

TUNABLE FILTERS FROM LYOTROPIC LIQUID CRYSTALS

Leonaș DUMITRAȘCU¹, Irina DUMITRAȘCU², Dana-Ortansa DOROHOI³

Influența unui câmp electric extern asupra birefringenței stratului de cristal lichid este exprimată folosind ecuația de dispersie a lui Cauchy. În articol sunt prezentate elementele de bază ale teoriei filtrelor interferențiale de tip Lyot - Ohmann și Wood, realizate în geometrie multistrat. Sunt stabilite expresiile corespunzătoare maximelor și semilățimii spectrale ale benzilor transmise.

The external field influence on the birefringence of the liquid crystalline layer is expressed by using the Cauchy dispersion equation. The basically theoretical elements of the polarization interferential filters of Lyot - Ohmann and Wood type, achieved in a multilayer geometry, are given in this paper. The expressions corresponding to the transmitted maxima and to the spectral bandwidths of the transmitted bands are established here.

Keywords: polarization interference filter, lyotropic liquid crystal, liquid crystal polarization filters, Lyot tunable filter, Wood tunable filter

1. Introduction

Liquid crystal tunable filters (LCTFs) are gaining wide acceptance in such diverse areas as optical fiber communications, astronomy, remote sensing, pollution monitoring, color generation for display and medical diagnostics [1]. The large aperture and imaging capability of liquid crystal tunable filters represent a distinct advantage over conventional dispersive spectral analysis techniques. The benefits of liquid crystal tunable filters over acousto-optic tunable filters include: low power consumption, low addressing voltage, excellent image quality and large clear aperture [1, 2].

Liquid crystal tunable filters (LCTFs) use electrically controlled liquid crystal elements to select a specific visible wavelength of light for transmission through the filter at the exclusion of all others. This type of filter is ideal for use with electronic imaging devices, such as charge-coupled devices (CCDs), because it offers excellent imaging quality with a simple linear optical pathway [1, 2].

¹ Phd.student, Faculty of Physics, "Al. I. Cuza" University, 11 Carol I Blvd., Iași -RO-700056, Romania

² Phd.student, Faculty of Physics, "Al. I. Cuza" University, 11 Carol I Blvd., Iași -RO-700056, Romania

³ Prof., Faculty of Physics, "Al. I. Cuza" University, 11 Carol I Blvd., Iași -RO-700056, Romania

There are two distinct types of LC-based polarization filters: discrete and continuous.

Both discrete and continuous tunable filters are based on a technique known as polarization interference [1-5]. Discretely tuned liquid crystal polarization filters (LC-PIFs) produce two complementary spectra. Multiple stages can be stacked to produce multiple pass bands, but at the expense of transmission efficiency and complexity.

Continuously tuned filters (CTFs) are best suited to low resolution applications. Higher resolution devices require so many stages, (resulting in loss and increased device thickness) as to be considered impractical.

2. Theoretical background

2.1. Transmission factor computation

The structures considered here are based on the work of Lyot, Ohmann [1-3, 5-6] and Wood [3, 7]. These filters are made up of a cascade of filter units, or stages, (Figs. 1 a) and b)) each stage requiring two linear polarizers (parallel or crossed) bounding a linear multiple-order retarder oriented at 45° .

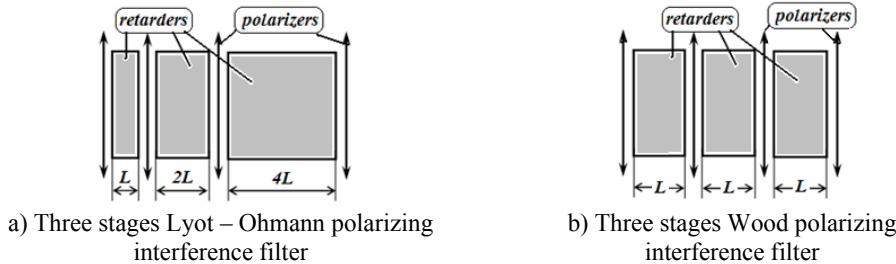
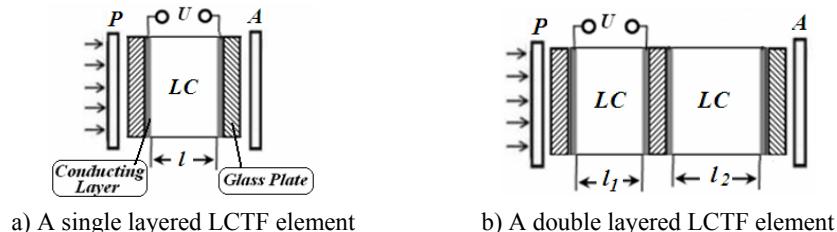


Fig. 1 Typical structures of the LC-PIFs

The stages are constructed of one or two liquid-crystal sheets combination (Figs. 2 a) and b)) [8].



a) A single layered LCTF element

b) A double layered LCTF element

Fig. 2 Schematic structure of a liquid crystalline tunable filter element

The spectral region passed by LCTFs is dependent upon the choice of

polarizers, optical coatings, and the liquid crystal layers characteristics (thicknesses, relative orientation, birefringence, dispersion of birefringence, etc.).

Retardation in birefringent crystals is dependent upon crystal thickness (l) and the refractive index difference between the ordinary (n_o) and extraordinary (n_e) light rays produced at the wavelength of incident illumination [1-5, 7]:

$$\Delta = l \cdot (n_e - n_o) \quad (1)$$

The propagation speed of the extraordinary and ordinary ray differs, and they emerge from the anisotropic stack with a phase delay that is dependent upon the wavelength of light (λ) entering the stack:

$$\Delta\varphi = \frac{2\pi \cdot l \cdot (n_e - n_o)}{\lambda} \quad (2)$$

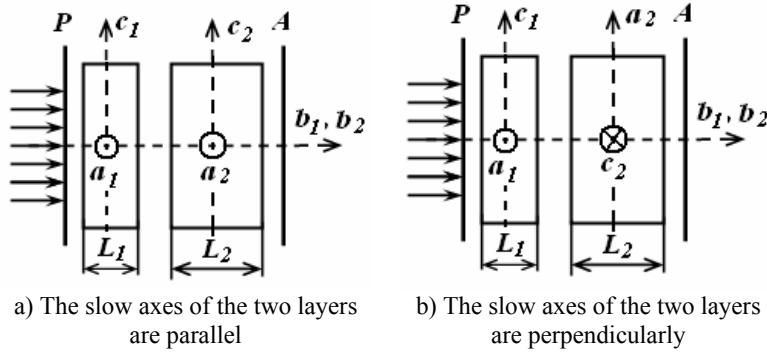


Fig. 3 Relative orientation of the main vibration direction of the two anisotropic layers

The phase delay for the device illustrated in Figs. 3 a) and b) can be calculated with the relation [8]:

$$\Delta\varphi = \frac{2\pi(l_1\Delta n_1(\lambda, E) \pm l_2\Delta n_2(\lambda))}{\lambda} \quad (3)$$

The transmission factor of light through the device (T) is dependent on the value of $\Delta\varphi$.

When the angle between the basic directions of the anisotropic layer and the transmission directions of the polarizers is $\theta = 45^\circ$ and the losses both by radiation absorption in the anisotropic layer and by reflection at the separation surfaces are neglected, the transmission factor of the device is given by one of the following relations [3,5,6]:

- for perpendicularly transmission directions of the two polarizer:

$$T = \frac{1}{2} \cos^2 \left(\frac{\Delta\varphi}{2} \right) = \frac{1}{2} \cos^2 \left(\frac{\pi(l_1\Delta n_1(\lambda, E) \pm l_2\Delta n_2(\lambda))}{\lambda} \right) \quad (4)$$

- for parallel transmission directions of the two polarizer:

$$T = \frac{1}{2} \sin^2 \left(\frac{\Delta\varphi}{2} \right) = \frac{1}{2} \sin^2 \left(\frac{\pi(l_1 \Delta n_1(\lambda, E) \pm l_2 \Delta n_2(\lambda))}{\lambda} \right) \quad (5)$$

Equations (4) and (5) show the transmission factor of a single stage PIF is an oscillatory function of the path-length difference.

2.1.1. Lyot – Ohmann LC-PIF transmission factor

Liquid crystal layers in a typical Lyot filter are often selected for a binary sequence of retardation so that the transmission value is maxim at the wavelength determined by the thickest crystal retarder. Other stages in the filter serve to block the transmission of unwanted wavelengths.

By cascading a series of these filter stages, a band pass filter can be synthesized. For the computational modeling presented below the retardation values were chosen in binary steps of the crystalline thickness layers: (l_1, l_2) , $(2l_1, 2l_2)$, and $(4l_1, 4l_2)$.

The global transmission factor of light through the three stages Lyot polarizing interference filter is given by the following equation [1-3, 5]:

$$T = \frac{1}{2} \cos^2 \left(\frac{\Delta\varphi}{2} \right) \cos^2 \left(2 \cdot \frac{\Delta\varphi}{2} \right) \cos^2 \left(4 \cdot \frac{\Delta\varphi}{2} \right) \quad (6)$$

Tuning the wavelength of peak transmission in the Lyot polarization interference filter (PIF) requires changing the path-length difference, or retardance, of each filter stage.

Application of an electric field in a lyotropic device produces an analog variable retardation. It is therefore quite obvious how a passive PIF can be retrofitted to be active with nematic LC.

The wavelength corresponding to the maxim value of the transmission factor and the band pass of the hybrid tunable filter can be calculated with the relations [5, 7]:

$$\lambda_{k,Max} = \frac{l_1 \Delta n_1 - l_2 \Delta n_2}{k} \quad (7)$$

$$\Delta\lambda = \frac{\lambda^2}{2(l_1 \Delta n_1 - l_2 \Delta n_2)} \quad (8)$$

2.1.2. Wood LC-PIF transmission factor

An important disadvantage of a Wood interferential filter with a single element is the equality of the bandwidth of the bands corresponding to the

transmission maxima and the width of the channel.

Consequently, the Wood interferential filter with a single stage has a modest selectivity. In order to improve their performances a multi-layer geometry is adopted here. This kind of filters is achieved by more identical layers (Fig. 1 b)).

Let us consider a device achieved from m identical elements so the transmission direction of the entrance polarizer in each element to be parallel to the transmission direction of the exit polarizer from the precedent layer. Then one from the mentioned above polarizer can be eliminated.

The transmission factor of the device becomes [3, 7]:

$$T = \frac{1}{2} \sin^2 m \left(\frac{\pi(l_1 \Delta n_1(\lambda, E) \pm l_2 \Delta n_2(\lambda))}{\lambda} \right) (m = 2, 3, 4, \dots) \quad (9)$$

The wavelengths corresponding to the maxims of transmission for a Wood interferential filter with a single stage are:

$$\lambda_{k,Max} = \frac{2(l_1 \Delta n_1 \pm l_2 \Delta n_2)}{4k \pm 1} \quad (k \in \mathbb{Z}) \quad (10)$$

The wavelengths corresponding to the minims of transmission are:

$$\lambda_{k,Min} = \frac{l_1 \Delta n_1 \pm l_2 \Delta n_2}{k} \quad (k \in \mathbb{Z}, k \neq 0) \quad (11)$$

The spectral bandwidth corresponding to the maxim from the wavelength $\lambda_{k,Max}$ is:

$$\Delta\lambda = |\lambda_{k,Max} - \lambda_{k,Min}| = \frac{\lambda_{k,Max}}{k} \cong \frac{\lambda_{k,Max}^2}{2(l_1 \Delta n_1 \pm l_2 \Delta n_2)} \quad (12)$$

Relation (12) permits the estimation of the retardation $(l_1 \Delta n_1 \pm l_2 \Delta n_2)$ necessary to eliminate one from the two neighboring lines separated by the spectral interval $\Delta\lambda$.

The number of the transmitted bands in the spectral range limited by the wavelengths λ_1 and λ_2 ($\lambda_1 < \lambda_2$) can be estimated by the relation [5, 7]:

$$N \cong \frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} (l_1 \Delta n_1 \pm l_2 \Delta n_2) \quad (13)$$

The wavelengths corresponding to the maxims and minims of transmission and the number of the transmitted bands in the considered spectral range λ_1 and λ_2 ($\lambda_1 < \lambda_2$) are the same with those obtained with a filter achieved by one element (relations (10), (11) and (13), respectively).

The transmission spectral bandwidth neighboring the maxim of k order is modified, (one decreases). It is given by relation [7]:

$$\Delta\lambda_{(m)} = \frac{4\pi(l_1\Delta n_1(\lambda, E) \pm l_2\Delta n_2(\lambda))\arcsin\frac{1}{2^m}}{(2k+1)^2\pi^2 - 4\left(\arcsin\frac{1}{2^m}\right)^2} \quad (14)$$

The relations (1) - (14) are the equations of the model used for simulation the tunable LC-PIFs.

2.2. Dispersion of the birefringence modeling

The optical properties of the liquid crystals correspond to those of the anisotropic crystals. The refractive index of the liquid crystals depends both on the propagation direction of the electromagnetic waves and on their polarization state. The most obvious indication of the anisotropy of the liquid crystals is their birefringence. The main refractive indices of a uniaxial material determine the values of the birefringence [9-12]:

$$\Delta n = n_e - n_o \quad (15)$$

Usually, the main refractive indices and the birefringence depend on the orientation order. So, the birefringence is considered as a measure of the degree of order in liquid crystalline samples [8-10, 13-14]. External mechanic, electric or magnetic fields can modify the order parameter of the liquid crystalline sample and the birefringence. Like the main refractive indices, the birefringence shows dispersion in the visible range.

The ordinary and the extraordinary refractive indices dispersion can be expressed by using the Cauchy formula [3-5, 7-8]:

$$n_{o,e}(\lambda) = A_{lo,e} + \frac{A_{2o,e}}{\lambda^2} + \frac{A_{3o,e}}{\lambda^4} \quad (16)$$

From relations (1) and (2) it results that the dispersion of the birefringence can be expressed by using the formula:

$$\Delta n(\lambda) = A_1 + \frac{A_2}{\lambda^2} + \frac{A_3}{\lambda^4} \quad (17)$$

where:

$$\begin{cases} A_1 = A_{1e} - A_{1o} \\ A_2 = A_{2e} - A_{2o} \\ A_3 = A_{3e} - A_{3o} \end{cases} \quad (18)$$

The fitting coefficients A_i ($i=1,3$) may depend of the electric field intensity. In the linear approximation they can be calculated with the following relations:

$$A_i(E) = A_i(0) + \alpha_i \cdot E \quad (i=1,3) \quad (19)$$

From relations (16) – (19) it results that to determine the fitting

coefficients A_i and α_i it is necessary to know the values of the refractive indices or for birefringence for three different wavelengths.

3. Experimental data

Liquid crystalline layer made from PPMAECOBA in TCM was kept in a special cell having interior conducting layers of SnO_2 deposited on the glass plates in order to permit the application of an external electrostatic field, perpendicularly oriented on the plate's surfaces. An orientation layer facilitates the initial orientation of the side chains polymers parallel to the plate surface. The field improves the degree of orientation and increases the birefringence of the sample [13-14].

The main values of the refractive index were measured with linearly polarized waves. The ordinary value (n_o) was measured with light having the electric field perpendicular to the optical axis and the extraordinary value (n_e) was measured with linearly polarized light having its electric field intensity parallel to the optical axis [13-14].

The measurements were made in the absence and in the presence of the external electrostatic field applied between the opposite wells of the cell and having a variable intensity. The results obtained are given in Fig. 4.

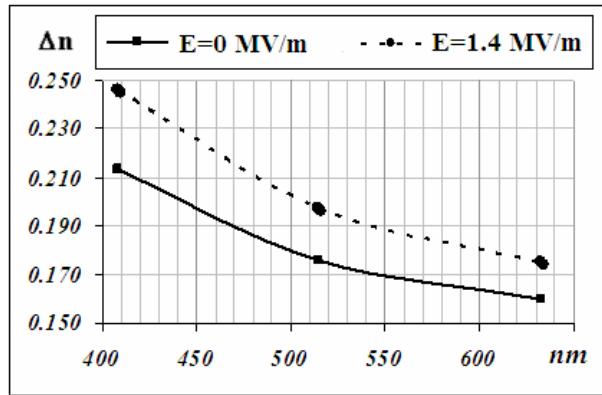


Fig. 4 PPMAECOBA in TCM birefringence in the absence and in the presence of the electrostatic field

Both the principal refractive indices and the birefringence of the studied samples decrease with the light wavelength increasing. This means that PPMAECOBA in TCM shows a normal dispersion [13-16].

Table 1
The Cauchy fitting coefficients for the ordinary and extraordinary refractive indices

Ordinary index			
i	A _{oi} (0)	A _{oi} (E)	α _{oi}
1	1.5638	1.5579	-5.5068·10 ⁻⁸
2	-3.6899·10 ⁻¹⁴	-3.7622·10 ⁻¹⁴	-6.7482·10 ⁻²¹
3	1.0165·10 ⁻²⁶	1.2097·10 ⁻²⁶	1.8032·10 ⁻³²
Extraordinary index			
i	A _{ei} (0)	A _{ei} (E)	α _{ei}
1	1.5339	1.5787	4.1814·10 ⁻⁷
2	3.7385·10 ⁻¹⁴	3.3321·10 ⁻¹⁴	-3.7932·10 ⁻²⁰
3	3.6800·10 ⁻²⁷	5.8926·10 ⁻²⁷	2.0651·10 ⁻³²

Table 2
The Cauchy fitting coefficients for the birefringence

i	A _i (0)	α _i
1	-0.0299	4. 7321×10 ^{- 7}
2	7. 4284×10 ^{- 14}	-3. 1184×10 ^{- 20}
3	-6. 485×10 ^{- 27}	2. 619×10 ^{- 33}

The Cauchy fitting coefficients for PPMAECOBA in TCM were calculated on the basis of the experimental data shown in Fig. 4, by using the relations (16) – (19) and they are listed in the Table 1 and Table 2.

4. Application interface description

The applet frame is composed by four regions (Figs. 5 a) and b)):

- (top-left) - Displaying zone of the transmission factor of the device versus light wavelength and the image of the visible transmitted spectrum;
- (bottom-left) - Displaying zone of the image of the visible spectrum in the absence of the device (for comparison);
- (top-right) - Displaying zone for the first stage of the filter;
- (bottom-right) - Displaying zone for controlling scrollbars of the application.

There are three scrollbars controls labeled for the electric field intensity (E) and for the two thicknesses (l_1 and l_2) of the lyotropic liquid crystalline layers that correspond to the first stage of the filter.

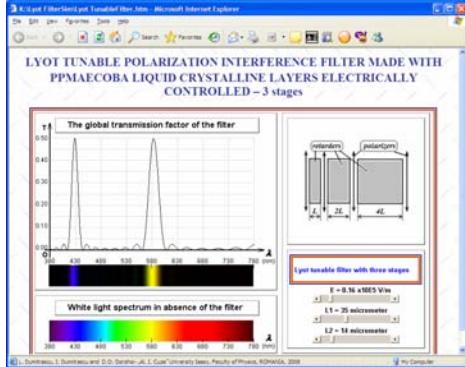


Fig. 5 a) The applet frame that simulate a Lyot PIF

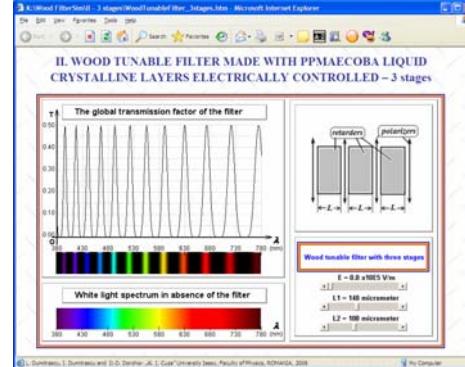


Fig. 5 b) The applet frame for a Wood multistage PIF simulation

5. Results and discussions

The simulation permits a continuous modification of the wavelength corresponding to the maximum of the transmission band by modifying

- the electric field intensity, and
- the thicknesses of the used anisotropic layers

When l_1 and l_2 thicknesses increase, the number of the transmission bands, obtained in the visible range, increases and their band pass decreases.

In the pictures below (Figs. 6 a) and b)) one can see the band pass increasing effect for the Wood tunable filter when the number of stages increases.

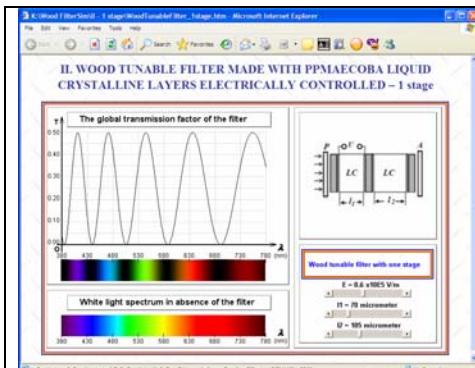


Fig. 6 a) Transmission factor of a single stage Wood PIF

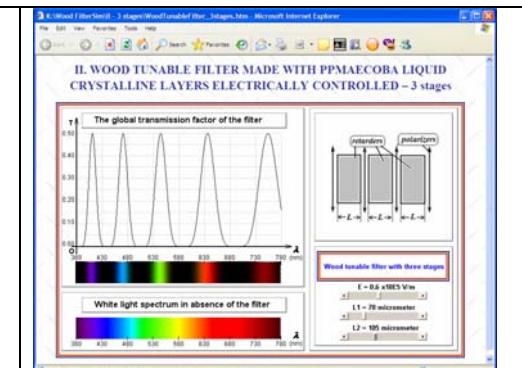


Fig. 6 b) Transmission factor of a triple stage Wood PIF

To improve the general (global) resolution of the LC-PIFs one can use a combination made of two modules consisting from both type of filters.

6. Conclusions

Our work demonstrates the possibility of using the PPMAECOBA in TCM to design optoelectronic devices such a Lyot or a Wood multistage polarizing interference filter.

The simulations demonstrate that the continuously tuned filters are, generally, best suited to low resolution applications.

To obtaining higher resolution devices one can use many stages for each type of presented filters or a combination between two filters (a Lyot and a Wood multistage PIF).

The simulation may be used in studying and designing the tunable polarization interference filters electrically controlled.

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