

## DYNAMICS OF THE TRANSLATIONAL 3-UPU PARALLEL MANIPULATOR

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*Matrix relations for dynamics analysis of a spatial translational 3-UPU parallel mechanism are established in this paper. Three identical legs connect the moving platform by universal joints. Knowing the translation motion of the platform, inverse dynamics problem is solved using an approach based on explicit equations of parallel robots dynamics, but it has been verified the results in the framework of the Lagrange equations with their multipliers. Finally, matrix equations offer expressions and graphs of simulation for the input forces and powers of three prismatic actuators.*

**Keywords:** Dynamics, Parallel mechanism, Platform, Virtual work

### 1. Introduction

Provided with closed-loop structures, the links of the parallel robots can be connected one to the other by spherical joints, universal joints, revolute joints or prismatic joints. Generally, the number of actuators is typically equal to the number of degrees of freedom and each leg is controlled at or near the fixed base [1]. Compared with traditional serial manipulators, the accuracy and precision in the direction of the tasks are essential for the parallel architectures, since the positioning errors of the tool could end in costly damage [2].

Recently, considerable efforts have been devoted to the dynamics analysis of parallel robots. The class of manipulators known as Stewart-Gough platform, used in flight simulators, focused great attention (Stewart [3], Di Gregorio and Parenti Castelli [4]). The Delta parallel robot (Clavel [5], Tsai and Stamper [6]) and the Star parallel manipulator (Hervé and Sparacino [7]) are equipped with three motors, which train on the mobile platform in a three-degrees-of-freedom general translational motion. Angeles [8], Wang and Gosselin [9] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three revolute actuators.

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In the present paper, two recursive matrix methods, already implemented in the inverse dynamics of parallel robots, are applied to the analysis of a spatial translational mechanism. It has been proved that the number of equations and computational operations reduces significantly by using a set of matrices for dynamics modelling.

## 2. Kinematics reviews

The 3-UPU parallel robot is a symmetrical structure composed of three kinematical chains of variable length with identical topology, all connecting the fixed base to the moving platform by means of universal joints. Each leg is made up of a cylinder and a piston connected together by a prismatic joint, as well as three prismatic actuators can drive the manipulator. The mechanism consists of a fixed base, a circular mobile platform and three legs with identical kinematical structure. Each limb connects the fixed base to the moving platform by two universal joints interconnected through a prismatic joint made up of a cylinder and a piston [10].

The first joint  $A_1$  ( $\alpha_A = 0$ ) is typically contained within the plane  $Ox_0y_0$ , whereas the positions of remaining joints  $B_1, C_1$  make the angles  $\alpha_B = 120^\circ$ ,  $\alpha_C = -120^\circ$  respectively, with the line  $OA_1$  of first leg. For the purpose of analysis, we assign a fixed Cartesian coordinate system  $Ox_0y_0z_0(T_0)$  at the centred point  $O$  of the fixed base platform and a mobile frame  $Gx_Gy_Gz_G$  on the mobile platform at its centre  $G$ . The angle  $\nu$  between  $Gx_G$  axis and the line  $GA_4$  is defined as the *twist* angle of the robot (Fig. 1).

The moving platform is initially located at a *central configuration*, where the platform is not translated with respect to the fixed base and the origin  $O$  of the fixed frame is located at an elevation  $OG = h$  above the mass centre  $G$ . To simplify the graphical image of the kinematical scheme of the mechanism, in what follows we will represent the intermediate reference systems by only two axes, so as is used in most of robotics papers [1], [2], [8].

The active leg  $A$ , for example, consists of a little cross of a fixed Hooke joint linked at the frame  $A_1x_1^Ay_1^Az_1^A$ , characterised by a negligible mass, which has the angular velocity  $\omega_{10}^A = \dot{\phi}_{10}^A$  and the angular acceleration  $\varepsilon_{10}^A = \ddot{\phi}_{10}^A$ , connected at a moving cylinder  $A_2x_2^Ay_2^Az_2^A$  of length  $l_2$ , mass  $m_2$  and tensor of inertia  $\hat{J}_2$ , which has a relative rotation around  $A_2z_2^A$  axis with the angle  $\varphi_{21}^A$ , so that  $\omega_{21}^A = \dot{\varphi}_{21}^A$ ,  $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$ . An actuated prismatic joint is as well as a piston of length  $l_3$ , mass  $m_3$  and tensor of inertia  $\hat{J}_3$ , linked to the  $A_3x_3^Ay_3^Az_3^A$  frame, having a relative

velocity  $v_{32}^A = \dot{\lambda}_{32}^A$  and acceleration  $\gamma_{32}^A = \dot{v}_{32}^A$ . Finally, a second little universal joint is introduced at the edge of a moving platform, which can be schematised as a circle of radius  $r$  and mass  $m_p$ .

At the central configuration, we also consider that the three sliders are initially starting from the same position  $l_1 = h / \sin \beta - l_3$  and that the angles of orientation of universal joints are given by

$$\alpha_A = 0, \alpha_B = \frac{2\pi}{3}, \alpha_C = -\frac{2\pi}{3}$$

$$\nu = \frac{\pi}{6}, (l_0 - r \cos \nu) \tan \delta = r \sin \nu, \quad r \sin \nu \tan \beta = h \sin \delta, \quad (1)$$

where  $\delta$  and  $\beta$  are two constant angles of rotation around the axes  $z_1^A$  and  $z_2^A$ .

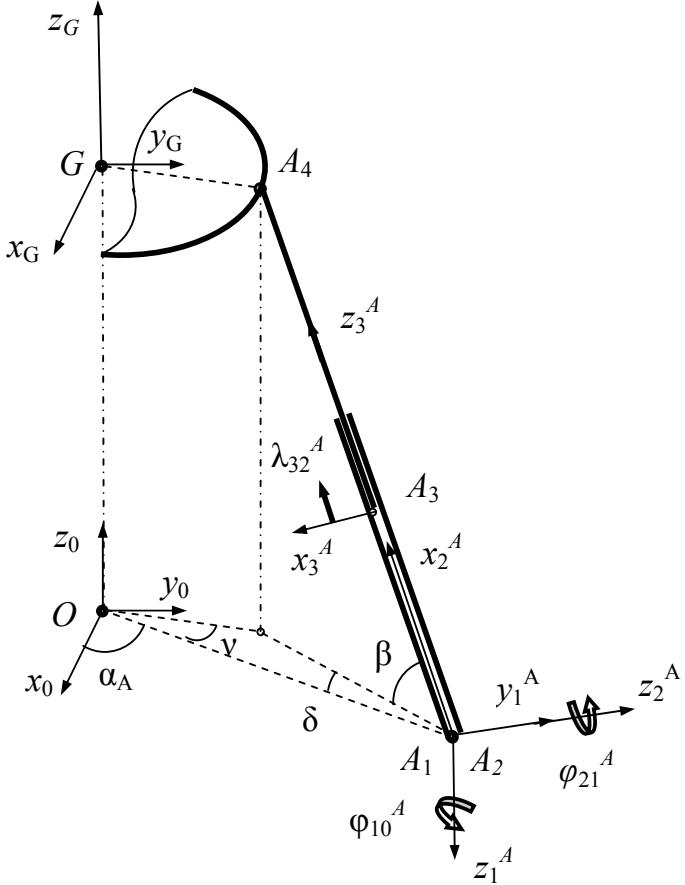


Fig. 1 Kinematical scheme of first leg  $A$  ( $\alpha_A = 0$ ) of parallel mechanism

Starting from the reference origin  $O$  and pursuing three legs  $OA_0A_1A_2A_3A_4$ ,  $OB_0B_1B_2B_3B_4$ ,  $OC_0C_1C_2C_3C_4$ , we obtain all transformation matrices

$$p_{10} = p_{10}^\varphi a_\delta \theta_3 a_\alpha^i, p_{21} = p_{21}^\varphi a_\beta \theta_1^T, p_{32} = \theta_2$$

$$p_{n0} = \prod_{\tau=1}^n p_{n-\tau+1, n-\tau} \quad (p = a, b, c), (i = A, B, C), (n = 2, 3), \quad (2)$$

where we denote the matrices:

$$p_{k,k-1}^\varphi = \text{rot}(z, \varphi_{k,k-1}^i), \quad a_\alpha^i = \text{rot}(z, \alpha_i), \quad a_\delta = \text{rot}(z, \delta)$$

$$\theta_1 = \text{rot}(x, \pi/2), \quad \theta_2 = \text{rot}(y, \pi/2), \quad \theta_3 = \text{rot}(y, \pi). \quad (3)$$

In the inverse geometric problem, the position of the mechanism is completely given through the coordinate  $x_0^G, y_0^G, z_0^G$  of the mass centre  $G$ . Consider, for example, that during three seconds the moving platform remains in the same orientation and the motion of the centre  $G$  along a *rectilinear trajectory* is expressed in the fixed frame  $Ox_0y_0z_0$  through the following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{h - z_0^G}{z_0^{G*}} = 1 - \cos \frac{\pi}{3} t. \quad (4)$$

Pursuing the kinematical modelling developed in our published paper [10], nine variables  $\varphi_{10}^A, \varphi_{21}^A, \lambda_{32}^A, \varphi_{10}^B, \varphi_{21}^B, \lambda_{32}^B, \varphi_{10}^C, \varphi_{21}^C, \lambda_{32}^C$  will be determined from the nine analytical equations.

Now, we compute the relative velocities  $\omega_{10}^i, \omega_{21}^i, v_{32}^i$  in terms of the angular velocity of the platform and velocity of centre  $G$ , starting from following *matrix conditions of connectivity* [11]:

$$\vec{V}_i = [Q_i]^{-1} \vec{P}_i, \quad (5)$$

where followings terms determines the contents of 3x3 invertible square matrix  $[Q_i]$  and the column matrix  $\vec{P}_i$ :

$$q_{j1}^i = \vec{u}_j^T \vec{p}_{10}^T \vec{u}_3 \vec{p}_{21}^T \{ \vec{r}_{32}^i + \vec{p}_{32}^T \vec{r}_{43}^i \}, \quad q_{j2}^i = \vec{u}_j^T \vec{p}_{20}^T \vec{u}_3 \{ \vec{r}_{32}^i + \vec{p}_{32}^T \vec{r}_{43}^i \}, \quad q_{j3}^i = \vec{u}_j^T \vec{p}_{20}^T \vec{u}_1 \quad (j = 1, 2, 3)$$

$$\vec{P}_i = [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^T. \quad (6)$$

Finally, the expressions of relative velocities are obtained from the column matrices

$$\vec{V}_i = [\omega_{10}^i \quad \omega_{21}^i \quad v_{32}^i]^T \quad (i = A, B, C). \quad (7)$$

Considering some independent *virtual motions* of the spatial mechanism, virtual displacements and velocities should be compatible with the virtual motions imposed by all kinematical constraints and joints at a given instant in time. Let us assume that the robot has successively three virtual motions determined by following sets of velocities:

$$\begin{aligned} v_{32a}^{Av} &= 1, v_{32a}^{Bv} = 0, v_{32a}^{Cv} = 0 \\ v_{32b}^{Av} &= 0, v_{32b}^{Bv} = 1, v_{32b}^{Cv} = 0 \\ v_{32c}^{Av} &= 0, v_{32c}^{Bv} = 0, v_{32c}^{Cv} = 1. \end{aligned} \quad (8)$$

The characteristic *virtual velocities* are expressed as functions of the pose of the mechanism at any time by the general kinematical equations (5).

Expressions of relative accelerations are obtained from the column matrix

$$\vec{\Gamma}_i = [\varepsilon_{10}^i \ \varepsilon_{21}^i \ \gamma_{32}^i]^T \quad (9)$$

using new conditions of connectivity:

$$\vec{\Gamma}_i = [Q_i]^{-1} \vec{S}_i, \quad (10)$$

where following terms determine the contents of column matrix  $\vec{S}_i = \dot{\vec{P}}_i - [\dot{Q}_i] \vec{V}_i$ :

$$\begin{aligned} s_j^i &= \vec{u}_j^T \vec{r}_0^G - \omega_{10}^i \omega_{10}^i \vec{u}_j^T p_{10}^T \vec{u}_3 \vec{u}_3^T p_{21}^T \{\vec{r}_{32}^i + p_{32}^T \vec{r}_{43}^i\} - \omega_{21}^i \omega_{21}^i \vec{u}_j^T p_{20}^T \vec{u}_3 \vec{u}_3^T \{\vec{r}_{32}^i + p_{32}^T \vec{r}_{43}^i\} - \\ &- 2\omega_{10}^i \omega_{21}^i \vec{u}_j^T p_{10}^T \vec{u}_3 p_{21}^T \vec{u}_3^T \{\vec{r}_{32}^i + p_{32}^T \vec{r}_{43}^i\} - 2\omega_{10}^i v_{32}^i \vec{u}_j^T p_{10}^T \vec{u}_3 p_{21}^T \vec{u}_1 - 2\omega_{21}^i v_{32}^i \vec{u}_j^T p_{20}^T \vec{u}_3 \vec{u}_1. \quad (11) \\ &\quad (j = 1, 2, 3) \end{aligned}$$

### 3. Inverse dynamics models

The dynamics analysis of parallel robots is complicated because the existence of a spatial kinematical structure, which possesses a large number of passive degrees of freedom, dominance of the inertial forces, frictional and gravitational components and by the problem linked to real-time control in the inverse dynamics. Considering all gravitational effects and neglecting the friction forces, the relevant objective of the inverse dynamics is to determine the input torques or forces, which must be exerted by the actuators in order to produce a given trajectory of the end-effector.

A lot of works have focused on the dynamics of Stewart platform. Dasgupta and Mruthyunjaya [12] used the Newton-Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. Tsai and Stamper [6] presented an algorithm to solve the inverse dynamics for a Delta manipulator using Newton-Euler equations and Lagrange formalism.

### 3.1 Explicit dynamics equations of parallel robot

Three independent pneumatic or hydraulic systems  $A, B, C$  that generate three input forces  $\vec{f}_{32}^A = f_{32}^A \vec{u}_3, \vec{f}_{32}^B = f_{32}^B \vec{u}_3, \vec{f}_{32}^C = f_{32}^C \vec{u}_3$ , which are oriented along the axes  $A_3 z_3^A, B_3 z_3^B, C_3 z_3^C$ , control the motion of three moving pistons of the legs.

The parallel robot can artificially be transformed in a set of three open chains  $C_i$  ( $i = A, B, C$ ) subject to the constraints. This is possible by imaginary cutting each joint for moving platform and taking its effect into account by introducing the corresponding constraint conditions.

The force of inertia and the resulting moment of inertia forces of a rigid body  $T_k^A$ , for example,

$$\vec{F}_{k0}^{inA} = -m_k^A [\vec{\gamma}_{k0}^A + (\tilde{\omega}_{k0}^A \tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A) \vec{r}_k^{CA}], \quad \vec{M}_{k0}^{inA} = -[m_k^A \tilde{r}_k^{CA} \vec{\gamma}_{k0}^A + \hat{J}_k^A \tilde{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A \hat{J}_k^A \tilde{\omega}_{k0}^A] \quad (12)$$

are determined with respect to the centre of joint  $A_k$ . On the other hand, the wrench of two vectors  $\vec{F}_k^{*A}$  and  $\vec{M}_k^{*A}$  evaluates the influence of the action of the weight  $m_k^A \vec{g}$  and of other external and internal forces applied to the same element  $T_k^A$  of the manipulator, for example:

$$\vec{F}_k^{*A} = m_k^A g a_{k0} \vec{u}_3, \quad \vec{M}_k^{*A} = m_k^A g \tilde{r}_k^{CA} a_{k0} \vec{u}_3 \quad (k = 1, 2, 3). \quad (13)$$

Pursuing the first leg  $A$ , two significant recursive relations generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A, \quad (14)$$

where one denoted

$$\vec{F}_{k0}^A = -\vec{F}_k^{inA} - \vec{F}_k^{*A}, \quad \vec{M}_{k0}^A = -\vec{M}_k^{inA} - \vec{M}_k^{*A}. \quad (15)$$

As example, starting from (14), we develop a set of recursive relations:

$$\vec{F}_3^A = \vec{F}_{30}^A, \quad \vec{F}_2^A = \vec{F}_{20}^A + a_{32}^T \vec{F}_3^A, \quad \vec{F}_1^A = \vec{F}_{10}^A + a_{21}^T \vec{F}_2^A$$

$$\vec{M}_3^A = \vec{M}_{30}^A, \quad \vec{M}_2^A = \vec{M}_{20}^A + a_{32}^T \vec{M}_3^A + \tilde{r}_{32}^A a_{32}^T \vec{F}_3^A, \quad \vec{M}_1^A = \vec{M}_{10}^A + a_{21}^T \vec{M}_2^A + \tilde{r}_{21}^A a_{21}^T \vec{F}_2^A. \quad (16)$$

Applying the explicit form of the *equations of parallel robots dynamics* [13], [14], a compact matrix relation results for the input force of first *prismatic actuator*, for example

$$\begin{aligned} f_{32}^A = & \vec{u}_3^T \{ \vec{F}_3^A + \omega_{10a}^{Av} \vec{M}_1^A + \omega_{21a}^{Av} \vec{M}_2^A + \omega_{10a}^{Bv} \vec{M}_1^B + \omega_{21a}^{Bv} \vec{M}_2^B + \\ & + \omega_{10a}^{Cv} \vec{M}_1^C + \omega_{21a}^{Cv} \vec{M}_2^C + v_{10a}^{Gv} \vec{F}_1^G + v_{21a}^{Gv} \vec{F}_2^G + v_{32a}^{Gv} \vec{F}_3^G \}. \end{aligned} \quad (17)$$

The relations (14)-(17) represent the *inverse dynamics model* of the 3-UPU parallel robot. The various dynamical effects, including the Coriolis and centrifugal forces coupling and the gravitational actions are considered in this explicit equation.

### 3.2. Equations of Lagrange

A solution of the dynamics problem of a 3-UPU parallel robot can be developed based on the Lagrange equations of second kind for a mechanical system with constraints. The generalized coordinates of the robot are represented by 12 parameters

$$\begin{aligned} q_1 &= x_0^G, q_2 = y_0^G, q_3 = z_0^G, q_4 = \varphi_{10}^A, q_5 = \varphi_{21}^A, q_6 = \lambda_{32}^A, \\ q_7 &= \varphi_{10}^B, q_8 = \varphi_{21}^B, q_9 = \lambda_{32}^B, q_{10} = \varphi_{10}^C, q_{11} = \varphi_{21}^C, q_{12} = \lambda_{32}^C. \end{aligned} \quad (18)$$

The Lagrange's equations with their nine multipliers  $\lambda_1, \lambda_2, \dots, \lambda_9$  will be expressed by 12 differential relations

$$\frac{d}{dt} \left\{ \frac{\partial L}{\partial \dot{q}_k} \right\} - \frac{\partial L}{\partial q_k} = Q_k + \sum_{s=1}^9 \lambda_s c_{sk} \quad (k = 1, 2, \dots, 12), \quad (19)$$

which contain following 12 generalized forces

$$\begin{aligned} Q_1 &= 0, Q_2 = 0, Q_3 = 0, Q_4 = 0, Q_5 = 0, Q_6 = f_{32}^A, \\ Q_7 &= 0, Q_8 = 0, Q_9 = f_{32}^B, Q_{10} = 0, Q_{11} = 0, Q_{12} = f_{32}^C, \end{aligned} \quad (20)$$

for example. A number of nine kinematical conditions of constraint are given by the relations (5):

$$\sum_{k=1}^{12} c_{sk} \dot{q}_k = 0 \quad (s = 1, 2, \dots, 9). \quad (21)$$

The components of the general expression of the Lagrange function  $L = L_p + \sum_{v=1}^3 (L_v^A + L_v^B + L_v^C)$  are expressed as analytical functions of the generalized coordinates and their first derivatives with respect to time:

$$\begin{aligned} L_p &= \frac{1}{2} m_p (\dot{x}_0^{G2} + \dot{y}_0^{G2} + \dot{z}_0^{G2}) - m_p g z_0^G, L_1^i = 0, L_2^i = \frac{1}{2} \vec{\omega}_{20}^{iT} \hat{J}_2^i \vec{\omega}_{20}^i - m_2^i g \vec{u}_3^T p_{20}^T \vec{r}_2^{Ci} \\ L_3^i &= \frac{1}{2} m_3^i \vec{v}_{30}^{iT} \vec{v}_{30}^i + \frac{1}{2} \vec{\omega}_{30}^{iT} \hat{J}_3^i \vec{\omega}_{30}^i + m_3^i \vec{v}_{30}^{iT} \vec{\omega}_{30}^i \vec{r}_3^{Ci} - m_3^i g \vec{u}_3^T \{ p_{20}^T \vec{r}_{32}^i + p_{30}^T \vec{r}_3^{Ci} \} \\ i &= (A, B, C), \quad p = (a, b, c). \end{aligned} \quad (22)$$

Angular velocities, joint's velocities, skew-symmetric matrices associated to the angular velocities and first derivatives of orthogonal matrices  $p_{k,k-1}$  are expressed as follows:

$$\begin{aligned} \vec{v}_{10}^i &= \vec{0}, \vec{v}_{20}^i = \vec{0}, \vec{v}_{30}^i = p_{32} \vec{\omega}_{20}^i \vec{r}_{32}^i + \lambda_{32}^i \vec{u}_3, \vec{\omega}_{10}^i = \dot{\varphi}_{10}^i \vec{u}_3 \\ \vec{\omega}_{20}^i &= \dot{\varphi}_{10}^i p_{21} \vec{u}_3 + \dot{\varphi}_{21}^i \vec{u}_3, \vec{\omega}_{30}^i = \dot{\varphi}_{10}^i p_{32} p_{21} \vec{u}_3 + \dot{\varphi}_{21}^i p_{32} \vec{u}_3 \\ \vec{\omega}_{10}^i &= \dot{\varphi}_{10}^i \vec{u}_3, \vec{\omega}_{20}^i = \dot{\varphi}_{10}^i p_{21} \vec{u}_3 p_{21}^T + \dot{\varphi}_{21}^i \vec{u}_3, \vec{\omega}_{30}^i = \dot{\varphi}_{10}^i p_{32} p_{21} \vec{u}_3 p_{21}^T p_{32}^T + \dot{\varphi}_{21}^i p_{32} \vec{u}_3 p_{32}^T \\ \dot{p}_{k,k-1} &= \dot{\varphi}_{k,k-1}^i \vec{u}_3^T p_{k,k-1}, \quad \dot{p}_{k,k-1}^T = \dot{\varphi}_{k,k-1}^i p_{k,k-1}^T \vec{u}_3, \quad \frac{\partial p_{k,k-1}}{\partial \varphi_{k,k-1}^i} = \vec{u}_3^T p_{k,k-1}, \quad \frac{\partial p_{k,k-1}^T}{\partial \varphi_{k,k-1}^i} = p_{k,k-1}^T \vec{u}_3 \end{aligned}$$

$$i = (A, B, C), p = (a, b, c). \quad (23)$$

A long calculus of the derivatives with respect to time  $\frac{d}{dt}\{\frac{\partial L}{\partial \dot{q}_k}\}$  ( $k = 1, 2, \dots, 12$ ) of some above functions leads to a system of 12 relations.

In the direct or inverse dynamics problem, after elimination of the nine multipliers, finally we obtain same expressions (17) for the three input forces.

As application let us consider same spatial parallel robot 3-UPU analysed in [10], which has the following geometrical and mechanical characteristics:

$$x_0^{G*} = 0.05 \text{ m}, y_0^{G*} = 0.05 \text{ m}, z_0^{G*} = 0.15 \text{ m}$$

$$m_2 = 1 \text{ kg}, m_3 = 0.75 \text{ kg}, m_p = 5 \text{ kg}$$

$$r = 0.2 \text{ m}, OA_1 = l_0 = 0.6 \text{ m}, l_2 = 1 \text{ m}, A_3A_4 = l_3 = 0.6 \text{ m}, h = 0.8 \text{ m}, \Delta t = 3 \text{ s}$$

$$\hat{J}_2 = \begin{bmatrix} 0.01 & & \\ & 0.325 & \\ & & 0.325 \end{bmatrix}, \hat{J}_3 = \begin{bmatrix} 0.325 & & \\ & 0.325 & \\ & & 0.01 \end{bmatrix}.$$

Using MATLAB software, a computer program was developed to solve the dynamics of the 3-UPU parallel robot. To develop the algorithm, it is assumed that the platform starts at rest from a central configuration and moves pursuing successively rectilinear translations. Furthermore, at the initial location, the moving platform is assumed to be located  $h = 0.8 \text{ m}$  lower the fixed base, namely  $t = 0$ :  $x_0^G = 0, y_0^G = 0, z_0^G = 0.8 \text{ m}$ .

Two examples are solved to illustrate the algorithm.

For the first example, the platform moves along the *vertical direction*  $z_0$  with variable acceleration while all the other positional parameters are held equal to zero. As can be seen from Fig. 2 and Fig. 3 it is proved to be true that all active forces and powers of three actuators are permanently equal to one another.

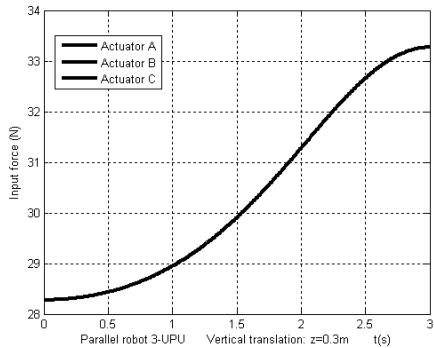


Fig. 2 Input forces of three actuators

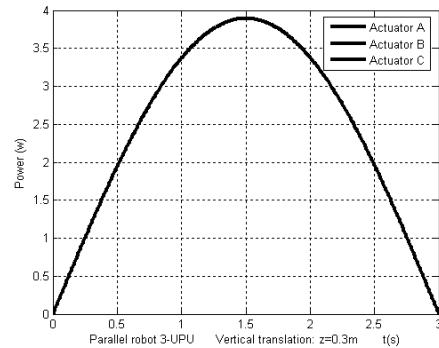


Fig. 3 Powers of three actuators

For the case when the platform's centre  $G$  moves along a *rectilinear horizontal trajectory* without any rotation of the platform, the graphs are plotted and illustrated in Fig. 4 and Fig. 5.

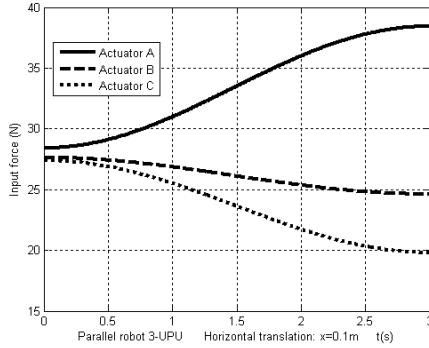


Fig. 4 Input forces of three actuators

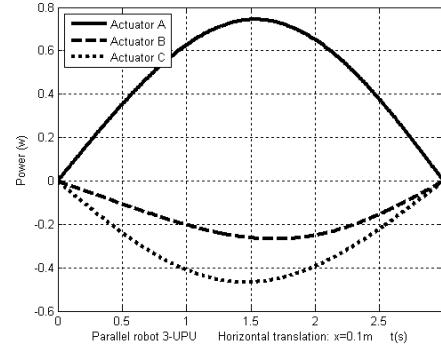


Fig. 5 Powers of three actuators

#### 4. Conclusions

Some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in the present paper. The dynamics models take into consideration the masses and forces of inertia introduced by all component elements of the parallel mechanism. The approach based on explicit equations of parallel robots dynamics can eliminate all forces of internal joints and establishes a direct determination of the time-history evolution of active forces and powers required by the actuators. The analytical calculations involved in the Lagrange formalism are very tedious, thus presenting an elevated risk of errors. The simulation certifies that one of the major advantages of the current matrix recursive formulation is the accuracy and a smaller processing time for the numerical computation.

Choosing the appropriate serial kinematical circuits connecting many moving platforms, the present method can be easily applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, with increased number of components of the mechanisms.

#### R E F E R E N C E S

- [1] *Tsai, L-W.*, Robot analysis: the mechanics of serial and parallel manipulator, Wiley, 1999
- [2] *Merlet, J-P.*, Parallel robots, Kluwer Academic, 2000
- [3] *Stewart, D.*, A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 180, pp. 371-378, 1965

- [4] *Di Gregorio, R., Parenti Castelli, V.*, Dynamics of a class of parallel wrists, ASME Journal of Mechanical Design, 126, 3, pp. 436-441, 2004
- [5] *Clavel, R.*, Delta: a fast robot with parallel geometry, Proceedings of 18<sup>th</sup> International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988
- [6] *Tsai, L-W., Stamper, R.*, A parallel manipulator with only translational degrees of freedom, ASME Design Engineering Technical Conferences, Irvine, CA, 1996
- [7] *Hervé, J-M., Sparacino, F.*, Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp.176-183, 1992
- [8] *Angeles, J.*, Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer, 2002
- [9] *Wang, J., Gosselin, C.*, A new approach for the dynamic analysis of parallel manipulators, Multibody System Dynamics, Springer, 2, 3, pp. 317-334, 1998
- [10] *Staicu, S., Popa, C.*, Kinematics of the spatial 3-UPU parallel robot, UPB Scientific Bulletin, Series D: Mechanical Engineering, 75, 3, pp. 9-18, 2013
- [11] *Staicu, S.*, Méthodes matricielles en cinématique des mécanismes, UPB Scientific Bulletin, Series D: Mechanical Engineering, 62, 1, pp. 3-10, 2000
- [12] *Dasgupta, B., Mruthunjaya, T.S.*, A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator, Mechanism and Machine Theory, Elsevier, 34, pp. 711-725, 1998.
- [13] *Staicu, S., Liu, X-J., Li, J.*, Explicit dynamics equations of the constrained robotic systems, Nonlinear Dynamics, Springer, 58, 1-2, pp. 217-235, 2009
- [14] *Staicu, S.*, Modèle dynamique en robotique, UPB Scientific Bulletin, Series D: Mechanical Engineering, 61, 3-4, pp. 5-19, 1999