

$f_t$	tube side friction coefficient
$H$	annual operating time ( $h/an$ )
$h_s$	shell side convective coefficient ( $W/m^2K$ )
$h_t$	tube side convective coefficient ( $W/m^2K$ )
$i$	annual discount rate (%)
$k$	thermal conductivity ( $W/mK$ )
$L$	tubes length ( $m$ )
$LMTD$	logarithmic mean temperature difference ( $K$ )
$m_s$	shell side mass flow rate ( $kg/s$ )
$m_t$	tube side mass flow rate ( $kg/s$ )
$N_t$	number of tubes
$n$	number of tube passes
$n_1$	numerical constant
$ny$	equipment life (yr)
$P$	pumping power (W)
$Pr_s$	shell side Prandtl number
$Pr_t$	tube side Prandtl number
$Q$	heat duty (W)
$Re_s$	shell side Reynolds number
$Re_t$	tube side Reynolds number
$R_{fs}$	shell side fouling resistance ( $m^2K/W$ )
$R_{ft}$	tube side fouling resistance ( $m^2K/W$ )
$S_t$	tube pitch ( $m$ )
$T_{ci}$	cold fluid inlet temperature ( $K$ )
$T_{co}$	cold fluid outlet temperature ( $K$ )
$T_{hi}$	hot fluid inlet temperature ( $K$ )
$T_{ho}$	hot fluid outlet temperature ( $K$ )
$U$	overall heat transfer coefficient ( $W/m^2K$ )
$v_s$	shell side fluid velocity ( $m/s$ )
$v_t$	tube side fluid velocity ( $m/s$ )
$\Delta h$	heat transfer difference ( $W/m^2K$ )
$\Delta P$	pressure drop ( $Pa$ )
$\Delta P_{\text{tube elbow}}$	tube elbow pressure drop ( $Pa$ )
$\Delta P_{\text{tube length}}$	tube length pressure drop ( $Pa$ )

## Greek Letters:

$\mu$	dynamic viscosity ( $Pa\cdot s$ )
$\rho$	density ( $kg/m^3$ )
$\eta$	overall pumping efficiency

## Subscripts:

c	cold stream
e	equivalent

---

h	hot stream
i	inlet
o	outlet
s	shell side
t	tube side
wt	tube wall
ws	shell wall

In the *STHE* model (see [?]) we will assume the inlet and outlet temperatures of the fluids and the flow rates as given design specifications, while shell inner diameter ( $D_s$ ), tube outer diameter ( $d_o$ ), baffles spacing ( $B$ ) and tubes length ( $L$ ) are assumed as the design variables. In dependence on the flow regime, the tube side heat transfer coefficient  $h_t$  is calculated according to the following correlations ([?]):

If  $Re_t < 2300$  (Stephan Preußer):

$$h_t = \frac{k_t}{d_i} \left[ 3.657 + \frac{0.0677(Re_t Pr_t \frac{d_i}{L})^{1.33}}{1 + 0.1 Pr_t (Re_t \frac{d_i}{L})^{0.3}} \right] \quad (14)$$

if  $2300 < Re_t < 10000$  (Gnielinski):

$$h_t = \left( \frac{k_t}{d_i} \right) \left[ \frac{\frac{f_t}{8} (Re_t - 1000) Pr_t}{1 + 12.7 \left( \frac{f_t}{8} \right)^{1/2} (Pr_t^{2/3} - 1)} \right] \left[ 1 + \left( \frac{d_i}{L} \right)^{0.67} \right] \quad (15)$$

and if  $Re_t > 10000$  (Sieder & Tate):

$$h_t = 0.027 \left( \frac{k_t}{d_i} \right) Re_t^{0.8} Pr_t^{1/3} \left( \frac{\mu_t}{\mu_{wt}} \right)^{0.14} \quad (16)$$

where  $f_t$  is the Darcy friction factor given by:

$$f_t = (1.82 \log_{10} Re_t - 1.64)^{-2} \quad (17)$$

$Re_t$  is the tube side Reynolds number given by:

$$Re_t = \frac{\rho_t v_t d_i}{\mu_t} \quad (18)$$

The flow velocity for tube side is given by:

$$v_t = \frac{m_t}{\frac{\pi}{4} d_i^2 \rho_t} \left( \frac{n}{N_t} \right) \quad (19)$$

where  $N_t$  is the number of tubes and  $n$  is the number of tube-passes. The number of tubes can be found approximately from the following equation:



$$N_t = C \left( \frac{D_s}{d_o} \right)^{n_1} \quad (20)$$

where  $C$  and  $n_1$  are coefficients taking values according to flow arrangement and number of passes.  $Pr_t$  is the tube side Prandtl number and is given by:

$$Pr_t = \frac{\mu_t C_{pt}}{k_t} \quad (21)$$

Also,  $d_i = 0.8d_o$ . Kern's formulation for segmental baffle shell and tube exchanger is used for computing shell side heat transfer coefficient  $h_s$ :

$$h_s = 0.36 \left( \frac{k_s}{d_e} \right) Re_s^{0.55} Pr_s^{1/3} \left( \frac{\mu_s}{\mu_{ws}} \right)^{0.14} \quad (22)$$

where,  $d_e$  is the shell hydraulic diameter and for square pitch it is computed by:

$$d_e = \frac{4 \left( S_t^2 - \frac{\pi d_o^2}{4} \right)}{\pi d_o} \quad (23)$$

while for triangular pitch it is computed by:

$$d_e = \frac{4 \left( 0.43 S_t^2 - \frac{0.5 \pi d_o^2}{4} \right)}{0.5 \pi d_o} \quad (24)$$

where  $S_t = 1.25d_o$ . The triangular and square tube pitch arrangements are presented in Fig. 2 ([?]) below for a: triangle and b: square patterns.

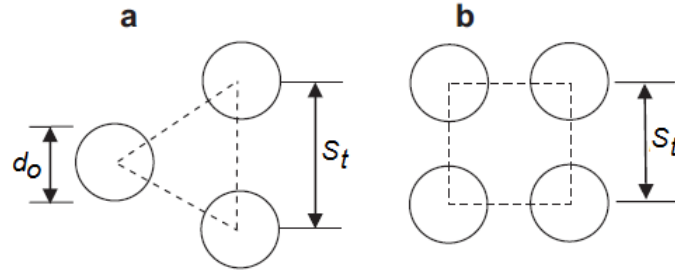


FIG. 1. Triangular and square tube pitch arrangements

Table 1 below gives the values of the numerical constants  $C$  and  $n_1$  for different flow arrangements and number of passes  $n$  ([?]).

Cross-section area normal to flow direction is determined by:

$$A_s = D_s B \left( 1 - \frac{d_o}{S_t} \right) = \frac{D_s B C_l}{S_t} \quad (25)$$

TABLE 1. Values of  $K_1$  and  $n_1$  coefficients for  $S_t = 1.25d_o$ 

No. of passes $n$	Tube pitch	$K_1$	$n_1$
1	triangular	0.319	2.142
2	triangular	0.249	2.207
4	triangular	0.175	2.285
6	triangular	0.0743	2.499
8	triangular	0.0365	2.675
1	square	0.215	2.207
2	square	0.156	2.291
4	square	0.158	2.263
6	square	0.0402	2.617
8	square	0.0331	2.643

where  $C_l = S_t - d_o$  is the shell side clearance. Flow velocity for the shell side can be obtained from:

$$v_s = \frac{m_s}{\rho_s A_s} \quad (26)$$

Reynolds number for shell side is given by:

$$Re_s = \frac{m_s d_e}{A_s \mu_s} \quad (27)$$

Prandtl number for shell side is given by:

$$Pr_s = \frac{\mu_s C_{ps}}{k_s} \quad (28)$$

The overall heat transfer coefficient  $U$  depends on both the tube side and shell side heat transfer coefficient and fouling resistances and is given by:

$$U = \frac{1}{\frac{1}{h_s} + R_{fs} + \frac{d_o}{d_i} \left( R_{ft} + \frac{1}{h_t} \right)} \quad (29)$$

Considering the cross flow between adjacent baffles, the logarithmic mean temperature difference  $LMTD$  is given by:

$$LMTD = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln \left( \frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}} \right)} \quad (30)$$

The correction factor  $F$  for the flow configuration involved is found as a function of dimensionless temperature ratio for most flow configurations of interest:

$$F = \sqrt{\frac{R^2 + 1}{R - 1}} \frac{\ln \left( \frac{1-P}{1-PR} \right)}{\ln \left( \frac{2-P(R+1-\sqrt{R^2+1})}{2-P(R+1+\sqrt{R^2+1})} \right)} \quad (31)$$

where  $R$  is the correction coefficient given by:

$$R = \frac{(T_{hi} - T_{ho})}{(T_{co} - T_{ci})} \quad (32)$$

and  $P$  is the efficiency given by:

$$P = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \quad (33)$$

Considering the overall heat transfer coefficient, the heat exchanger surface area  $A$  is calculated by:

$$A = \frac{Q}{U \times F \times LMTD} \quad (34)$$

The heat transfer rate is given by:

$$Q = m_h C_{ph}(T_{hi} - T_{ho}) = m_c C_{pc}(T_{co} - T_{ci}) \quad (35)$$

Based on total heat exchanger surface area  $A$ , the necessary tube length  $L_1$  is given by:

$$L_1 = \frac{A}{\pi d_o N_t} \quad (36)$$

It is important to note that here we have an equality constraint since the value of  $L$  used in the calculation of  $h_t$  in equations (??) and (??) has to be the same as the value of  $L_1$  calculated in the last equation above:  $L_1 = L$ .

The pressure drop allowance in a heat exchanger is the static fluid pressure which may be expended to drive the fluid through the exchanger. Tube side pressure drop include distributed pressure drop along the tube length and concentrated pressure losses in elbows and in the inlet and outlet nozzles:

$$\Delta P_t = \Delta P_{\text{tube length}} + \Delta P_{\text{tube elbow}} = \frac{\rho_t v_t^2}{2} \left( \frac{L}{d_i} f_t + p \right) n \quad (37)$$

Different values of constant  $p$  are considered by different authors. Here we will assume  $p = 2.5$ , as it is recommended by Sinnott ([?]). The shell side pressure drop is:

$$\Delta P_s = f_s \left( \frac{\rho_t v_t^2}{2} \right) \left( \frac{L}{B} \right) \left( \frac{D_s}{d_e} \right) \quad (38)$$

where:

$$f_s = 2b_0 Re_s^{-0.15} \quad (39)$$

and  $b_0 = 0.72$  valid for  $Re_s < 40000$ . Considering  $\eta$  the pumping efficiency, the pumping power is computed by:

$$P = \frac{1}{\eta} \left( \frac{m_t}{\rho_t} \Delta P_t + \frac{m_s}{\rho_s} \Delta P_s \right) \quad (40)$$

The total cost  $C_{tot}$ , taken as the objective function, includes capital investment  $C_i$ , energy cost  $C_e$ , annual operating cost  $C_o$  and total discounted operating cost  $C_{od}$ , with:

$$C_{tot} = C_i + C_{od} \quad (41)$$

Adopting Hall's correlation, the capital investment  $C_i$  is computed as a function of the exchanger surface area:

$$C_i = a_1 + a_2 A^{a_3} \quad (42)$$

where  $a_1 = 8000$ ,  $a_2 = 259.2$  and  $a_3 = 0.93$  for exchanger made with stainless steel for both shell and tubes. The total discounted operating cost related to pumping power to overcome friction losses is computed from the following equations:

$$C_o = PC_e H \quad (43)$$

$$C_{od} = \sum_{x=1}^{ny} \frac{C_o}{(1+i)^x} \quad (44)$$

## 6. Results and discussion

TABLE 2. Process parameters and physical properties for *Case Study #1*

Fluid location	Shell side	Tube side
Fluid	Distilled water	Raw water
Mass flow $m$ (kg/s)	22.07	35.31
Inlet temperature $T_i$ (K)	33.9	23.90
Outlet temperature $T_o$ (K)	29.40	26.70
Density $\rho$ (kg/m <sup>3</sup> )	995	999
Heat capacity $C_p$ (J/kgK)	4.18	4.18
Viscosity $\mu$ (Pas)	0.00080	0.00092
Wall viscosity $\mu$ (Pas)	-	-
Thermal conductivity $k$ (W/mK)	0.62	0.62
Fouling factor $R_f$ (m <sup>2</sup> K/W)	0.00017	0.00017

The efficiency of the proposed optimization method *DMSACO* (combined with the *PSLO* method for local optimization) will be evaluated by comparison on two case studies solved previously by Kern ([?]) and respectively Sinnot et al. ([?]) in the original design solutions, Caputo et al. using *GA* ([?]), Patel and Rao using *PSO* ([?]), Sahin et al. using *ABC* ([?])

TABLE 3. Process parameters and physical properties for *Case Study #2*

Fluid location	Shell side	Tube side
Fluid	Methanol	Sea water
Mass flow $m$ (kg/s)	27.80	68.90
Inlet temperature $T_i$ (K)	95	25
Outlet temperature $T_o$ (K)	40	40
Density $\rho$ (kg/m <sup>3</sup> )	750	995
Heat capacity $C_p$ (J/kgK)	2.84	4.20
Viscosity $\mu$ (Pas)	0.00034	0.00080
Wall viscosity $\mu$ (Pas)	0.00038	0.00052
Thermal conductivity $k$ (W/mK)	0.19	0.59
Fouling factor $R_f$ (m <sup>2</sup> K/W)	0.00033	0.00020

TABLE 4. Parameters of the optimal shell and tube heat exchangers for *Case Study #1* using different optimization methods

Parameter	Original ([?])	GA ([?])	PSO ([?])	ABC ([?])	BBO ([?])	DMSACO
$D_s$ (m)	0.387	0.62	0.0181	1.0024	0.55798	0.5115
$L$ (m)	4.880	1.548	1.45	2.4	1.133	1.237
$B$ (m)	0.305	0.440	0.423	0.354	0.5	0.50
$d_o$ (m)	0.019	0.016	0.0145	0.0103	0.01	0.01
$S_t$ (m)	0.023	0.020	0.0187	-	0.0125	0.0125
$C_l$ (m)	0.004	0.004	0.0036	-	0.0025	0.0025
$N_t$	160	803	894	704	1565	1471
$v_t$ (m/s)	1.76	0.68	0.74	0.36	0.898	0.9560
$Re_t$	36409	9487	9424	—	7804	8305.1474
$Pr_t$	6.2	6.2	6.2	—	6.2	0.62026
$h_t$ (W/m <sup>2</sup> K)	6558	5043	5618	4438	9180	5113.216
$f_t$	0.023	0.031	0.0314	—	0.0337	0.03314
$\Delta P_t$ (Pa)	62812	3673	4474	2046	4176	4165.91
$A_s$ (m <sup>2</sup> )	0.0236	0.0541	0.039	-	0.0558	0.05115
$D_e$ (m)	0.013	0.015	0.0103	-	0.0071	0.00711
$v_s$ (m/s)	0.94	0.41	0.375	0.12	0.398	0.62
$Re_s$	16200	8039	4814	—	3515	3834.324
$Pr_s$	5.4	5.4	5.4	—	5.4	5.3935
$h_s$ (W/m <sup>2</sup> K)	5735	3476	4088.3	5608	4911	5150.664
$f_s$	0.337	0.374	0.403	—	0.423	0.4176
$\Delta P_s$ (Pa)	67684	4365	4271	2716	5917	0.62
$U$ (W/m <sup>2</sup> K)	1471	1121	1177	1187	1384	1217.857
$A$ (m <sup>2</sup> )	46.6	62.5	59.2	54.72	55.73	57.17
$C_i$ (€)	16549	19163	18614	17893	18059	18295.74
$C_o$ (€/an)	4466	272	276	257.82	203.68	316.59
$C_{od}$ (€)	27440	1671	1696	1584.2	1251.5	1945.31
$C_{tot}$ (€)	43989	20834	20310	19478	19310	20241.06

and Hadidi and Nazari using *BBO* ([?]). The following two case studies were investigated:

TABLE 5. Parameters of the optimal shell and tube heat exchangers for *Case Study #2* using different optimization methods

Parameter	Original ([?])	GA ([?])	PSO ([?])	ABC ([?])	BBO ([?])	DMSACO
$D_s$ (m)	0.894	0.830	0.81	1.3905	0.801	0.67824
$L$ (m)	4.830	3.379	3.115	3.963	2.040	2.42787
$B$ (m)	0.356	0.500	0.424	0.4669	0.500	0.50
$d_o$ (m)	0.020	0.016	0.015	0.0104	0.010	0.010
$S_t$ (m)	0.025	0.020	0.0187	-	0.0125	0.01250
$C_l$ (m)	0.005	0.004	0.0037	-	0.0025	0.0025
$N_t$	918	1567	1658	1528	3587	2741
$v_t$ (m/s)	0.75	0.69	0.67	0.36	0.77	1.00486
$Re_t$	14925	10936	10503	—	7642.497	10000.002
$Pr_t$	5.7	5.7	5.7	—	5.7	5.6949
$h_t$ (W/m <sup>2</sup> K)	3812	3762	3721	3818	4314	5985.17
$f_t$	0.028	0.031	0.0311	—	0.034	0.03144
$\Delta P_t$ (Pa)	6251	4298	4171	3043	6156	6801.569
$A_s$ (m <sup>2</sup> )	0.0320	0.0831	0.0687	-	0.0801	0.0678
$D_e$ (m)	0.014	0.011	0.0107	-	0.007	0.0711
$v_s$ (m/s)	0.58	0.44	0.53	0.118	0.46	0.5465
$Re_s$	18381	11075	12678	—	7254.007	8571.76
$Pr_s$	5.1	5.1	5.1	—	5.1	5.082
$h_s$ (W/m <sup>2</sup> K)	1573	1740	1950.80	3396	2197	2408.26
$f_s$	0.330	0.357	0.349	—	0.379	0.3702
$\Delta P_s$ (Pa)	35789	13267	20551	8390	13799	19203.66
$U$ (W/m <sup>2</sup> K)	615	660	713.9	832	755	830.505
$A$ (m <sup>2</sup> )	278.6	262.8	243.2	-	229.95	209.109
$C_i$ (€)	51507	49259	46453	44559	44536	41510.01
$C_o$ (€/an)	2111	947	1038.7	1014.5	984	1241.93
$C_{od}$ (€)	12973	5818	6778.2	6233.8	6046	7631.18
$C_{tot}$ (€)	64480	55077	53231.1	50793	50582	49141.18

*Case Study #1:* This study used a heat exchanger with a heat load of 0.415 MW with distilled water (on shell side) and raw water (on tube side). This case study was taken from Kern ([?]). The original project assumed a heat exchanger with two passes on the tube side and one pass on shell side, and a triangular tube pitch ( $C = 0.249$  and  $n_1 = 2.207$ , see Table 1). The process parameters and physical properties for *Case Study #1* are given in Table 2 ([?]).

*Case Study #2:* This study used a heat exchanger with a heat load of 4.34 MW with methanol (on shell side) and brackish water (on tube side). This case study was taken from Sinnott et al. ([?]). The original project assumed a heat exchanger with two passes on the tube side and one pass on shell side, and a triangular tube pitch ( $C = 0.249$  and  $n_1 = 2.207$ , see Table 1). The process parameters and physical properties for *Case Study #2* are given in Table 3 ([?]).

Common parameters used for both case studies were: number of years  $n_y = 10\text{yrs}$ , annual discount rate  $i = 10\%$ , energy cost  $C_e = 0.12\text{€/kWh}$ , annual work hours  $H = 7000\text{h/yr}$ , and pumping efficiency  $\eta = 0.8$ . The constraints applied in both cases were:

$$0.1 \leq D_s \leq 1.5, \quad (45)$$

$$0.01 \leq d_o \leq 0.61, \quad (46)$$

$$0.05 \leq B \leq 0.5, \quad (47)$$

$$0.2 \leq L \leq 20, \quad (48)$$

$$0.5 \leq v_t \leq 2.5, \quad (49)$$

$$0.2 \leq v_s \leq 1.5, \quad (50)$$

$$0.2 \leq \frac{B}{D_s} \leq 1, \quad (51)$$

$$L = L_1 \quad (52)$$

The method parameters for *DMSACO* in both cases were:  $p = 3$ ,  $N = 200$ ,  $\epsilon_g = 0.001$ ,  $\alpha_v = 0.9$ ,  $N_l = 30$ ,  $\epsilon_l = 10^{-5}$ .

The comparative results for *Case Study #1* are given in Table 4 below. In *Case Study #1* we observe for method *DMSACO* better results than the ones reported for the original project, *GA* and *PSO*, but weaker than the ones reported for *ABC* and *BBO*. The number of function evaluations for method *DMSACO* in *Case Study #1* was  $NFE = 25860750$  ( $NFE/N = 129303.75$ ), and the computing time on a PC i7 computer at 3 MHZ with 4 processors (8 hyper-threads) in parallel computing was 5.792sec.

The comparative results for *Case Study #2* are given in Table 5. In *Case Study #2* we observe for method *DMSACO* better results than the ones reported for the original project, *GA*, *PSO*, *ABC* and *BBO*. The number of function evaluations for method *DMSACO* in *Case Study #2* was  $NFE = 29672220$  ( $NFE/N = 148361.1$ ), and the computing time on a PC i7 computer at 3 MHZ with 4 processors (8 hyper-threads) in parallel computing was 6.723sec.

## 7. Conclusions

This paper is proposing a new global optimization method Distributed MultiStart Ant Colony Optimization (*DMSACO*) inspired from the Multi-start global optimization methods and the Ant Colony Optimization (*ACO*) optimization methods for continuous functions. As local optimization method the *DMSACO* method uses a new proposed Particle Swarm Local Optimization (*PSLO*) method. Both new proposed methods are suitable for parallel computing implementation on networked multicore computers (clusters, grids, clouds). The *DMSACO* method converges to the global optimization solution in a controlled manner, providing the ability to pre-calculate the number of

iterations based on the required precision and the given volumetric decrease rate, in this way making possible to estimate in advance the execution time of the method. The new proposed methods are applied for solving the problem of design and economic optimization of shell-and-tube heat exchangers. The results are compared to those of other published studies the combined methods *DMSACO* and *PSLO* demonstrating a high degree of competitiveness.

## REFERENCES

- [1] *G.F. Hewitt*, Heat Exchanger Design Handbook, Begell House, New York, 1998.
- [2] *A. Hadidi, A. Nazari*, Design and economic optimization of shell-and-tube heat exchangers using biogeography-based (BBO) algorithm, Applied Thermal Engineering, **51**(2013), 1263-1272, Elsevier.
- [3] *W.M. Rosenhow, P.J. Hartnett*, Handbook of Heat Transfer, McGraw-Hill, New York, 1973.
- [4] *A.C. Caputo, P.M. Pelagagge, P. Salini*, Heat exchanger design based on economic optimization, Applied Thermal Engineering, **28**(2008), 1151-1159, Elsevier.
- [5] *V.K. Patel, R.V. Rao*, Design Optimization of shell-and-tube heat exchanger using particle swarm optimization technique, Applied Thermal Engineering, **30**(2010), 1417-1425, Elsevier.
- [6] *A.S. Sahin, B. Kilic, U. Kilic*, Design and economic optimization of shell and tube heat exchangers using Artificial Bee Colony (ABC) algorithm, Energy Conversion and Management, **52**(2011), 3356-3362.
- [7] *R.V. Rao, V. J. Savsani*, Mechanical Design Optimization Using Advanced Optimization Techniques, Springer Series in Advanced Manufacturing, Springer-Verlag London, 2012.
- [8] *V.C. Mariani, A.R.K. Duck, F.A. Guerra, L.d.S. Coelho, R.V. Rao*, A chaotic quantum-behaved particle swarm approach applied to optimization of heat exchangers, Applied Thermal Engineering, **42**(2012), 119-128, Elsevier.
- [9] *J. Kennedy, R.C. Eberhart*, Particle Swarm Optimization, Proceedings IEEE International Conference on Neural Networks, 1942-1948, Perth, Australia, 1995.
- [10] *K. Socha, M. Dorigo*, Ant Colony Optimization for Continuous Domains, European Journal of Operational Research, **185**, 1155-1173, 2008.
- [11] *M. Schlüter, J. A. Egea, J. R. Banga*, Extended Ant Colony Optimization for non-convex Mixed Integer Nonlinear Programming, Computers & Operations Research, **36**, 2217-2229, 2009.
- [12] *D.Q. Kern*, Process Heat Transfer, McGraw-Hill, New York, 1950.
- [13] *R.K. Sinnott, Coulson and Richardson*, Chemical Engineering, in Chemical Engineering Design, vol. **6**, Butterworth-Heinemann, 2005.