

## **METHODS FOR GREEN TIMES ALLOCATION IN UNDER-SATURATED SIGNALIZED INTERSECTION**

Mircea Augustin ROSCA<sup>1</sup>, Aura RUSCA<sup>2</sup>

*În lucrare sunt analizate metode de alocare a duratelor de verde diferitelor combinații de mișcări ale traficului de autovehicule într-o intersecție izolată pentru optimizarea structurii ciclului de semaforizare. Două dintre aceste metode sunt legate în mod explicit de o funcție de optimizare - întârzierea minimă la nivelul întregii intersecții și suma minimă a gradelor de saturație corespunzătoare direcțiilor ce acced în intersecție (ambele asigurând condiții de optim social). De asemenea, sunt examineate și metode care impun anumite condiții: întârziere egală între utilizatorii (echitate) și grade de saturare egale ale mișcărilor critice corespunzătoare fazelor de semaforizare. Folosind metode uzuale bazate pe raportul debit/capacitate de servire  $q/c$  al mișcărilor critice, precum și pe cele care iau în considerare întârzierile mișcărilor individuale prin intersecție se studiază dependența funcțiilor obiectiv de mărimea duratelor de verde și a modului în care acestea pot atinge o valoare optimă.*

*In this paper, we intend to analyze some methods of allocation of green times for various combinations of vehicles traffic movements in an isolated intersection in order to optimize the traffic signal timing. Two of the studied methods are explicitly linked to an optimization function - minimum delay across intersections and minimum sum of saturation degrees corresponding to the intersection's approach legs (both of them ensuring the social optimal condition). Moreover, certain methods are being analyzed which impose some conditions: equal delay between users (equity) and equal saturation degrees of the critical movements corresponding to traffic light phases. Using usual methods based on the flow to capacity ratio  $q/c$  of critical movements, and those that take into account individual movements delays through intersection, we study objective functions dependence on the size of green times and also the way how they reach an optimal value.*

**Keywords:** traffic congestion, green times allocation, critical movement

### **1. Introduction**

Traffic congestion is one of the major drawbacks of modern life, eager to satisfy the need for unhindered mobility. It is a price that people pay for the various benefits derived from agglomeration of population and economic activity in a dense urban area, in case that there is a weak correlation between land use and transportation in a land development strategy.

<sup>1</sup> Assist., University POLITEHNICA of Bucharest, Romania, e-mail: [augustin.rosca@gmail.com](mailto:augustin.rosca@gmail.com)

<sup>2</sup> PhD Student, University POLITEHNICA of Bucharest, Romania

Most of the various measures against congestion attempt to reduce the transport infrastructure demand/supply ratio. We mention here the following measures:

- Reshaping/ modernizing/ development of the urban network infrastructure;
- An efficient use of the existent network infrastructure;
- Transport Demand Management, meaning the social mobility diminishing (the number of vehicle-km on the existent infrastructure) and also traffic peaks.

There are multiple difficulties to ensure a continuous increase of transport infrastructure capacity, according to the car user needs, as follows: the urban land is scarce and there is no space for new development; the road infrastructure works are very expensive, especially in a historical urban area, where a lot of cultural heritage have to be preserved. Even if temporarily, by developing the infrastructure, the regime of free flow would be reached, because of the generating effects on the car traffic, the congestion will be back inevitably, even worst in a certain time, [1].

A better use of the existent infrastructure includes many familiar traffic management techniques such as those designed to minimize capacity-reducing factors (for example, turning, parking and loading regulations) or to maximize the use of existing road networks (for example, improved traffic signal control).

Traffic signal controls are implemented for reducing or eliminating conflicts at intersections. Signals accomplish this by allocating green times among the various users at the intersections. Signal controls vary from simple methods, which determine the timing settings on a time-of-day/day-of-week basis, to complex algorithms, which calculate the green time allocation in real time based on traffic volumes.

In the 1950s, Webster conducted a series of experiments on an isolated fixed-time intersection. Two traffic signal timing strategies came from his study. One is signal phase splits. Webster demonstrated, both theoretically and experimentally, that pre-timed signals should have their critical phases timed for the equal degrees of saturation for a given cycle length to minimize the delay. The other strategy of study was related to the minimum delay cycle length equation. In this case, Webster's study assumed that the effective green times of the phases were in relation with their respective flow ratios, [2].

Papageorgiou, [3], presented three possibilities for influencing traffic conditions via traffic lights operation at isolated intersection:

- Stage specification: the specification of the optimal number and constitution of stages can have a major impact on intersection capacity and efficiency (especially for complex intersections).

- Split cycle: the relative green duration of each stage (as a part of the cycle time) that should be optimized according to the traffic flow.
- Cycle time: longer cycle times increase the intersection capacity (the proportion of the constant lost times becomes smaller); on the other hand, longer cycle times may increase vehicle delays in under-saturated intersections (longer waiting times during the red phase).

For a single intersection, there are two main fixed-time strategies, stage-based and phase-based. The first strategies determine the optimal split and cycle times. The second strategies determine also the optimal staging, which may be an important feature for complex intersections [4]. Well known examples of stage-based strategies are SIGSET [5] (which seeks to minimize the total delay) and SIGCAP [6] (which seeks to maximize the intersection's capacity) proposed by Allsop in 1971 and 1976. Phase-based approaches [7] solve a similar problem, suitably extended to consider different staging combinations in order to optimize the total delay or system capacity.

In other studies, minimizing total delay for all vehicles is proposed as the optimal solution to the signal timing problem, often in combination with other measures such as stops and fuel consumption minimizing. Many signal timing models offer this type of optimization [8].

In this paper, we study several methods. Two methods are explicitly linked to the optimization function such as: minimum delay across intersections and minimum sum of saturation degrees corresponding to the directions that accede to the intersection (both associated with a social optimal). The other two impose certain conditions, on one side - the equal delay between users (equity) and on the other side – the equal saturation degrees of the critical movements corresponding to traffic light phases. Using usual methods based on the flow to capacity ratio  $q/c$  of critical movements, and those that take into account individual movements delays through intersection, we study the objective functions dependence on the size of green times and also the strategy to obtain an optimal value.

## 2. Model of traffic flow in intersection

During this paper we use the terms and notations from Table 1.

Table 1

### List of terms and notations used in this article

$c_i$   $s_i T_{g_i} / T_C$  = serving capacity of traffic lane  $i$  during the allocated green phase (equivalent vehicle/h)

$d_1$  uniform delay (s/equivalent vehicles)

$d_2$  random delay which takes into account the appearance of queues for a certain period (s/ equivalent vehicles)

$d_3$  residual demand delay which takes into account a possible traffic congestion previously to the period of analysis (s/Equivalent vehicles)  
 $d_i$  average delay per vehicle (s/Equivalent vehicles)  
 $f_p$  adjustment factor for group arrivals, during the green phase  
 $I$  adjustment factor that takes into account the upstream intersections  
 $k$  incremental delay factor that takes into account the type of signal controller settings  
 $k_f$  adjustment factor for coordinated traffic systems  
 $\lambda_i$   $T_{g_i}/T_C$  = the proportion of the cycle allocated to vehicles motion during the green phase  
 $q$  arrivals flow (Equivalent vehicles/h)  
 $q_v$  flow on the period corresponding to green phase (Equivalent vehicles/h)  
 $s_i$  saturation flow corresponding to traffic lane  $i$  (Equivalent vehicle/h)  
 $T$  period of analysis (evaluation) (h)  
 $t_1$  duration for the motion of the queue (start-up lost)  
 $t_2$  duration to clear the intersection (clearance lost)  
 $T_C$  traffic signal cycle length (s)  
 $T_{gi}$  effective duration of green phase  $i$  (s)  
 $T_p$  time lost for the beginning of movement of queue ( $t_1$ ) and for the clearance of the intersection ( $t_2$ )  
 $T_y$  duration of yellow phase (s)  
 $x_i$   $q_i/c_i$  = degree of saturation corresponding to traffic lane  $i$   
 $y_i$   $y_i = q_i/s_i$  flow ratio corresponding to the group of traffic lanes  $i$   
 $Y$  sum of critical lane group flow ratios

We define the delay of an approach vehicle to a signalized intersection as the difference between travel time for crossing the intersection and travel time of the same vehicle when the traffic signal is absent.

In Fig. 1, we give an example, in order to study the trajectories of successively arriving vehicles at a signalized intersection. In the first part of the diagram of space-time coordinates, the trajectories of successively vehicles arriving from a direction at signalized intersection are depicted.

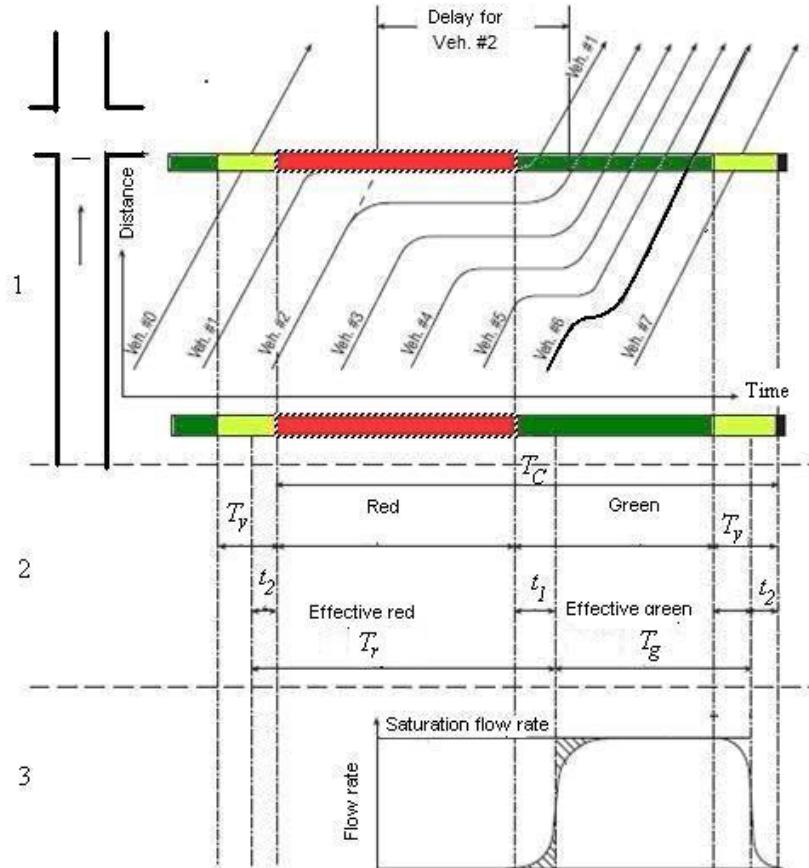


Fig. 1. The trajectories of vehicles passing through a signalized intersection [8]

The vehicles numbered from 1 to 5 record delays because of braking, stationary and speeding up phases. The vehicle number 6, which reaches the intersection almost at the same time with the queue dissipation, has only braking and acceleration delays, and is forced to reduce speed to maintain a safe distance from vehicles ahead. The last vehicle entering the intersection before the start of red traffic light phase has no delays; it joins the squad cars that are already moving at maximum speed. The second part of the diagram shows the information of electric traffic lights corresponding to one direction and the durations:  $t_1$  for the motion of the queue,  $t_2$  to clear the intersection (to avoid any possibility of conflict of vehicles which entered the intersection and should evacuate it during that phase, respectively  $T_y - t_2$  which represent the extending effect of green phase into the yellow one. The traffic lights operating cycle is defined as the time between two successive appearances of the same indication of traffic signal lights.

Thus, cycle is composed of effective green and effective red period. Effective green period corresponds to vehicle motion at the saturation flow.

Traffic lights cycle length is given by:

$$T_C = \sum_{i=1}^n T_{g_i} + T_p \quad (1)$$

$$\sum_{i=1}^n T_{g_i} = T_C - T_p \quad (2)$$

Thus, we obtain [9]:

$$\sum_{i=1}^n \lambda_i = \sum_{i=1}^n T_{g_i} / T_C = 1 - T_p / T_C \quad (3)$$

So that all requests for traffic to be properly served on the traffic lights cycle length, the next inequality must be satisfied [10]:

$$\sum_{i=1}^n q_i T_C \leq \sum_{i=1}^n T_{g_i} s_i \quad (4)$$

$$\sum_{i=1}^n y_i \leq \sum_{i=1}^n \lambda_i \quad (5)$$

If we have the equation (3), the next inequality (5) can determine the allowable domain for the variation of parameters  $\lambda_i$ .

For an intersection with two phases and for a given traffic demand  $(q_1, q_2)$ , feasible values  $(\lambda_1, \lambda_2)$  are situated in the shaded area (Fig. 2).

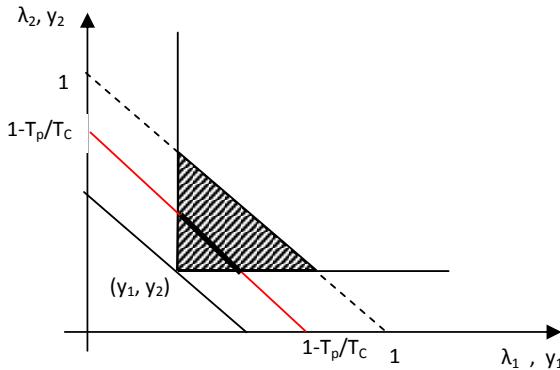


Fig. 2. The dependency  $(\lambda_1, \lambda_2)$  for a given traffic demand  $(q_1, q_2)$  (authors)

If the value of cycle time  $T_C$  is set, then  $(\lambda_1, \lambda_2)$  are situated on the thickened segment that satisfies the relation (3). Also, relations (3) and (5) set the minimum value for traffic signal cycle length [11]:

$$T_{C_{\min}} = \frac{T_p}{1 - \sum_{i=1}^n y_i} \quad (6)$$

This expression gives the minimum cycle length necessary for the intersection to operate at an acceptable level, but it does not necessarily minimize the average vehicle delay.

### 3. Methods for allocating green period on traffic lights phases

The improvement of urban traffic can be achieved by optimizing traffic lights into intersections in terms to set the signal cycle length and to allocate green periods for different movements or various combinations of movements for traffic vehicles from intersection approaches. For more accurate optimization, there are considered four methods of which two are explicitly linked to a optimization function - that point out minimum delay for the entire intersection and the minimum sum of saturation degrees corresponding to directions that accede to intersection (both associated with a social optimum) and the other remaining two require certain conditions - equal delay between users (fairness) and equal saturation degrees of the critical movements corresponding to traffic signal phases. Further, in order to compare these methods, we considered a simplified case of two-direction traffic intersection. The signalized intersection has two phases and only the passage ahead of vehicles is being allowed.

#### 3.1. Equal degrees of saturation of critical movements corresponding to traffic signal phases

In the case of the crossing of two traffic flows, the referred condition  $x_i = x_j$  becomes:

$$\frac{q_i}{s_i \lambda_i} = \frac{q_j}{s_j \lambda_j} \quad (7)$$

If we assume that saturation flows for the two directions is the same ( $s$ ) then:

$$\lambda_i / \lambda_j = y_i / y_j \quad (8)$$

By replacing in (8) the expression (3), we obtain a first relationship between the green periods of different phases of traffic signal cycle:

$$\lambda_i = \frac{y_i}{\sum y_i} \left(1 - \frac{T_p}{T_C}\right) \quad (9)$$

In the case of the intersection between two traffic flows, of which one is dominant in relation to the other, one can allocate the minimum green time period to the secondary road and assigning the remaining green time period to the main road. This implies that the saturation degree for the secondary road is  $x_2 = 1$ .

We obtain:

$$\begin{aligned}\lambda_1 &= 1 - T_p / T_C - \lambda_2 \\ \lambda_2 &= y_2\end{aligned}\tag{10}$$

These equations (10) represent a particular case of equations (9).

### 3.2. Minimum average delay per vehicle

The average delay of vehicles for passing through the intersection is:

$$d = \frac{\sum d_i q_i}{\sum q_i}\tag{11}$$

The first approximate relationship for determination of the average delay of a vehicle in a signalized intersection was obtained by Webster [12] through a combination of theoretical approaches and numerical simulation.

$$d_i = \frac{T_C (1 - \lambda_i)^2}{2(1 - \lambda_i x_i)} + \frac{x_i^2}{2q_i(1 - x_i)} - 0.65 \left( \frac{T_C}{q_i^2} \right)^{1/3} x_i^{2+5\lambda_i}\tag{12}$$

The first term of expression (12) is the deterministic component of the delay assuming uniform arrivals and service times, i.e., a queuing system of the type D/D/1. The second is the stochastic component of the delay. The last term is a correction factor based on the results of some simulations and represent 10% of the sum of the first two terms. Thus,

$$d_i = 0.90 \left( \frac{T_C (1 - \lambda_i)^2}{2(1 - \lambda_i x_i)} + \frac{x_i^2}{2q_i(1 - x_i)} \right)\tag{13}$$

Highway Capacity Manual 2000 [8] estimated the average delay by:

$$d_i = k_f d_1 + d_2 + d_3\tag{14}$$

$$d_1 = \frac{T_C}{2} \frac{(1 - T_{g_i} / T_C)^2}{1 - \min(1, x_i) T_{g_i} / T_C}\tag{15}$$

$$d_2 = 900T \left[ (x_i - 1) + \sqrt{(x_i - 1)^2 + \frac{8kIx}{cT}} \right]\tag{16}$$

$$k_f = \frac{(1 - q_v / q) f_p}{1 - T_{g_i} / T_C}\tag{17}$$

where,

- $I$  adjustment factor that takes into account the upstream intersections ( its value is 1 for isolated intersections)
- $k$  incremental delay factor that takes into account the type of signal controller settings (value is 0.5 for isolated intersections)

To minimize the delay (11) we must consider the aim of determining the optimal values of green times ( $\lambda_1, \lambda_2, \dots$ ) that correspond to various demands ( $y_1, y_2, \dots$ ). Thus, in the expressions given above for  $d_i$ , the saturation degrees  $x_i$  and traffic flows  $q_i$  must be seen as functions of relevant variables:  $x_i \rightarrow y_i/\lambda_i$ ,  $q_i \rightarrow y_i s_i$ . As a result, for example,  $d_i$  given by (13) as a function of ( $\lambda_i, y_i$ ) becomes:

$$d_i = d_i(\lambda_i, y_i) = 0.9 \left( \frac{T_C(1-\lambda_i)^2}{2(1-y_i)} + \frac{y_i}{2s_i(\lambda_i - y_i)\lambda_i} \right) \quad (18)$$

In the case of intersections with two flows, in view of relation (3), the minimizing function (11) becomes:

$$d = \frac{s_1 y_1}{s_1 y_1 + s_2 y_2} d_1(\lambda_1, y_1) + \frac{s_2 y_2}{s_1 y_1 + s_2 y_2} d_1(1-\lambda_1 - T_p/T_C, y_2) \quad (19)$$

The value of  $\lambda_1$  which accomplish the minimum average delay for vehicles is given by the equation:

$$\frac{\partial d}{\partial \lambda_1} = 0 \quad (20)$$

Based on equations (20) and (3), we obtain the proportion of the cycle allocated to phases  $\lambda_1, \lambda_2$  which minimize the average delay per vehicle that transit through intersection.

### 3.3. Equal average delay for users (equity)

The appropriate allocation of green times corresponding to saturation degrees of critical movements (method 3.1), originally proposed by Webster, can lead to huge delays on directions with low traffic volume. Establishing some green periods which have as effect an average delay equal between users requires satisfying the next sequence of equations:

$$d_1 = d_2 = \dots = d_i = \dots$$

where  $d_i$  represents, as above, the average delay per vehicle (s/equivalent vehicles) in the direction  $i$ . Of course, as above, the average delay must be expressed accordingly with relevant variables, respectively  $d_i = d_i(\lambda_i, y_i)$ .

In the case of a two phases traffic lights we have to solve a single equation:

$$d_1(\lambda_1, y_1) = d_2(1 - \lambda_1 - T_p/T_C, y_2) \quad (21)$$

which determines  $\lambda_1$  and then  $\lambda_2 = 1 - T_p/T_C - \lambda_1$  (at fixed flow ratios  $(y_1, y_2)$ ).

In section 4, we give an example in order to determine the values  $(\lambda_1, \lambda_2)$  and to reveal their dependence on  $(y_1, y_2)$ .

Consequently, we may obtain maximum threshold for the average delay which must not be exceeded in case of direction with lowest traffic volume. This threshold is chosen accordingly to green times.

### 3.4. Minimum sum of degrees of saturation

We propose here to choose values  $(\lambda_1, \lambda_2)$  using the minimum sum of degrees of saturation (22) for all directions getting into intersection.

$$\min \sum x_i \quad (22)$$

In the case of two phase's traffic lights and with the relevant variables, the previous relationship becomes:

$$\min\left(\frac{y_1}{\lambda_1} + \frac{y_2}{\lambda_2}\right) \quad (23)$$

$$\min\left(\frac{y_1}{\lambda_1} + \frac{y_2}{1 - \lambda_1 - T_p/T_C}\right) \quad (24)$$

Derivation of the sum for degrees of saturation (24) in relation with  $\lambda_1$  establishes the relationship of dependence between  $\lambda_1$  and  $\lambda_2$  (26).

$$-\frac{y_1}{\lambda_1^2} + \frac{y_2}{(1 - \lambda_1 - T_p/T_C)^2} = 0 \quad (25)$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{y_1}{y_2}} \quad (26)$$

Equations (26) and (3) offer the cycle split accordingly to minimum sum of degrees of saturation.

## 4. Example

We perform here a comparison between the methods presented above for determining the green time of traffic signal phases, in case of a signalized intersection between two directions with two-phases (Figure 3). The duration of cycle is considered as  $T_C = 80$ s, lost time per cycle is considered as  $T_p = 8$ s and the sum of critical lane group flow ratios  $Y = 0.8$ .

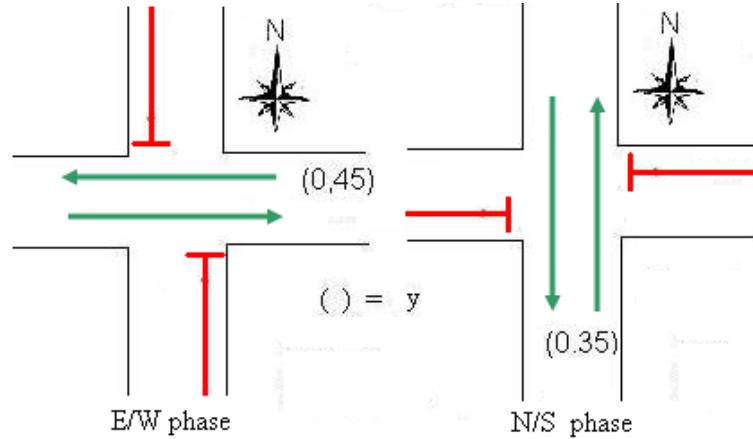
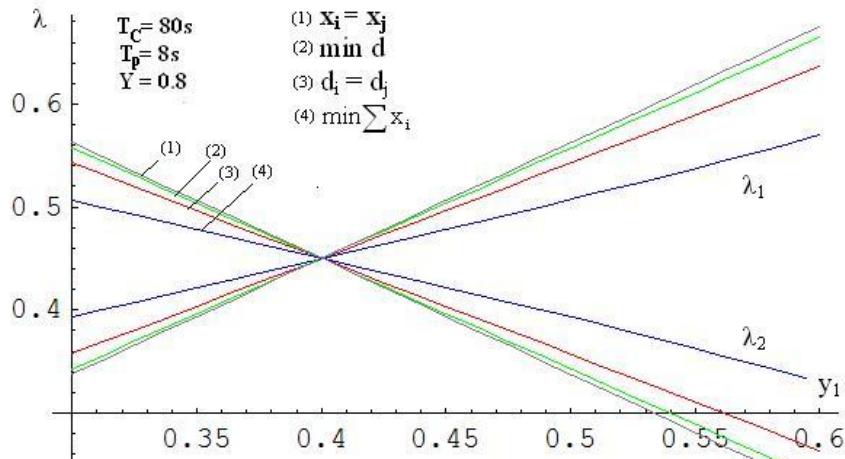


Fig. 3. Cross-intersection with two phases (authors)

The cycle time was taken close to the optimal value  $C_{opt} = (1,5T_p + 5)/Y = 85s$  [12] and it is considered that the two arteries N/S, E/W have the same characteristics, so we have  $s_1 = s_2 = 0,5 \text{ vehicles/s}$ . These are the conditions for the  $\lambda_1$ , respectively  $\lambda_2 = 1 - \lambda_1 - T_p/T_C$  determination, in case of flow ratio  $y_1 \in [0.2, 0.7]$ . For all four cases, the corresponding graphs are drawn in Figure 4 with the software Mathematica 8 [13].

Those two arteries were assumed to have the same features ( $s_1 = s_2$ ) and hence it results that: regardless the methods we are using at equal flow ratio  $y_1 = y_2$ , the values  $\lambda_1 = \lambda_2$  (in our case 0.4 respectively 0.45); this result is observed in both Figs. 4 and 5.

Fig. 4. The dependency  $\lambda_1$  and  $\lambda_2$  in relation with the flow ratio on a direction, for the methods 3.1-3.4 (authors)

It can be noticed that in Fig. 4,  $\lambda_1, \lambda_2$  seem to depend linearly on  $y_1$  for all four methods. For the first method (equalizing degrees of saturation) we have linear dependence due to relation (9).

In the case that the minimum sum of degree of saturation method is considered,  $\min (x_1+x_2)$ , the resemblance of linear dependence is caused by the fact that the linear approximation of  $\lambda_1(y_1)$  around the point of  $y_1 = y_2 = Y/2$  was appropriately chosen.

Indeed from equation (25) we have:

$$\lambda_1(y_1) = (1 - T_p/T_C) \frac{\sqrt{y_1}}{\sqrt{y_1} + \sqrt{Y - y_1}} \quad (27)$$

which developed around  $Y/2$  leads to:

$$\lambda_1(y_1) = (1 - T_p/T_C) \left[ 1 + \frac{1}{Y} (y_1 - \frac{Y}{2}) + \frac{1}{Y^3} (y_1 - \frac{Y}{2})^3 + \dots \right] \quad (28)$$

The absence of quadratic term makes the linear approximation to be adequate: in our case, it has the following expression

$$\lambda_{1lin}(y_1) = 0.45 + 0.5625(y_1 - 0.4) \quad (29)$$

The numerical evaluation shows that the variation from the exact value is less than .0005 (meaning 0.05%) on the interval [0.2, 0.7]. However, for linear behavior of the other two methods we don't have yet an explanation.

In order to study the asymmetry of  $(\lambda_1, \lambda_2)$  in figure 5 we represent the ratio  $\lambda_1/\lambda_2$  as a function of  $\lambda_1$ ; we note that we obtain the same behavior and even similar values for all four methods.

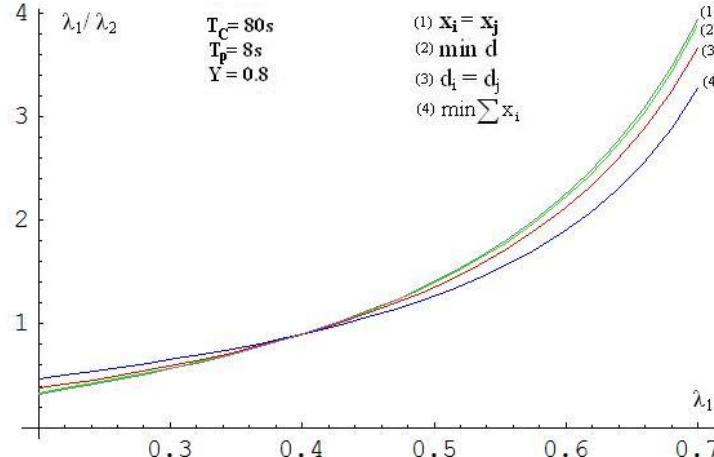


Fig. 5. The dependency  $\lambda_1/\lambda_2$  in relation with  $\lambda_1$  for the methods 2.1-2.4 (authors)

The delay is the most important variable for users, and that is the way we represent in Fig. 6 for all four methods for comparison reason. The lowest values are obtained using the second method (minimum average delay), as it is expected. The similar values are obtained using the first (equal saturation degree).

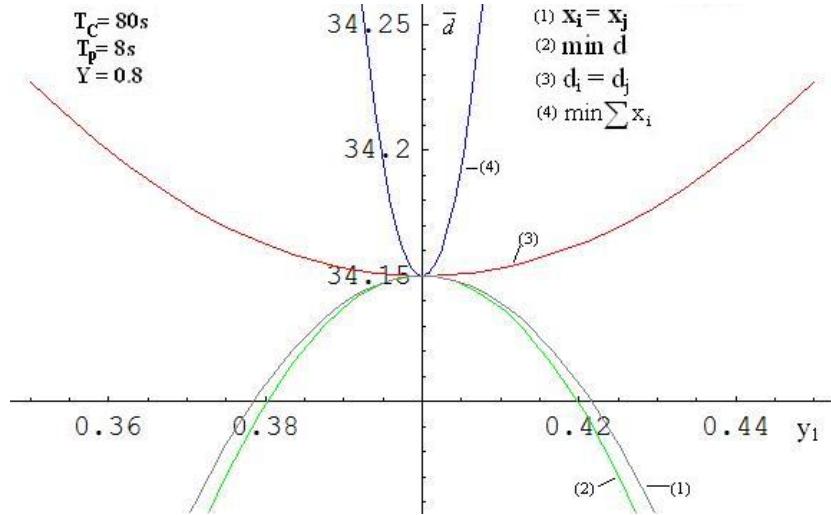


Fig. 6. The average duration in relation with flow ratio in one direction (authors)

The other two methods give more unsatisfactory results, but for flow ratios contained in the interval  $[0.35, 0.45]$  all four methods are giving values between 34s and 34.5s, that means practically similar values. Even for the small or higher flow ratios, except for the third method (equal delays, equity), the other methods give similar values. The first method is preferable against the other methods because it gives values very close to the second method's values and it is easier for implementation.

## 5. Conclusions

Mobility needs of all socio-economic activities in a territory are leading to the development of transport and traffic flows on transport infrastructure networks[1].

Traffic congestion can vary since demand (day of week, time of day, season, recreational, special events, evacuations, special events) and capacity (incidents, work zones, weather) are changing [14].

Unlike newly built urban areas where traffic optimization can be considered since at the design stage, in the old urban areas (usually central places) the existent infrastructure is drastically limiting the measures that could be taken

to improve traffic. In fact, with few exceptions, are accessible only measures for traffic control are accessible. An essential component of traffic control is the optimization of traffic lights intersections in terms of the duration of cycle and allocation of green times for different movements or various combinations for traffic movements of cars from one access direction into intersection.

The comparison made between different methods to allocate green time into an intersection in an unsaturated regime shows that a method giving better results does not exist, but the first method (equalization of degrees of saturation) is easy to implement and gives very similar results to the second method (minimum delay) which is preferably to use because it corresponds to a social optimum and has direct significance for car users.

## B I B L I O G R A P H Y

- [1] *S.Raicu*, Sisteme de Transport (Transport Systems), Ed. AGIR, 2007
- [2] *D.Cheng, , C.J. Messer, Z. Tian, and J. Liu*, Modification of Webster's Minimum Delay Cycle Length Equation Based on HCM 2000, the 81st Annual Meeting of the Transportation Research Board in Washington, D.C., 2003
- [3] *M. Papageorgiou, C.Diakaki, V.Dinopoulou, A.Kotsialos, Y.Wang*, Review of Road Traffic Control Strategies, Proceeding of the IEEE, vol.91, 2003, pp.2043-2067.
- [4] *M. C. Fu, W. C. Howell* Application of perturbation analysis to traffic light signal timing. in: Proceedings of IEEE Conference on Decision and Control, 2003, pp. 4837-4840.
- [5] *R. B.Allsop*, SIGSET: A computer program for calculating traffic capacity of signal controlled road junctions, Traffic Engineering & Control, 1971, vol. 12, pp. 58-60.
- [6] *R. B.Allsop*, SIGCAP: A computer program for assessing the traffic capacity of signal controlled road junctions, Traffic Engineering & Control, 1976, vol. 17, pp.338-341.
- [7] *G.Improta, G. E. Cantarella*, Control systems design for an individual signalized junction, Transportation Research B, 1984, vol. 18, pp. 147-167.
- [8] *Transportation Research Board*, Highway Capacity Manual, National Research Council, Washington D.C., 2000
- [9] *C.F.Daganzo*, Fundamentals of Transportation and Traffic Operations, Pergamon-Elsevier, Oxford, U.K., 1997
- [10] *R. Akçelik*, Traffic Signals: Capacity and Timing Analysis, Australian Road Research Board (ARRB) Research Report 123, Victoria, Australia, 1981.
- [11] *N. H.Gartner*, Area traffic control and network equilibrium in Traffic Equilibrium Methods, (Editor: M.A. Florian), Lecture Notes in Economics and Mathematical Systems, Vol. 118, Springer-Verlag, pp. 274-297, 1976
- [12] *F.V.Webster, B. M. Cobbe*, Traffic Signals, Road Research Technical Paper No. 56. HMSO London UK., 1966
- [13] *Mathematica 8*, Wolfram Research, User Guide, 2010
- [14] *M.Roșca, E. Roșca*, Essential concepts of road traffic congestion, 5th International Conference on Manufacturing Science and Education MSE, 2011