

## **METHOD OF CALIBRATION OF HIGH ACCURACY LINE STANDARDS WITH NOMINAL LENGTH BETWEEN 0.1 mm TO 1000 mm USING AN INTERFERENTIAL COMPARATOR UNCERTAINTY BUDGET**

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*Lucrarea descrie o metodă modernă de etalonare a riglelor de înalță exactitate, utilizând o instalatie unică în România, denumita comparator interferential pentru rigle și face o estimare a erorilor care apar în procesul de masurare. Contributia autorilor constă în analiza surselor de erori, datorita riglei, comparatorului interferential, echipamentului electronic și temperaturii, estimarea bugetului de incertitudine în conformitate cu specificațiile standardului SR ENV 13005:2003 „Ghid pentru exprimarea incertitudinii de masurare” și oferă o soluție constructivă modernă a comparatorului interferential pentru creșterea exactității de masurare a riglelor etalon, constând în înlocuirea microscopului de vizare cu un echipament compus din microscop de vizare și cameră CCD, dotat cu soft pentru captarea și analiza imaginii.*

*The paper describes the method used for the calibration of high accuracy line standards, using a modern installation named interferential comparator, unique in Romania and the estimation of errors which appear in measurement of line standards with interferential comparator. The contribution of the authors consist in the analyses of the sources of errors which appear during the measurement, due to the line standard, interferential comparator, electronic equipment and temperature, the estimation of the budget of measurement uncertainty, according to the specifications of “Guide to the expression of uncertainty in measurements, SR ENV 13005:2003” and finding a better construction solution of interferential comparator for increasing the performances of measure of line standards, consisting in replacing of the optical microscope with a modern viewing microscope with CCD camera and soft for capture and analyze the image.*

**Key words:** line standard of high accuracy, interferential comparator for line standards, uncertainty budget

### **Introduction**

The article describes a modern method of calibration of line standards with high accuracy, using an interferential comparator with laser source He-Ne stabilized in frequency and makes an analyze of sources of errors which appeared during the measurement.

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To increase the accuracy of measurements of standard reference lines, our length laboratory tried to improve the performances of installation used for measure standard lines. For this purpose, the old optical microscope was replaced by an ensemble microscope – CCD camera and soft dedicated for capture and analyze of image, which can magnified the image from 50X to 400X. For decrease the effect of vibration, the whole installation has a pneumatic suspension.

Also, the article presents the evaluation of components of standard uncertainties and the estimation of the budget of uncertainty and offers some solutions for increase the uncertainty of measurement.

### **1. Method of calibration**

Line standards, in conformity with transmission scheme of unit measurement of length, are length measures underlying the basement of the calibration of some apparatus having scale with line marks.

In our country, the transmission of unit of length to standard lines of high accuracy is made using an interferential comparator, with laser source He – Ne stabilized in frequency that has the following advantages:

- uses as measurement unit, the wavelength of He-Ne laser stabilized in frequency, that can take the measurement unit directly from the national standard of length as He-Ne laser stabilized in frequency which is connected to the cesium standard of frequency and the light velocity in vacuum;
- high reproducibility of measurement unit ( $10^{-10} \dots 10^{-8}$ );
- digital resolution with small increments (from 0.01  $\mu\text{m}$  to 0.1 $\mu\text{m}$ );
- facility of laser alignment with measurement direction.

For the validation of the method it was used a metal line of marks, made by SIP Suisse, serial number 46, with H section, which was calibrated by BIMP in 1987 and the results obtained using interferential comparator were compared with the results from BIPM certificate number 1/1988. The results of the measurement are given in table 1.

Table 1

## Comparative results between BIPM certificate and INM certificate

Nominal length mm	Measured length at BIPM mm	Measured length at INM mm	Differences between INM certificate and BIPM certificate μm
100	100,01131	100,01122	0,09
200	200,03072	200,03061	0,19
300	300,04632	300,04629	0,03
400	400,06209	400,06219	- 0,10
500	500,07379	500,07361	- 0,18
600	600,09246	600,09239	- 0,07
700	700,11190	700,11182	- 0,08
800	800,12264	800,12269	- 0,05
900	900,14423	900,14434	- 0,11
1000	1000,15723	1000,15720	- 0,03
Uncertainty measurement INM			
$0.2 \mu\text{m} + 0.5 \cdot 10^{-6}L$ , L in meter			

The results obtained by Length Laboratory of INM Romania integrate in the limits of measurement uncertainty, declared by INM for measurement capability in Euromet comparisons.

Interferential comparator is a unique reference standard installation, designed and built by Length Laboratory of INM Romania. It is a longitudinal comparator, based on the cinematic method, according to Abbe principle: the reference line measures must be aligned on the same longitudinal axis with laser beam on the measurement direction, in the scope to eliminate the first order errors. During the measurements, the line is put into a closed box situated on the granite plane base that offers a constant temperature and isolates according the environmental temperature of the room. The installation is equipped with a view microscope made by Hirox Japan, type CX – 5040SZ, which permits the magnification of image from 50X to 400X and a field of view from 6,1 mm to 0,78 mm, a CCD camera made by Lumenera Scientific Canada with a maxim resolution of 1280x1024 pixels and soft for the capture of the image. The interferometer comparator is presented in figure 1.

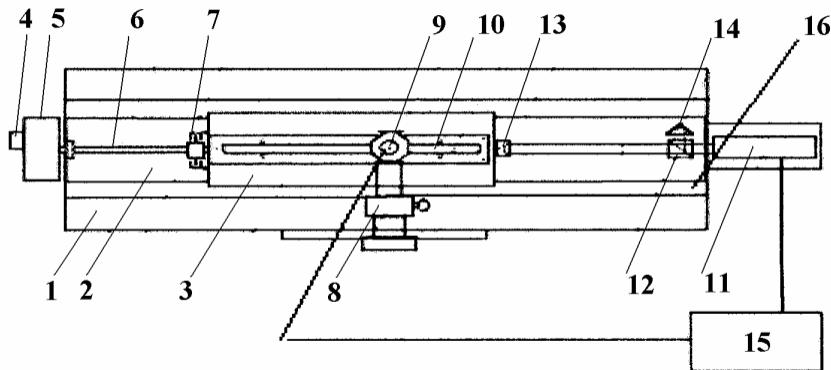


Fig. 1. The interferometer comparator

- 1 - granite plane base supported on the pneumatic suspension
- 2 - rectitude guide line, section I
- 3 - carriage for line, on the pneumatic gas suspension
- 4 - pneumatic gas skids of the carriage
- 5 - variable speed motor with shift reduction unit
- 6 - rolling screw for carriage displacement
- 7 - screw and gaze static system for coupling nut on the carriage
- 8 - microscope support
- 9 - microscope for viewing the marks of lines and CCD Camera
- 10 - reference line measure to calibrate
- 11 - stabilised frequency HeNe laser
- 12 - Michelson interferometer
- 13 - measuring retro reflector
- 14 - reference retro reflector
- 15 - electronic system control and data acquisition - PC for data laser system
- 16 - closed box

The view microscope has a vertical optical axe and it is fixed on the granite plane base with a rigid support, equipped with vertical and transversal adjusters for the viewing of the marks of the line.

Laser interferometer stabilised in frequency permits the measurement of the displacement of the line by counting the interferential fringes which appear in Michelson interferometer when the mobile mirror of interferometer from the carriage that supports is displaced. Measurement method is a direct comparison method.

Method of measurement consists in the displacement of the reference line measure along the measurement direction. Speed of displacement is about maxim 10 mm/min. Marks' viewing of line measure is made using a view microscope with a CCD camera and is captured with a special soft dedicated to this equipment. The image of the mark appears magnified in the computer monitor.

The line is put on the mobile arm of the interferometer and the distance between marks is directly measured in length units.

The appearance of a fringe in the field of view of interferometer is determined by a deviation of optical path from measurement arm of interferometer equal with  $\lambda/2$ , where  $\lambda$  is the wavelength in the propagating medium at medium's temperature, pressure and humidity of the laser He-Ne radiation, used as monochromatic source of light.

Measured length of the line standard is determined by the formula:

$$L_{20^0, 760 \text{ torr}} = L_{tp} + (\Delta L) \lambda + \alpha L_n (20^0 - t), \quad (1)$$

where:

$L_{20^0, 760 \text{ torr}}$

– line length at  $20^0\text{C}$  and pressure of 760 torr;

$L_{tp}$

– measured length displaced by electronic block in measurement conditions (pressure  $p$ , temperature  $t$ )

$\alpha_H$

– thermal coefficient of expansion of the line;  $\alpha = 8,302 \cdot 10^{-6}\text{C}^{-1}$ ;

$L_n$

– nominal length in meter;

$t$

– line temperature indicated by electronic thermometers for  $U = 0,05^0\text{C}$  and resolution  $0,01^0\text{C}$ .

$(\Delta L) \lambda$

- correction of the wavelength due to the medium conditions.

## 2. Study of the errors which appear at the line standards measurement using interferential comparator

These errors can be caused by:

- errors due to the geometry of the measurement system, as comparator and viewing microscope – CCD camera ensemble;
- errors due to the line standard (supporting of the line, thermal coefficient of expansion);
- temperature errors;
- errors due to the laser source.

### 2.1 Geometrical errors

are produced by:

- a) deviations from the ideal geometrical model, which suppose as the line axe and the microscope axe to be in the same vertical plane, what means as the viewing of the marks to be made perpendicular on the mark (Fig.2).

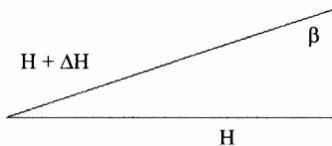


Fig.2.

Laying the line on the view axe of the laser interferometer, the principle of Abbe is respected, so the errors produced by the deviations from the rectilinear of

the carriage carrier – line displacement are only second order errors. If the viewing axe is tilted about  $\beta = 5'$  from the normal, for a focusing height of the image of 6 mm, the error is equal to:

$$H = (H + \Delta H) \sin \beta$$

$$H = 0.008 \text{ mm}$$

But this error is a systematic and constant error all along the line, so it can be eliminated.

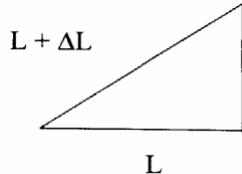


Fig. 3.

$$L = 1 \text{ m} - \text{line length}$$

$$L + \Delta L = 1000,0000004 \text{ mm} \quad (2)$$

$$(\delta l_r) = 0$$

c) error due the winding of the guide ( $\delta l_s$ )

From experimental evaluations results a maxim winging of  $0.5 \mu\text{m} / 1\text{m}$  that introduces a measurement error equal with  $0.12 \mu\text{m}$ .

d) viewing error of the microscope ( $\delta l_{vm}$ )

Viewing of the line marks is made using the microscope and the magnification of the image of the mark is projected in the computer's monitor. CCD camera permits a magnification from 50X to 400X. If consider the width of the mark equal to 0.01 mm, for a magnification of 200X, this will be projected on the screen with a width of 2 mm. Because of the resolution of the camera (1280x1024 pixels) the framing of the middle of the mark can be done with an uncertainty better then  $0.004 \mu\text{m}$ , for a rectangular distribution.

e) due to the electric motor which displaces the line carrier - line ensemble ( $\delta l_v$ )

During the process of measuring, can appear vibrations which can produce non alignment of the line. Experimental it was obtained a value due to the vibration smaller then  $0.5 \text{ nm}$ .

## 2.2 Errors due to the line standards

a) error due to the setting of the line

Supporting mode of the line, universal adopted, consists in the setting line on two supports, one fixed and one mobile, adjustable in height and length, to permit the setting of the line in Bessel points, situates at  $0.2203L$  from edges. This

setting assures the minim shortage of the line and a negligible error due the deformation of the line.

b) error due the non uniformity of the thermal coefficient of expansion of material of the line

At  $20^{\circ}\text{C}$ , the thermal coefficient of expansion has a value equal to  $8.302 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ . Its variation along the line has  $\Delta\alpha = 0.5 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1}$  limits. Line temperature measured inside close box is  $20^{\circ}\text{C} \pm 0.03^{\circ}\text{C}$ .

Assignment uncertainty for a rectangular distribution is:

$$(\Delta\alpha \cdot \Delta t) / \sqrt{3} = 0.5 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1} \cdot 0.03^{\circ}\text{C} / \sqrt{3} = 0.0087 \cdot 10^{-6}. \quad (3)$$

### 2.3 Errors due to the temperature

These types of errors come from calibration of thermometers, readings errors (resolution of thermometers) and the non uniformity of the temperature along the line.

a) expended measurement uncertainty of the thermometers, according to the calibration certificate is equal to  $U_t = 0.05^{\circ}\text{C}$ . For a extended factor  $k = 2$  and a normal distribution type B, standard uncertainty is equal to:  $u_t = 0.025^{\circ}\text{C}$ ;

b) measurement uncertainty due to the thermometer's resolution, for a trapezoidal distribution is equal to:

$$u_c = 0.01^{\circ}\text{C} / \sqrt{12} = 0.003^{\circ}\text{C} \quad (4)$$

c) measurement uncertainty due to the non uniformity of the temperature along the line, for  $\Delta t = 0.03^{\circ}\text{C}$  and  $\alpha = 8.302 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1}$  and for a rectangular distribution is:

$$u_g = \alpha L \Delta t / \sqrt{3} = 0.08 \cdot 10^{-6} \text{ }^{\circ}\text{C} / \sqrt{3} = 0.143 \cdot 10^{-6} \text{ }^{\circ}\text{C} \quad (5)$$

### 2.4 Errors due to the laser interferometer

a) error due to the influence of environmental parameters with refractive index

Corrections of the wavelength of interferometer radiation are determined by environmental conditions in which the laser radiation travels, respective temperature, pressure and environment humidity, according to Owens-Edlen formula:

$$(\Delta L)_\lambda = (1 - n/n_N) \cdot L, \quad (6)$$

where:  $L$  – measured length;  $n$  – refractive index of air;  $n_N$  – refractive index of air at  $t = 20^{\circ}\text{C}$ ,  $p = 760$  torr and  $u = 10$  torr (pressure of water vapors in air).

Consider that measurement uncertainty of the wavelength depends by measurement uncertainty of pressure and temperature, which for laser interferometer Hewlett – Packard have the following values:  $u_p = 1.12$  torr;  $u_t = 0.2^{\circ}\text{C}$ ;  $u_u = 0.1$  torr.

The wavelength of laser at  $t = 20^{\circ}\text{C}$ ,  $p = 760$  torr and  $u = 10$  torr is:  $\lambda = 0,6328 \mu\text{m}$ .

For  $t = 20.2^{\circ}\text{C}$ ,  $p = 761.12$  torr and  $u = 10.1$  torr, applying Owens - Edlen formula, results the following value for the wavelength of laser:

$$(\Delta L)_\lambda = 0.021566 \cdot 10^{-6} L, \text{ where } L \text{ is measured in meter.} \quad (7)$$

Results that the measurement error of wavelength according to the measurement uncertainty due to the environmental parameters ( $u_p = 1.12$  torr,  $u_t = 0.2^{\circ}\text{C}$ ,  $u_u = 0.1$  torr) is equal to  $0.22 \cdot 10^{-6} L$ ,  $L$  is measured in meter.

- b) error due to the flatness deviation of retro – reflectors' mirrors is equal to  $\lambda/8 = 0.08 \mu\text{m}$
- c) cosines error between incident laser beam and reflective laser beam.

Experimental it was obtained a maxim error equal to  $0.002 \cdot 10^{-6} L$ ,  $L$  in meter.

- d) delay error for data acquisition due to the inertia and swinging of the system about the measurement point, in case that retro-reflector displaces during the time when the measurement takes place with a time equal to 25 ns. We can estimate that the analyses of the measurements can be made in a time equal to 1/20 from the time necessary to make the measurement and the error depends by the velocity of the retro-reflector. If suppose a maxim velocity about 10mm/m, the error is given by the formula:

$$E_{\text{achiz}} = v_{\text{retroreflector}} \Delta t / 20 = 0.012 \mu\text{m} \quad (8)$$

- e) error due to the electronic block, according to the signal filter and the counter of the fringes.

System resolution is 0.3 nm, and the counter accuracy can be situated between 1.3 ... 1.5 increments. Multiply the both values, can be estimated an error about 0.45 nm.

### 3. Uncertainty budget

According to the errors which appear at the measurement of the line standards using interferential comparator for line standards, results the following budget of uncertainty (Table 2):

$$u_c = \sqrt{(0.083)^2 + (0.255L)^2} \mu\text{m} \quad (9)$$

For  $k = 2$ , expended uncertainty has value equal to:  $U = [0,17; 0,51L] \mu\text{m}$ , where  $L$  is nominal length in meter.

Table 2.

Uncertainty source	Uncertainty compound	Distribution of probability	Sensitive coefficient	Standard uncertainty ( $\mu\text{m}$ )
winding of the guide	$u(\delta l_s)$	rectangular	1	0,069
viewing microscope	$u(\delta l_{vm})$	normal type B	1	0,004
thermal coefficient of expansion	$u(\alpha)$ $\alpha = 8,302 \cdot 10^{-6} \text{C}^{-1}$	rectangular	$L \cdot \Delta t$ $\Delta t = 0,03 \text{ }^{\circ}\text{C}$	0,009L
temperature: -calibration of thermometer - reading error - non uniformity of temperature along the line	$u(t_l)$ $u(t_c)$ $u(t_g)$	normal type B trapezoidal rectangular	$\alpha L$ $\alpha L$ $\alpha L$	0,208L 0,025 L 0,143L
Wavelength correction due to the environmental parameters	$u(\delta\lambda)$	normal type B	1	0,022L
deviation of the flatness of the mirrors of retro-reflectors	$u(\Delta p)$	rectangular	1	0,046
cosines angle	$u(\phi)$	rectangular	1	0,002L
time of data acquisition	$u(v)$	rectangular	1	0,007
electronic block	$u(e)$	rectangular	1	0

## Conclusions

According to the acquisition of the viewing microscope made by Hirox Japan, together a CCD camera having a resolution of 1280x1024 pixels and soft for the capture of the image and the replacement of the optical microscope which the interferential comparator was equipped in past, it was obtained an improvement of expended uncertainty from  $U = [1; 5L] \mu\text{m}$  to  $U = [0,17; 0,51L] \mu\text{m}$ , where  $L$  is nominal length in meter.

Analyzing the uncertainty compounds, for obtaining better results, could be necessary acquire thermometers having a resolution equal to  $0,001 \text{ }^{\circ}\text{C}$  and expended uncertainty equal to  $0,005 \text{ }^{\circ}\text{C}$ , that can contribute to the decreasing of the terms depending by temperature.

But even in this condition, applying the method of calibration using interferential comparator we can obtain high accuracy measurements for line standards with marks, which can demonstrate our length laboratory capability for this kind of measurements at international requires.

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