

## SEMI-EMPIRICAL METHODS FOR BEARINGS DIAGNOSIS

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*Vibrăriile, în special generate de sistemele mecanice, posedă o trăsătură extrem de importantă care poate fi exploatată pentru diagnosticarea defectelor: codificarea informației despre starea de funcționare a sistemului care le produce. Pentru a decodifica această informație, trebuie cunoscute a priori principalele proprietăți ale vibrăriilor care conduc la identificarea și izolarea defectelor. Scopul articolului este de a evidenția aceste proprietăți și de a prezenta câteva metode elementare de diagnoză automată a defectelor prin prelucrarea vibrăriilor.*

*Vibrations, especially generated by mechanical systems, exhibit a fortunate and extremely useful feature in fault diagnosis: encoding information about the health state of the system that produced them. In order to decode that information, the main properties of vibration yielding identification and isolation of faults have to a priori be known. The paper goal is to review such properties and to present some elementary methods of automatic fault diagnosis by vibrations processing.*

**Keywords:** vibrations processing, spectral analysis, bearings fault diagnosis

### 1. Introduction

Like in medicine, faults prevention remains a demanding task, which requires both *self-anticipation* from the system and *intelligent approach* from the user. Usually, a self-anticipatory system is transmitting information about its behavior through *anticipating signals*. For example, human or animal muscles have different electrochemical activity just before they are damaged, due to high intensity and long effort [1]. Another example is issued from mechanical systems, for which the vibrations act as anticipating signals [2], [3], [4], [5]. Their intimate structure changes some time before a failure occurs [5]. But this change is so fast and sometimes so difficult to distinguish, that, without special detection and decoding techniques, it could be ignored. These techniques focus on the extraction of vibration main characteristics (features), in order to classify the possible faults. In general, the strategy adopted within a fault detection method starting from vibrations consists of the following stages: signal acquisition, signal analysis (in

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order to extract features), features grouping, faults classification (eventually adaptively, through a continuously learning mechanism), fault identification (if defects exist).

Vibration acquired from mechanical systems is interesting mainly for its capacity to encode information about threatening defects or faults. Several distinct efforts in detection of such defects can be noticed, but only in the last few decades the vibration has became crucial for automating this process. The first methods of fault detection and diagnosis (fdd) were rather empirical. A trained observer or listener referred to as (*expert*) *analyst* can detect flaws, by simply “watching” or “listening” some machinery. Other subsequent attempts became more systematic and are based on the monitoring of some specific parameters. For example, in a mechanical system, one watches: the lubricant temperature, the oil cleanliness, the noise level of acoustic emission, etc.

Modern and efficient methods in early detection of defects are using *Signal Processing (SP)* techniques [7]. Differently from many typical SP applications, where the noise attenuation is a fundamental requirement, when using vibrations for fdd, exactly the noise is the most concerned part in the analysis. This is due to the fact that not the natural oscillations of machinery could encode the defective behavior, but the noise corrupting them. Moreover, the applications revealed that the *signal-to-noise ratio (SNR)* is extremely small for vibrations encoding information about defects. Therefore, the models of vibration used in fdd are actually models of their noisy parts, encoding information about defect types and severity [8].

One of the most interesting applications in fdd is concerned with bearings, due to their simple structure and large integration within mechanical systems [9], [10], [11]. By inspecting the spectrum of vibration acquired from bearings, some researchers believed that its irregular shape is mainly due to the environmental noise and correlation between different components. Hence, techniques to “remove” the white noise and decorrelate the data, based on SP concepts such as: *auto-correlation*, *backstrum*, or *cepstrum* were introduced first. Although the irregularities are only slightly attenuated, an analyst could easier perform the diagnosis, which is the result of an ad hoc fault classification, by simply inspecting the spectrum. Moreover, the analyst is usually able to improve the accuracy of classification for every new investigated case. What rationale is employed by the expert? – that is a question with a difficult answer. Modeling such rationales often requires non conventional approaches. For example, a very interesting approach combining statistics and pattern recognition has been introduced in [12]. This is in fact an attempt of human reasoning automating, which resulted in a quite efficient and simple fdd algorithm, though with unavoidable limitations. A different approach is introduced in [13], where one assumes the largely accepted idea that *human reasoning is also fuzzy*. This means

a solution to a problem could be issued even from unclear, vague or ambiguous information, i.e. from information, which is strongly affected by uncertainty. Usually, the analyst selects the most plausible diagnostic, according to the available data. Therefore, the analyst's experience is crucial for diagnosis accuracy. Unfortunately, the analyst has to cope not only with external perturbations corrupting the data, but also with his/her own subjectivism. In order to increase objectivity, the analysis relies on simple statistical assessments. The reasoning hidden behind data analysis could thus be automated by performing a combination between *spectral statistics* and *fuzzy clustering* (in entropy sense [14]). In this way, both subjectivism and perturbations influence decrease.

This article is not concerned with advanced fdd methods based on vibration analysis, but rather with the most employed of them in bearings industry, referred to as *semi-empirical*. It constitutes a survey on the most encountered bearing abnormalities and also on the industrial employed fdd methods. Within the next section, the main defect types of bearings are revised, in order to reveal their variety. Sections 3 presents some semi-empirical fdd methods, based on statistics and spectral analysis. The article is completed by conclusions and a references list.

## 2. Bearings typical defects

The rich experience acquired during utilization of semi-empirical fdd methods led to a number of very useful insights concerning the bearing defects.

In case of defect free bearings, the friction forces and the generated vibration are independent on the rotation angles of rolling elements. But, if a defect starts to develop, the amplitude of vibration could basically be modulated following 2 types of defect trajectories: periodic (or train of impulses, due mostly to defects on races, rolling elements (if tubular) and cage); quasi-random (due mostly to defects on rolling elements (if balls) and of lubrication). Moreover, in case of defects, vibration statistical parameters are very sensitive to magnitude and direction of applied load and to relative speed between races, as reveals the study in [12]. If the defect trajectory is periodic, then the period of vibration amplitude changing encodes the defect type, whereas the modulation amplitude encodes the degree of severity. Otherwise (and especially for lubrication defects), the severity degree is difficult to estimate.

The abnormal functioning of bearings could succinctly be described as follows:

- a. Defects develop following an evolution difficult to predict. Also, they can convert from a type to another.

- b. The severity degree of a defect could better be emphasized by another defect. It *could decrease in time*, after reaching some value, due mostly to excessive wearing. But, in general (and especially at early stage of defect evolution), the severity degree increases in time (even if decreased before).
- c. Indications about a defect could vanish and not appear again over a large period of time.
- d. Multiple defects are developing more rapidly than single ones.
- e. Defects of lubrication induce a faster apparition and development of other defects than in case of appropriate lubrication.
- f. Short time before failure, the harmonic amplitudes in vibration spectrum could strongly be attenuated or even vanish.
- g. After a while, some defects (especially the multiple point ones) induce harmonic components over the limit of 2 kHz.
- h. When the bearing is new and defect free, the vibration could look as if it would have defects, caused by the lack of usage and imperfections on rolling surfaces.
- i. The defects on rolling elements develop faster. Actually, the life time of a bearing with defects on rolling elements is no greater than 25% of its MTBF (*Mean Time Before Failure*). (MTBF is a standard parameter, statistically evaluated, specified by manufacturers together with bearings constructive parameters and types.)

Regarding the last remark (i), after testing more than 5000 bearings, one has concluded in [15] that the failure occurred as follows: for about 26% of bearings, when the first defect appeared on the outer race; for about 28% of bearings with lubrication defects; for about 44% of bearings, when the first defect appeared on the inner race; for about 76% of bearings, when the first defect appeared on rolling elements. Obviously, rolling elements are by far the most exposed parts to defects. The evolution of a defect from single point to multiple points and from incipient to medium or high severity degree is illustrated in Fig. 1.

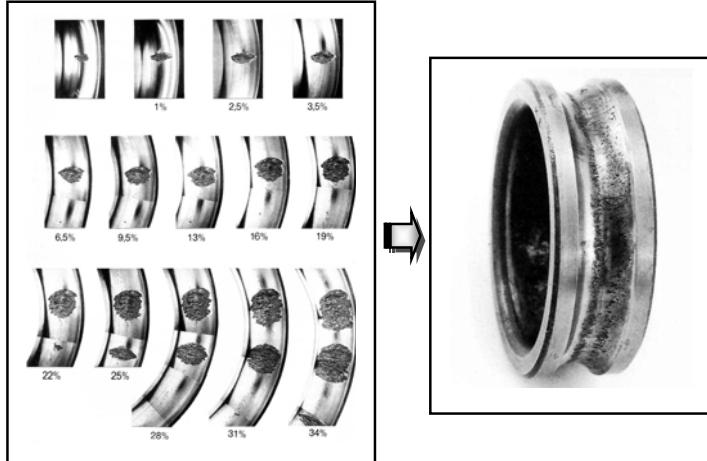


Fig. 1. Development of a defect on inner race.

The defect consists of cracks on the inner race and appeared very early during the life time of bearing (before 1% of MTBF). One can see to the left the different instants of defect evolution, in percentages of MTBF. The severe multiple defects to the figure right put the bearing in state of advanced wearing out. The forces that originate vibrations in bearings are mostly due to: irregularities of rolling surfaces; excessive friction; shock pulses and ruptures of lubrication layer; variation in stiffness; rotor self oscillations; interaction/collision with other constructive parts. These forces induce mechanical constraints on bearing constructive parts, which constitute the premise of defect apparition and development. Thus, irregularities of rolling surfaces induce shaft vibrations. They encode information about possible defects around some critical frequencies, related to the natural ones described in [8]. Usually, set of 5 natural oscillation frequencies could be derived: the ball pass frequency on the outer race ( $\nu_{out}$ ); the ball pass frequency on the inner race ( $\nu_{in}$ ); the cage rotation frequency with respect to the outer race ( $\nu_{cout}$ ); the cage rotation frequency with respect to the inner race ( $\nu_{cin}$ ); the ball rotation frequency ( $\nu_b$ ).

The friction forces are seen as sets of short shock pulses randomly distributed in time, duration and shape. They induce strong vibrations in range 2-10 kHz and very weak vibrations beyond 10 kHz, up to 30 kHz, by means of bearing resonance. Normally, when the defects are incipient or missing, these vibrations could not include any related information, because the defect shock pulses intensity could not overpass (too much) the intensity of friction shock pulses. This information starts to significantly be encoded only when defect pulses have an intensity of more than about 10 times bigger than friction pulses. Shock

pulses and ruptures of lubrication layer produce 2 types of bearing oscillations: forced and natural. Forced oscillations are distributed over a wide range of frequencies, whereas the natural ones concentrate in narrow sub-bands around some specific frequencies (related to the natural ones) and have fast decay. The variation in stiffness is almost due to variation of loads applied on bearing. The rotor self oscillations are due to excessive distance between engine and bearing. In general, the corresponding vibration has strong harmonic components at  $\nu_r / 2$  and  $2\nu_{cout}$ . The interaction with other constructive parts induces vibrations that encode information about defects of other mechanisms. They also could damage a healthy bearing, if not cancelled.

The most important forces that could damage a mechanical system are described for example in [16]. They appear mostly when: the bearing is wrongly mounted (the races are misaligned, the shaft is wobbling, etc.); the loads are very irregular, the bearing has intrinsic constructive defects; the lubrication is insufficient or overrated, etc. The raw vibration could thus encode information about 3 main categories of possible defects:

- a. Installation defects: misalignment of races; non-uniform radial tension.
- b. Defects during bearing operation: wear of outer race; cracks, spalls, cavities on outer or inner race; wear of inner race; wear of rolling elements; cracks, spalls, chops on rolling elements; wear of cage (or other cage defects such as: deformation, local ruptures, etc.); inappropriate lubrication (too much or too few lubricant).
- c. Defects of other machinery parts (especially under dynamic loads): rotor revolution (shaft wobbling); jointed coupling effects; gear teeth defects; gear interaction defects.

For example, in Figure 2, races misalignment is produced by wrong bearings positions or by excessive load. After a while, an eccentric groove is practically created on inner race, which is associated to excessive wearing out.

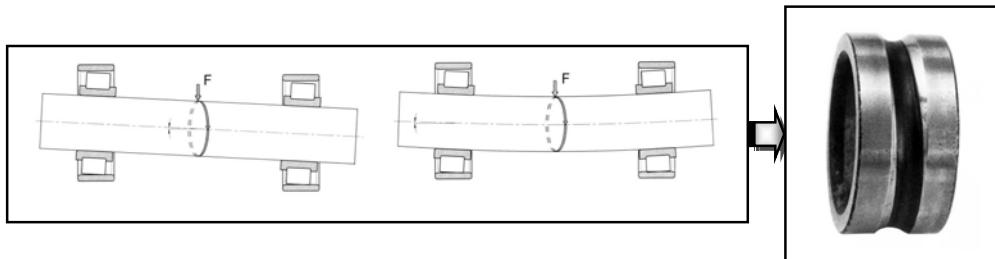


Fig. 2. Effect of races and/or axes misalignment.

Another example is depicted in Figure 1 above, where cracks and wear on inner race were pictured. Some defects on outer race are illustrated in Figure 3.

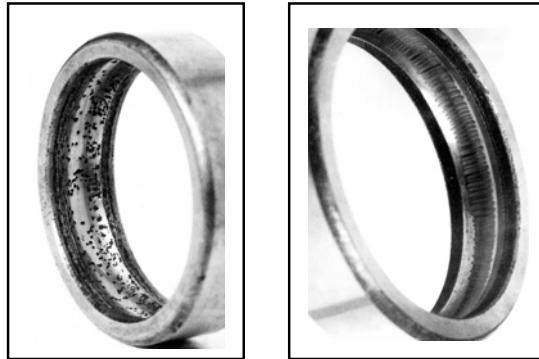


Fig. 3. Defects on outer race.

To the left, the cracks were produced by rust (the bearing worked in a corrosive environment), whereas to the right one could see how an excessive and non uniform dynamic load created transversal grooves only on a sector of race.

The rolling elements can also be affected by chops, wear or cracks, like in Fig. 4.

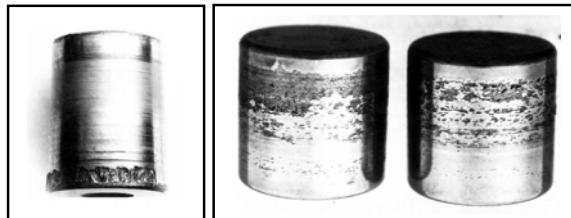


Fig. 4. Chops, wear or cracks on tubular elements.

The last example is depicted in Figure 5 and concerns the cage wearing, also due to excessive loads. In general, cage defects lead to loss of clearance between rolling elements, which involves non uniform wearing and local quasi random impacts between rolling elements and cage.

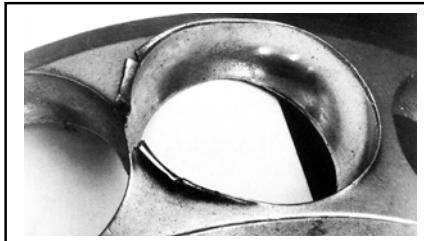


Fig. 5. Cage wearing out and local deformation.

The defects involve different behaviors of raw vibration, depending on their nature. Thus, for example:

- a. Cavities, spalls, cracks or chops on rolling surfaces (including rolling elements) produce uniform or non uniform shock pulses that activate the sensor resonance, which results in vibration enveloping [6].
- b. The wearing out of rolling surfaces and elements or cage produces non-uniform friction, although without visible shocks. Apparently, these defects are not activating the sensor resonance, but the effect in vibration is the same: enveloping. Therefore, one considers that variations of friction forces could be modeled by a train of microscopic shock pulses.
- c. The ruptures in lubrication layer and/or the loss of its purity are the most difficult defects to diagnose by only using the raw vibration. They could or could not produce shock pulses. Shocks appear especially when the lubricant is mixed with metallic parts or another impurities or when the shaft is wobbling (eventually combined with another defects). In the absence of shocks, self sustained oscillations could appear, especially when the lubricant is insufficient. When the lubricant is overrated, the friction forces increase, due to viscosity, which produce a low frequency modulation of vibration.
- d. When the clearance between rolling elements is lost (due to cage defects), shock pulses could appear as well (especially because of local impacts between cage and rolling elements or even between rolling elements themselves).

### 3. Spectral-statistical diagnosis

Once the vibration has been acquired and after performing some preliminary processing operations (in order to remove accidental samples and to reduce environmental noise), the main problem is to extract the information

concerning the type and severity degree of defects. This goal is not easy to approach. However, during the last 50 years, several solutions have been proposed.

The oldest attempt is based on the classical *Fourier Analysis* of vibration data. The vibration spectrum is usually constructed by spectral estimators with appropriate windows (it seems that, in this case, the *Hanning window* is the most suitable) [7]. The analyst could detect some defects even at their early stage of development by direct inspection of vibration spectrum. But, because the frequency content of vibration components are mixed in an unknown way (resulting in a very irregular spectrum), many defects are hidden and could not be detected, even by an experienced analyst. Moreover, the defect severity degree is practically impossible to estimate with good accuracy only by inspection.

Take, for example, a tested bearing. If vibration data  $v$  are consistent (few thousands of rotations), the vibration spectrum  $|V|$  looks like in Figure 6.

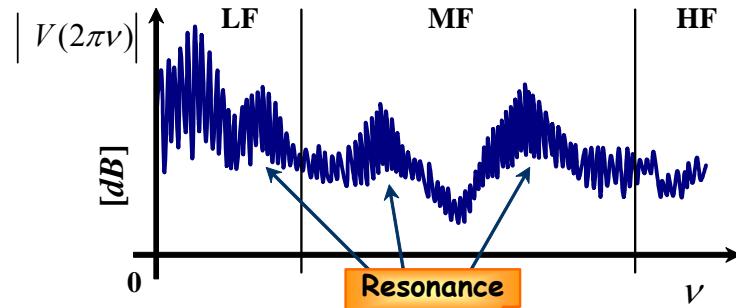


Fig. 6. Overall vibration spectrum.

Two cases could be discussed here. When the bearing is defect free, the spectral energy is mainly concentrated inside the low frequency sub-band (LF) encoding information about bearing oscillations and their natural frequencies (derived from bearing geometry and depending on shaft rotation speed). Few multiples of natural frequencies are replicated within spectrum, but their power have an exponential decay (due to damping). In case of defective bearing, the idea that the defect noise is basically generated by visible or microscopic quasi-random shocks has been largely accepted today (even in case of wearing). Shocks are modeled by trains of impulses and force the sensor to resonate. Usually, sensors resonance appears at (very) high frequency, but, by convolution with a train of impulses, it is replicated towards low and middle frequency as well. In Figure 6, this phenomenon is suggested by energy concentration around some peaks located in middle frequency sub-band (MF). Usually, a resonance peak is mixed with basic LF spectrum as well, such that it could hardly be distinguished. The high frequency sub-band (HF) rather encodes information about resonance corrupted

by environmental noises. The spectrum could change (even dramatically), depending on the applied load, sensors locations, shaft speed, bearing mounting, etc. Because of this sensitive behavior, the information about defects encoded by vibration spectrum is extremely difficult to read.

By inspecting the spectrum, some researchers believed that its irregular shape is due to the environmental noise (white or colored). A simple technique to remove the white noise component from data is to operate with *auto-correlation sequence* instead of the genuine vibration data. This sequence is estimated as follows, for  $N$ -length vibration data series,  $\{v[n]\}_{n \in 0, \overline{N-1}}$ :

$$r_v[k] = \frac{1}{N_k} \sum_{n=0}^{N-k-1} v[n]v[n+k], \quad (1)$$

where  $N_k$  is either  $N$  or  $N - k$  and  $k \in 0, \overline{N/4}$  (in order to avoid introducing supplementary computational noises). Hence, instead of vibration and its spectrum, one operates with the auto-correlation sequence and the power spectral density (psd), but the irregularities are only slightly smoothed.

Another different spectral concepts are also used. For example, two successive Fourier Transforms (FT) are applied on  $v$ , resulting in a new concept: the *backstrum*,  $\hat{V}$ . This is defined like below for continuous time signals:

$$\hat{V}(\Lambda) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} V(\Omega) e^{-j\Lambda\Omega} d\Omega, \quad \forall \Lambda \in \mathbb{R}, \quad (2)$$

where  $V$  is the FT of  $v$  and  $\Lambda$  is referred to as *quefrency* (instead of *frequency*). The defect information extraction is good enough, but it requires sophisticated neuro-fuzzy classifiers and good interpretation skills from human operator, as proven in [17].

The vibration model of convolution between two vibration components (a harmonic resonance one and a defect encoding one), described in [8], inspired other authors to replace the spectrum by the concept of *cepstrum* ( $\hat{v}$ ) and to use the *homomorphic deconvolution* [7], in order to separate the components. The cepstrum is defined as the inverse Fourier Transform of logarithmic  $V$ :

$$\hat{v}(\tau) \stackrel{\text{def}}{=} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \ln[V(\Omega)] e^{+j\Omega\tau} d\Omega, \quad \forall \tau \in \mathbb{R}. \quad (3)$$

Thus, by accounting the Convolution Theorem, the cepstra of the two components are added each other. The *deconvolution* (i.e. the component separation) can now be realized by appropriately filtering the cepstrum. This method is quite effective for some single point defects, but multiple defects are still hidden and the severity degree is difficult to estimate. Moreover, the cepstral methods have shown that the superposition hypothesis does not hold in any case.

The limitations of methods mentioned so far are mainly due to the fact that the sensor intimate behavior is ignored. The sensor is only seen as a quasi-linear device that distorts insignificantly the crude vibration. The resonance effect produced by defects is not actually accounted. The cepstrum method advanced one step in the direction of considering this effect, but did not go farther. The Fourier Analysis could be completed by a statistical approach. Statistical parameters of vibration itself and/or of the associated spectrum are sometimes very useful when inspecting a spectrum. Using statistics to extract information about defects from raw vibration is not a new idea. Many analysts perform diagnosis with the help of some parameters such as the *root mean square* (RMS) or the *peak value* of vibration or its spectrum.

A quasi complete statistical set includes the following 12 parameters: *peak (to valley)* ( $\Delta v$ ); *average* ( $\bar{v}$ ); *absolute average* ( $|\bar{v}|$ ); *energy* ( $\mathcal{E}_v$ ); *normalized energy or variance* ( $\mathcal{E}_v^N$ ); *root mean square* ( $RMS_v$ ); *peak to average ratio* ( $PAR_v$ ); *crest factor* ( $CF_v$ ); *impulse factor* ( $IF_v$ ); *shape factor* ( $SF_v$ ); *clearance factor* ( $CLF_v$ ); *Kurtosis* ( $\kappa_v$ ). Their definitions are listed next, for  $N$ -length vibration data series,  $\{v[n]\}_{n=0, N-1}$ , but they could be used for any numerical data:

$$\begin{aligned} \Delta v &\stackrel{\text{def}}{=} \frac{1}{2} \left[ \max_{n=0, N-1} \{v[n]\} - \min_{n=0, N-1} \{v[n]\} \right]; \quad \bar{v} \stackrel{\text{def}}{=} \frac{1}{N} \sum_{n=0}^{N-1} v[n]; \quad |\bar{v}| \stackrel{\text{def}}{=} \frac{1}{N} \sum_{n=0}^{N-1} |v[n]|; \quad \mathcal{E}_v \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} |v[n]|^2; \\ \mathcal{E}_v^N &\stackrel{\text{def}}{=} \frac{1}{N} \sum_{n=0}^{N-1} |v[n]|^2; \quad RMS_v \stackrel{\text{def}}{=} \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (v[n] - \bar{v})^2}; \quad PAR_v \stackrel{\text{def}}{=} \frac{1}{|\bar{v}|} \max_{n=0, N-1} \{|v[n]| \}; \quad CF_v \stackrel{\text{def}}{=} \frac{\Delta v}{RMS_v}; \\ IF_v &\stackrel{\text{def}}{=} \frac{\Delta v}{|\bar{v}|}; \quad SF_v \stackrel{\text{def}}{=} \frac{RMS_v}{|\bar{v}|}; \quad CLF_v \stackrel{\text{def}}{=} N \Delta v / \left( \sum_{n=0}^{N-1} \sqrt{|v[n]|} \right)^2; \quad \kappa_v \stackrel{\text{def}}{=} \frac{\frac{1}{N} \sum_{n=0}^{N-1} (v[n] - \bar{v})^4}{RMS_v^4}. \end{aligned} \quad (4)$$

The first 6 parameters are concerned with energetic characteristics, while the other 6 quantify different properties of graphical shape. Obviously, the whole set is redundant. But selecting a non redundant sub-set of statistical parameters is complicated and, often, not really necessary. Two of them could however be removed: the (absolute) average and the (normalized) energy.

Usually, the values of parameters defined in (4) are compared to standard values corresponding to defect free systems. Their biases could indicate the desired information about defects (including estimations of severity degree). Although the number of parameters to account is large enough, no one is able to extract all necessary information about defects. Therefore, the analysis becomes complicated. A promising attempt of automating the faults classification starting from 4 of these parameters ( $\Delta v$ ,  $RMS_v$ ,  $IF_v$ ,  $\mathcal{K}_v$ ) has been introduced in [12].

The 4 parameters define a 3D space, where the coordinates are:

$$\frac{\Delta v}{RMS_v^0}, IF_v, \mathcal{K}_v.$$

Here,  $RMS_v^0$  stands for RMS of defect free vibration. The 3D space is transformed in a so called *features space*, with 2 dimensions. The *features* are: Kurtosis  $\mathcal{K}_v$  (which is sensitive to energy distribution and to impulsive behavior) and *data spikiness*, defined as:

$$\mathfrak{F}_v \stackrel{\text{def}}{=} \ln \left[ \frac{\Delta v}{RMS_v^0 \mathcal{K}_v} + IF_v \right]. \quad (5)$$

These 2 features are considered representative for discriminating between defects. Six faults classes are isolated inside the features space, by continuously training a pattern recognition classifier with data acquired from bearings. The method introduced in [12] is computationally simple and has been implemented in an experimental faults detector for bearings at Canadian Railways. Its drawbacks are however obvious: only single point defects are accurately detected; the severity degree could not be estimated; raw vibration acoustic data require sophisticated denoising techniques, etc.

In [13], the diagnosis method is different. A fuzzy classifier can make the distinction between different fault classes, through a holonic mechanism, starting from the biases of 10 parameters (4). Results are quite surprising (even the severity degree of defects could be estimated), but the method still requires a trained analyst.

#### 4. Conclusion

The research concerning fdd by means of vibrations is by far more advanced than described within this article. However, the industry is still fully using semi-empirical diagnosis methods, due to their simplicity.

The paper aimed to review the most important defect types of bearings and the most industry employed methods of fdd based on vibration analysis. The vibration analysis mainly considers the noise component of the vibration signal to encode the abnormal functioning of the bearings. The type and the severity degree of the defects could be determined by applying several techniques based on different spectral concepts (Fourier analysis, auto-correlation sequence, backstrum, cepstrum) combined with statistical approaches based on statistical parameters studies.

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