

EXPERIMENTAL RESULTS ABOUT THE STICK-SLIP PHENOMENON WITH APPLICATION TO THE DISC-BRAKE FRICTION MATERIALS COUPLE USED IN THE AUTOMOTIVE DOMAIN

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The stick-slip phenomenon is considered to be one of the causes of the noises and vibrations that appear during the braking process of vehicles. The present paper is composed of two experimental studies. The first one aims to analyze from the statistical point of view the stick-slip phenomenon between the couple of friction materials of the automotive disc brakes to see if it is stable and to demonstrate that the test results are repeatable. The other study aims to observe the different forms of sliding that appear during the braking process and the evolution of the friction coefficient by varying the sliding speed, the contact pressure and the system rigidity.

Keywords: stick-slip, friction coefficient, automotive disc-brake friction materials, statistical analysis.

1. Introduction

One of the most important systems of the automobiles is the braking system. A vast array of problems that arise in the braking system of the automobiles are of tribological nature.

In the automobile brake system, the braking speed between the disc and pads or between the drum and shoes is variable, so in the low and very low speeds domain, as a result of the elasticity of the brake system components, the stick-slip phenomenon appears, which can be one of the causes of the noises and vibrations that appear during the braking process [1-8].

The stick-slip phenomenon is described as a series of intermittences in the braking process, caused by the differences between the values of the kinetic friction coefficient (μ_k) and those of the static friction coefficient (μ_s).

Usually, for the stick-slip phenomenon to occur, the static friction coefficient (μ_s) between the two contact surfaces of the brake disc and brake pad must be greater than the kinetic friction coefficient (μ_k) [1, 8, 9].

The variation of the friction coefficient(μ) versus time (t) for the stick-slip phenomenon can be observed in Fig. 1, where: μ_{smax} – maximum static friction

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coefficient, μ_s – static friction coefficient, μ_k – kinetic friction coefficient, μ_m – mean value of the friction coefficient and μ_v – friction coefficient amplitude (stick-slip amplitude).

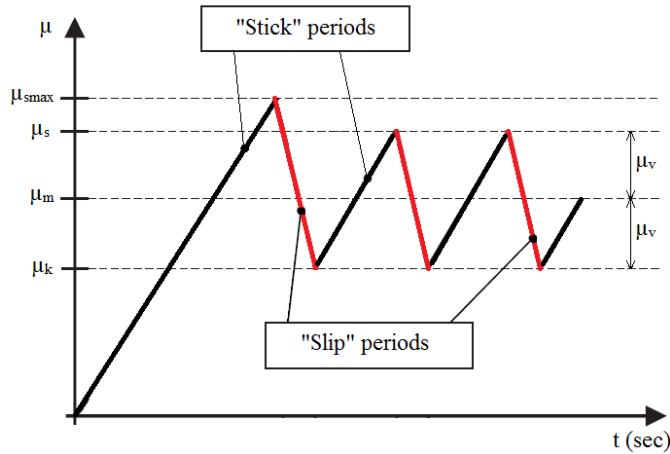


Fig. 1. Variation of the friction coefficient versus time

The amplitude of the stick-slip phenomenon is influenced by the rigidity characteristics of the brake system, by the relative speed between the brake disc and brake pad and by the local state of contact between the two surfaces in friction [10, 11, 12].

In a previous study [8], it was established that sliding can have various forms: stick-slip motion, driven oscillations mixed with stick-slip motion, harmonic oscillations, damped harmonic oscillations and smooth sliding.

The present paper aims to determine whether the stick-slip phenomenon for the automotive disc brake system friction materials couple is stable and to demonstrate that the test results are repeatable. Another objective of the paper is to reveal the various sliding forms that appear during braking and the influence of the relative sliding speed, of the contact pressure and of the system rigidity on their occurrence. Another goal of the paper is to see the influence of these parameters (speed, pressure, rigidity) on the evolution of the friction coefficient.

2. Testing procedure

The tests presented in the current paper have been conducted on a Bruker (former CETR) UMT-2 Tribometer in order to determine the evolution of the friction coefficient (COF) of the friction materials couple used for the automotive disc brake system. For these tests the UMT-2 Tribometer was equipped with a linear motion set-up, as shown in Fig. 2. This test method involves a pin shaped upper specimen that comes in contact with a rectangular shaped lower specimen under a prescribed

set of conditions. The upper specimen moves in the vertical plane, creating the loading force, while the lower specimen has a linear motion in the horizontal plane, thus creating the friction force.

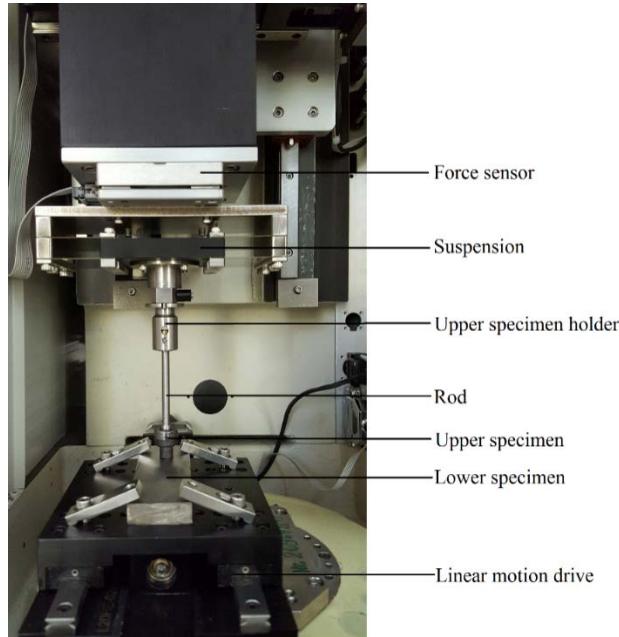


Fig. 2. Overview of the UMT-2 Tribometer set-up

The force sensor equipped on the tribometer was a model DFH-20 two-dimensional force sensor used to measure the sliding and breakaway friction force between the upper and lower test specimens as well as measuring and controlling the loading force. The suspension between the force sensor assembly and the upper specimen holder was necessary to maintain a constant load by compensating for variations in the distance between the force sensors and the surface of the lower specimen when the lower specimen is in motion. The upper specimen was mounted in the holder by the means of a connecting rod that was screwed in the back of the specimen. The upper holder connects the upper specimen to the suspension, while the lower specimen is connected to a model L20HE reciprocating linear motion drive.

In order to observe the influence of the system rigidity on the variation of the friction coefficient three connecting rods of different lengths were used. The three connecting rods and the two specimens used for the tests can be observed in Fig. 3. The upper specimen is a pin manufactured out of a real semi-metallic automotive brake pad and has a diameter in the contact zone of 8.6 mm, an upper diameter of 15.6 mm and a length of 18 mm. The lower specimen is a 95.5x32.2x10.9 mm rectangular shaped block manufactured out of a real

automotive grey cast iron brake disc. Thus, knowing the geometry of the two specimens, the contact between the two of them is a flat circular surface with a nominal area of 58.088 mm^2 . The rods have the diameter of 5 mm, while their total lengths are: 13.6 mm (short rod), 63.1 mm (medium rod) and 113.6 mm (long rod).

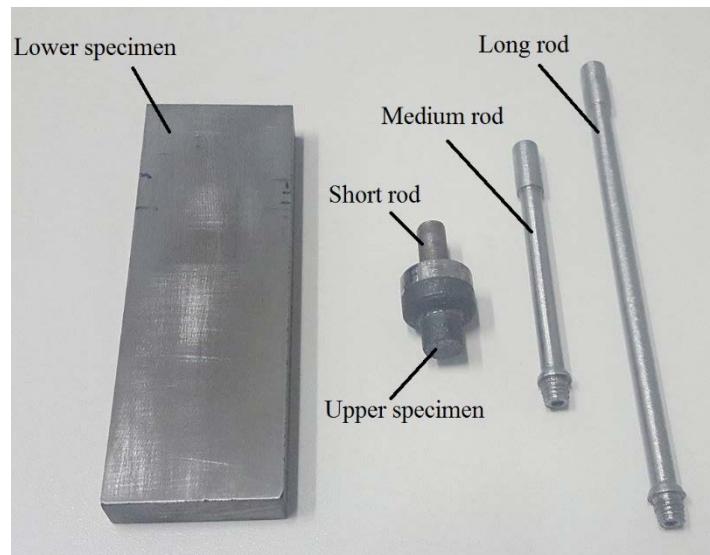


Fig. 3. Specimens and rods used for the tests

It is known that the friction process is influenced by the nature of the contact surfaces and by the functional parameters of the braking system: relative sliding speed, contact pressure and system rigidity [11, 12, 13]. These parameters influence the evolution of friction coefficient and the occurrence of the stick-slip phenomenon during the braking process.

To determine the influence of the relative sliding speed on the friction process the tests have been conducted at four speeds with an exponential growth, covering a very wide range: 0.004 mm/s, 0.04 mm/s, 0.4 mm/s and 4 mm/s. Also, the tests have been conducted for three normal loads (25 N, 50 N and 100 N), which allowed us to see the influence of the contact pressure (0.430 MPa, 0.861 MPa and 1.722 MPa) on the friction process. The typical nominal pressure at the pad surface for a front car brake during normal, relatively soft braking is just above 1.2 MPa, but in extreme situations the pressure could be close to 10 MPa [14].

As mentioned before, three connecting rods of different lengths were used in order to observe the influence of the system rigidity on the variation of the friction coefficient and the stick-slip occurrence. The system rigidity was determined directly from the tests by measuring the displacement of the upper specimen and the tangential force that determined this displacement. Therefore, the system rigidity for the long rod was 7.159 N/mm, for the medium rod 31.405 N/mm

and 79.904 N/mm for the short rod. For example, Fig. 4 presents the results for the system rigidity determination for the long rod.

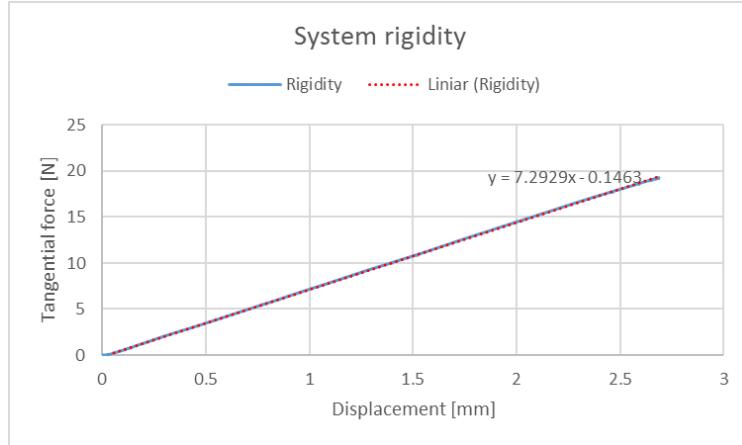


Fig. 4. System rigidity determination for the long rod

3. Results and discussions

For the experimental research presented in the current paper, two sets of tests have been conducted.

The first set was designed to determine the statistic parameters and also the repeatability of the tests when the stick-slip phenomenon occurred during the friction process. This was accomplished by repeating a test of a certain normal load and relative sliding speed 10 times and calculating the average values ($\bar{\mu}$), the standard deviation (σ) and the coefficient of variation (CV) for the specific values of the friction coefficient: μ_{smax} , μ_s , μ_k and μ_v . Also, with the help of the Kolmogorov-Smirnov test it was possible to determine the distribution law of the experimental results and the intervals in which future results will be obtained. The selected normal load was 100 N, which gives a contact pressure between the two samples of 1.722 MPa, while the selected relative sliding speed was 0.016 mm/s. For these tests the long connecting rod was used, resulting in a system rigidity of 7.159 N/mm. The results for these tests are presented in Fig. 5, where the variation of the friction coefficient (COF) vs. time can be seen for each of the 10 tests.

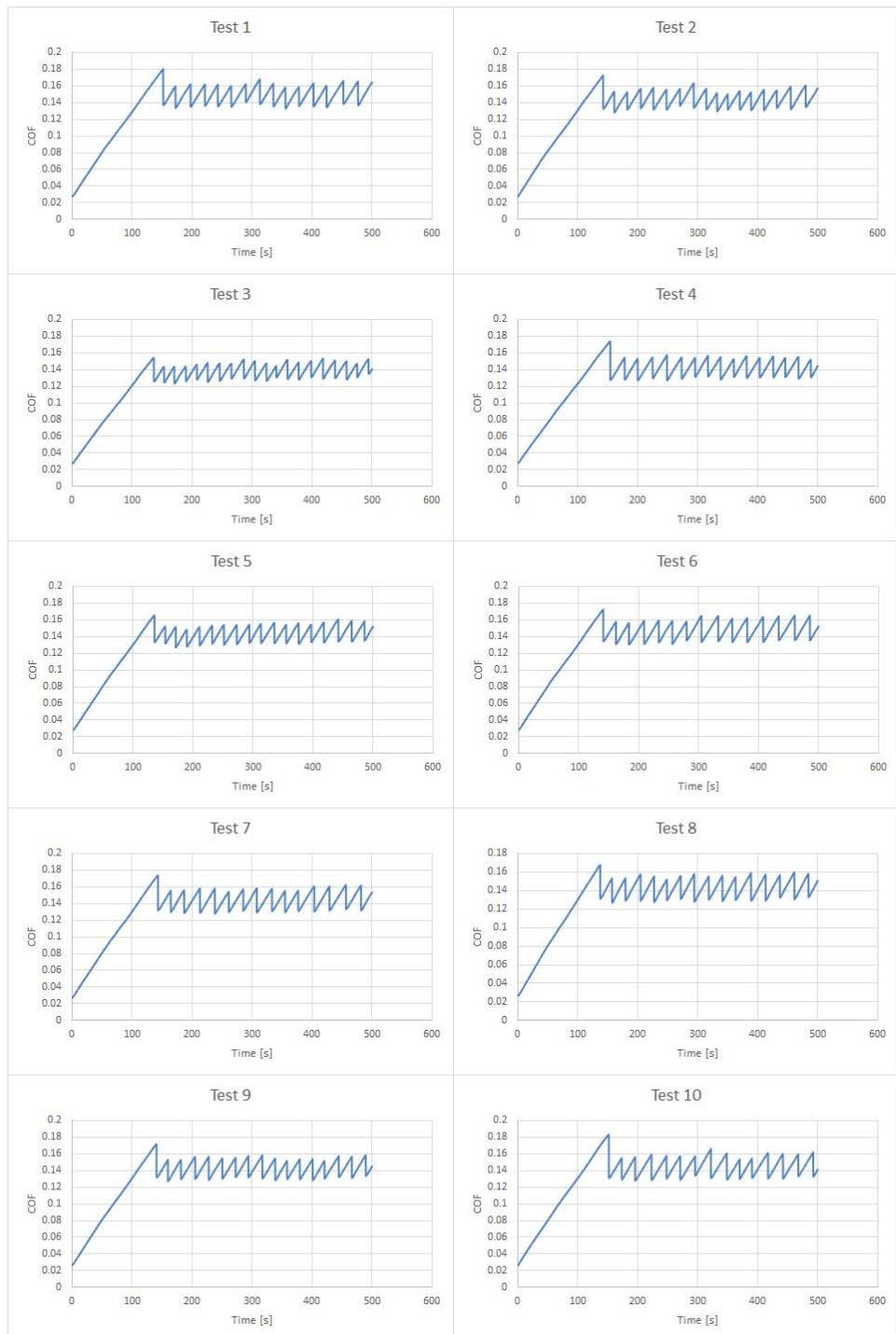


Fig. 5. Test results for the determination of the statistic parameters

For each of the 10 tests the first 10 stick-slip periods were selected and for these periods the values of μ_s and μ_k were noted and $\mu_v = (\mu_s - \mu_k)/2$ was calculated. Then their average values, noted with μ_{smed} , μ_{kmed} and μ_{vmed} , were calculated and also the value of μ_{smax} was noted. Furthermore, the mean for the values of these four coefficients was calculated. All of these values are presented in Table 1.

Table 1

Average values for the friction coefficient

Test	μ_{smax}	μ_{smed}	μ_{kmed}	μ_{vmed}
1	0.180	0.161	0.136	0.0128
2	0.173	0.156	0.132	0.0121
3	0.154	0.147	0.126	0.0104
4	0.174	0.155	0.129	0.0129
5	0.166	0.153	0.130	0.0113
6	0.173	0.160	0.133	0.0139
7	0.174	0.157	0.130	0.0135
8	0.168	0.155	0.129	0.0131
9	0.172	0.155	0.130	0.0128
10	0.183	0.158	0.130	0.0138
Mean($\bar{\mu}$)	0.172	0.156	0.130	0.0126

The standard deviation is a measure that is used to quantify the variation of a set of data values from the central tendency. It is calculated as [15, 16]:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}, \quad (1)$$

while the coefficient of variation is one of the most used indicators for the variation analysis and allows us to see the repeatability of the stick-slip phenomenon during the tests. It is defined as the ratio of the standard deviation and the mean [15, 16]:

$$CV = \frac{\sigma}{\bar{x}}, \quad (2)$$

where \bar{x} will be replaced by μ_{smed} , μ_{kmed} and μ_{vmed} .

The standard deviation, the coefficient of variation and their mean for μ_{smed} , μ_{kmed} and μ_{vmed} for each individual test are presented in Table 2.

Table 2

Standard deviation (σ) and coefficient of variation (CV) for each test					
Test	$\sigma(\mu_s)$	$\sigma(\mu_k)$	$\sigma(\mu_v)$	$CV(\mu_s)$	$CV(\mu_k)$
1	0.0028	0.0022	0.0012	0.0173	0.0165
2	0.0032	0.0021	0.0017	0.0202	0.0157
3	0.0025	0.0019	0.0010	0.0173	0.0151
4	0.0014	0.0017	0.0010	0.0089	0.0129
5	0.0023	0.0017	0.0008	0.0152	0.0134
6	0.0029	0.0015	0.0012	0.0179	0.0113
7	0.0014	0.0010	0.0009	0.0090	0.0078
8	0.0019	0.0018	0.0012	0.0126	0.0137
9	0.0021	0.0015	0.0010	0.0132	0.0113
10	0.0035	0.0015	0.0018	0.0220	0.0118
Mean	0.0024	0.0017	0.0012	0.0154	0.0129

The coefficient of variation for the static friction coefficient (μ_s) is between 0.9% and 2.2 % (1.55% in average), for the kinetic friction coefficient (μ_k) is between 0.78% and 1.65% (1.29% in average) and for the stick-slip amplitude is between 6.83% and 13.69% (9.46% in average). These values indicate that the stick-slip phenomenon that appears during the friction tests for the automotive disc-brake friction materials couple is a stable phenomenon.

In order to analyze the repeatability of the tests, the standard deviation and the coefficient of variation for μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} have been calculated. These values can be seen in Table 3.

Table 3

Standard deviation (σ) and coefficient of variation (CV) for all tests		
	σ	CV
μ_{smax}	0.0076	0.0430
μ_{smed}	0.0038	0.0241
μ_{kmed}	0.0024	0.0182
μ_{vmed}	0.0011	0.0839

Hereinafter, with the help of the Kolmogorov-Smirnov test, the distribution law of the experimental results for μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} was established. The Kolmogorov-Smirnov test verifies the compliance between a theoretical distribution $F(x)$ and an experimental one $F_n(x)$. After processing with the values grouped in intervals the maximum value of the difference is determined [16]:

$$d_n = F_n(x) - F(x), \quad (3)$$

and for an adopted significance level (α) the following Kolmogorov-Smirnov equation will be written:

$$K(\lambda) = 1 - \alpha = P\left(d_n \leq \frac{\lambda}{\sqrt{n}}\right), \quad (4)$$

Where $n = 10$, in our case, is the number of experimental results for μ_{smax} , μ_{smed} , μ_{kmed} and, respectively μ_{vmed} and λ , for the adopted level of significance, is the root of the equation:

$$Ec(\lambda, \alpha) = K(\lambda) + \alpha - 1. \quad (5)$$

If $d_n > \lambda/\sqrt{n}$, the hypothesis of the compliance between the empiric distribution and the theoretical one is accepted. If $d_n < \lambda/\sqrt{n}$, then the compliance hypothesis is rejected.

The Kolmogorov-Smirnov function is:

$$K(\lambda) = \sum_{-\infty}^{\infty} (-1)^k \cdot e^{-2k^2\lambda^2}, \quad (6)$$

where $k = 1, 2, \dots, n$.

In order to check the compliance of the experimental distribution of μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} , three theoretical distributions laws have been analyzed: normal (Gauss), exponential and Weibull. Thus, $F(x)$ will be:

$$F(x) = \begin{cases} F_G = 0.5 + \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}} dz, & \text{for Gauss distribution;} \end{cases} \quad (7a)$$

$$F_e = 1 - e^{\lambda x}, \quad \text{for exponential distribution;} \quad (7b)$$

$$F_W = 1 - e^{-(\frac{x-\gamma}{\eta})^\beta}, \quad \text{for Weibull distribution,} \quad (7c)$$

where η , β and γ are respectively the scale, form and position parameters which define the Weibull law.

By analyzing the normal distribution law (Gauss), the exponential distribution law and the Weibull distribution law, for a significance level $\alpha = 10^{-7}$ ($\lambda = 2.899$), the experimental results for μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} are compliant with the normal (Gauss) distribution law.

Using the 3σ rule, according to the normal (Gauss) distribution law:

- $\bar{\mu}_{smax} - 3\sigma < \mu_{smax} < \bar{\mu}_{smax} + 3\sigma$, where $\bar{\mu}_{smax} = 0.172$ and $\sigma = 0.0076$;
- $\bar{\mu}_{smed} - 3\sigma < \mu_{smed} < \bar{\mu}_{smed} + 3\sigma$, where $\bar{\mu}_{smed} = 0.156$ and $\sigma = 0.0038$;
- $\bar{\mu}_{kmed} - 3\sigma < \mu_{kmed} < \bar{\mu}_{kmed} + 3\sigma$, where $\bar{\mu}_{kmed} = 0.130$ and $\sigma = 0.0024$;
- $\bar{\mu}_{vmed} - 3\sigma < \mu_{vmed} < \bar{\mu}_{vmed} + 3\sigma$, where $\bar{\mu}_{vmed} = 0.0126$ and $\sigma = 0.0011$.

In consequence, the probability that any experimental results obtained for μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} will be inside the above specified intervals is 99.74%. The probability that they will not be in these intervals is very low (0.26%) and

according to the practical certitude principle this event can be considered impossible [16].

Another important statistical parameter is the coefficient of variation. The obtained values for the coefficient of variation for the specific values of the friction coefficient, presented in Table 3, indicate that the linear friction tests for the automotive disc-brake friction materials couple when the stick-slip phenomenon occurs are repeatable.

The second set of tests was designed to determine the influence of the system rigidity, the relative sliding speed and the contact pressure between the two specimens on the evolution of the friction coefficient and on the movement type that occurs during the braking process. For this, as mentioned above, the tests were conducted at four different relative sliding speeds (0.004 mm/s, 0.04 mm/s, 0.4 mm/s and 4 mm/s), for three different normal loads (25 N, 50 N and 100 N) which give three different contact pressures (0.430 MPa, 0.861 MPa and 1.722 MPa) and with three different system rigidities (7.159 N/mm, 31.405 N/mm and 79.904 N/mm). The results are presented in function of the normal load. With all of these taken into consideration, the results for the 25 N tests are presented in Fig. 6.

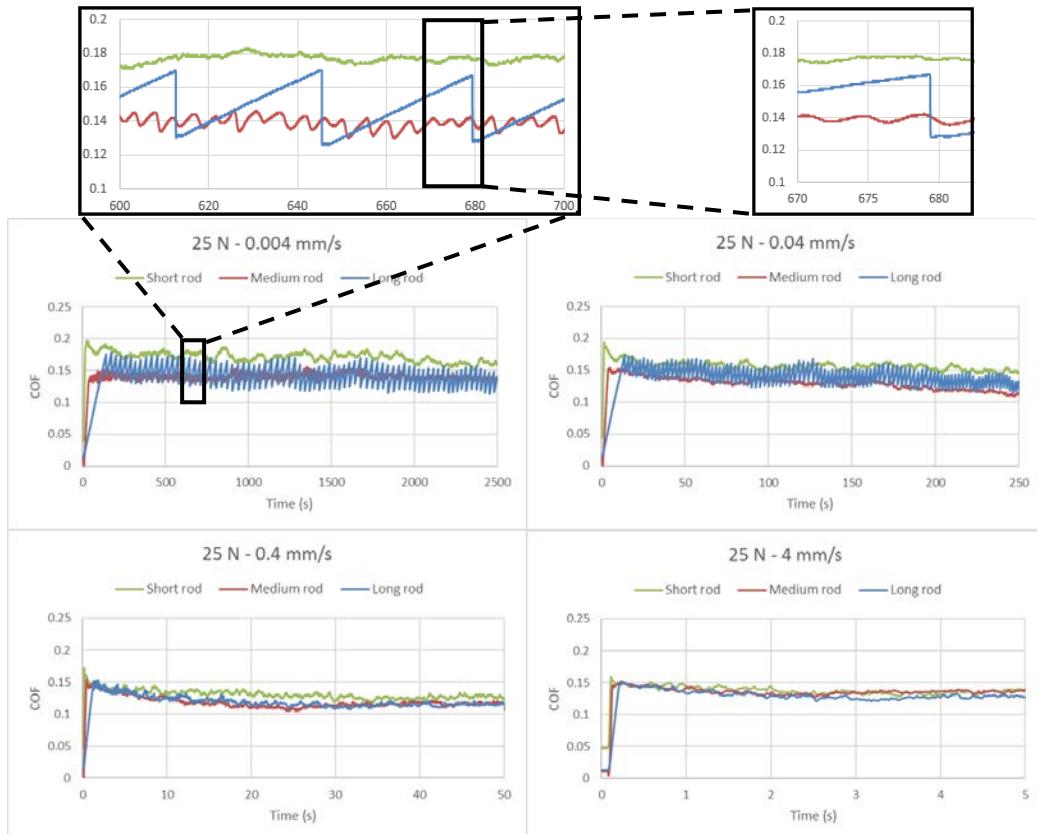


Fig. 6. The variation of the friction coefficient for the 25 N tests

In the top part of Fig. 6 there is a detail of the 25 N – 0.004 mm/s graph, that gives a better view of the evolution of the friction coefficient. As seen from the graphs for the 25 N (0.430 MPa) tests, the high rigidity of the system (short rod – 79.904 N/mm) determines a smooth sliding movement, with no occurring stick-slip, for the whole selected speed range. This is not valid for the other two system rigidities. For the medium system rigidity (medium rod – 31.405 N/mm), in the beginning of the 0.004 mm/s test there is stick-slip motion mixed with driven oscillations, while in the end of the tests the movement mostly takes the form of smooth sliding, with some areas where there are harmonic damped oscillations. The other three tests for the medium system rigidity present smooth sliding. For the low system rigidity (long rod – 7.159 N/mm), there is stick-slip movement for the two lower sliding speeds, while for the other two the movement is in the form of smooth sliding, with the mention that in the beginning of the 0.4 mm/s test there is a very short period of stick-slip mixed with driven oscillations.

The graphs for the 50 N (0.861 MPa) tests are presented in Fig. 7, where in the top part there is a detail of the last part of the 0.004 mm/s tests.

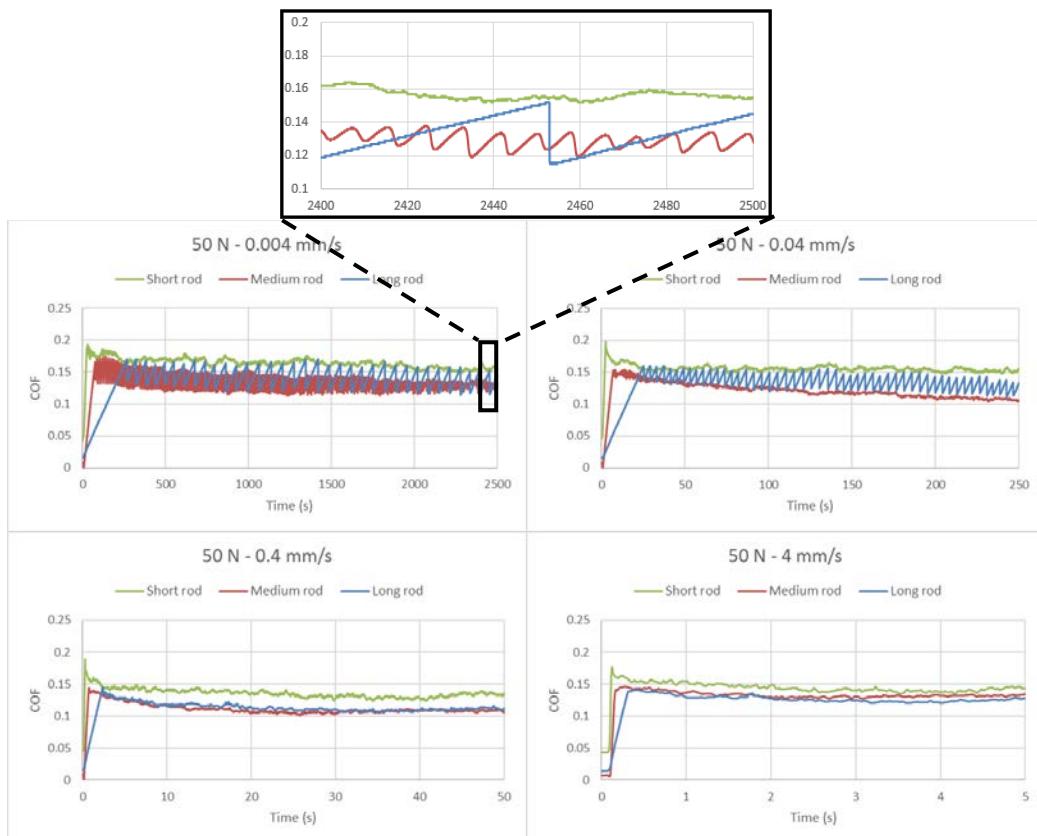


Fig. 7. The variation of the friction coefficient for the 50 N tests

As in the 25 N tests, all four 50 N tests for the high system rigidity present movement in the form of smooth sliding. The medium system rigidity tests for the 0.4 mm/s and 4 mm/s sliding speeds also have smooth sliding, while for the 0.04 mm/s sliding speed there is a short period in the beginning of the test that presents damped harmonic oscillations, after which the movement takes the form of smooth sliding. For the 0.004 mm/s sliding speed, when the medium connecting rod was used, stick-slip occurs almost throughout the whole test, with the exception of the ending part, where the stick-slip motion is mixed with driven oscillations, as seen in the detail graph. For the low system rigidity, the 0.004 mm/s and 0.04 mm/s tests have stick-slip motion, while the 0.4 mm/s and 4 mm/s tests have smooth sliding.

Fig. 8 presents the variation of the friction coefficient versus time for the 100 N (1.722 MPa) tests. In the top part of the figure, as in the other two cases, there is a detail of the 0.004 mm/s tests.

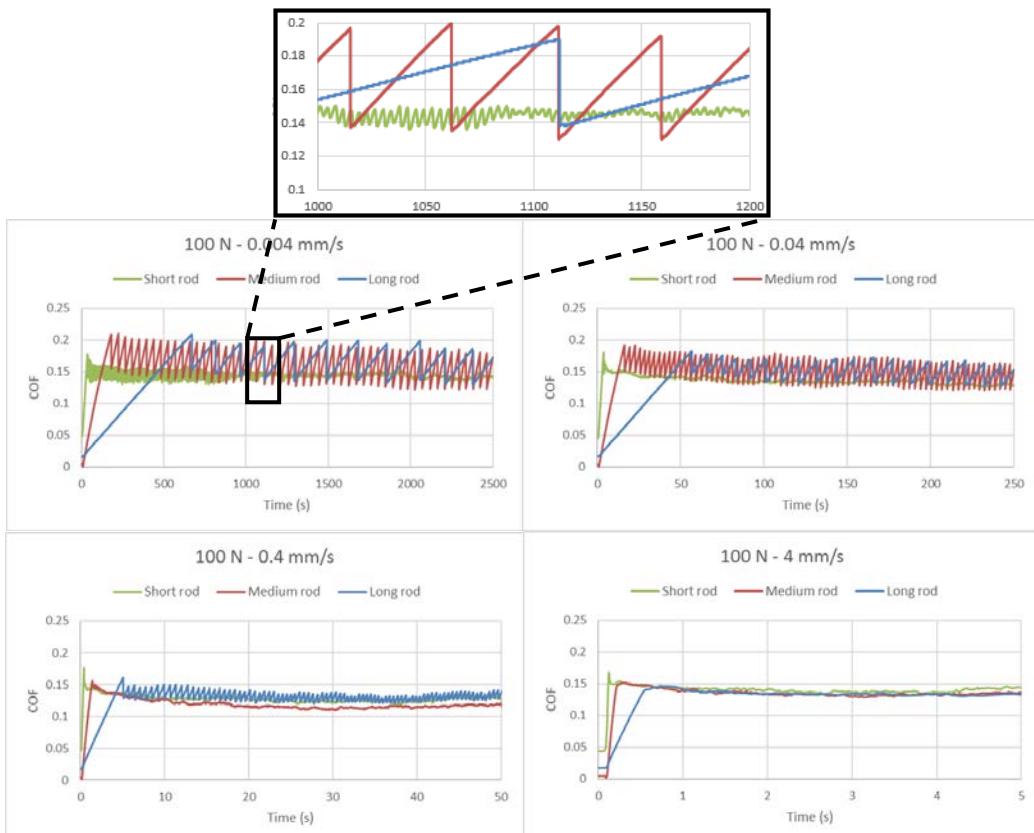


Fig. 8. The variation of the friction coefficient for the 100 N tests

For the 100 N normal load, the 0.004 mm/s tests present stick-slip motion for the low and medium system rigidities throughout their whole duration, while

for the high system rigidity, the stick-slip phenomenon appears at the beginning of the test and afterwards the movement takes the form of damped harmonic oscillations and smooth sliding. The 0.04 mm/s tests also present stick-slip motion for the low and medium system rigidities, while for the high system rigidity there is smooth sliding throughout the whole test. Smooth sliding also appears for the 0.4 mm/s tests in the case of the medium and high system rigidities, while the low rigidity given by the long connecting rod determines the occurrence of the stick-slip phenomenon. As in the case of the other two normal loads, the 4 mm/s tests for the 100 N normal load have smooth sliding throughout their whole duration, so it can be said that for this sliding speed, the selected normal loads and the system rigidities used for the current tests have no influence on the movement type.

The next step in the analysis which takes into consideration the influence of the relative sliding speed, of the contact pressure (normal load) and the system rigidity on the variation of the friction coefficient, was to determine the average value of the friction coefficient. These values are presented in Table 4 for each of 36 conducted tests.

Table 4

Average value of the friction coefficient

Long rod (Low system rigidity - 7.159N/mm)		Sliding speed [mm/s]			
		0.004	0.04	0.4	4
Normal force [N] (Contact pressure [MPa])	25 (0.430)	0.1424	0.1406	0.1193	0.1308
	50 (0.861)	0.1416	0.1355	0.1137	0.1267
	100 (1.722)	0.1660	0.1513	0.1316	0.1355
Medium rod (Medium system rigidity - 31.405N/mm)		Sliding speed [mm/s]			
Normal force [N] (Contact pressure [MPa])	25 (0.430)	0.1410	0.1295	0.1182	0.1354
	50 (0.861)	0.1340	0.1216	0.1111	0.1327
	100 (1.722)	0.1623	0.1515	0.1191	0.1349
Short rod (High system rigidity - 79.904N/mm)		Sliding speed [mm/s]			
Normal force [N] (Contact pressure [MPa])	25 (0.430)	0.1712	0.1564	0.1307	0.1373
	50 (0.861)	0.1645	0.1558	0.1358	0.1443
	100 (1.722)	0.1444	0.1363	0.1278	0.1411

The average value of the friction coefficient was calculated for the data recorded after μ_{smax} , when the movement was stabilized. These values were compared graphically in Fig. 9, where column a) presents the influence of sliding on the friction coefficient function of normal force (contact pressure) and column b) function of system rigidity.

As seen for all the contact pressures and system rigidities used for the current tests, the average value of the friction coefficient decreases with the increase of the relative sliding speed from thousands of mm/s to tenths of mm/s, while the increase from tenths of mm/s to mm/s causes an increase of the average friction

coefficient. It is known that when the friction coefficient decreases with the increase of the relative sliding speed, the stick-slip phenomenon can occur [10, 11, 17, 18, 19, 20]. Therefore, in the case of the current experiments (selected system rigidities and contact pressures), the stick-slip phenomenon is characteristic to sliding speeds up to tenths of mm/s, fact that is confirmed by the experimental results.

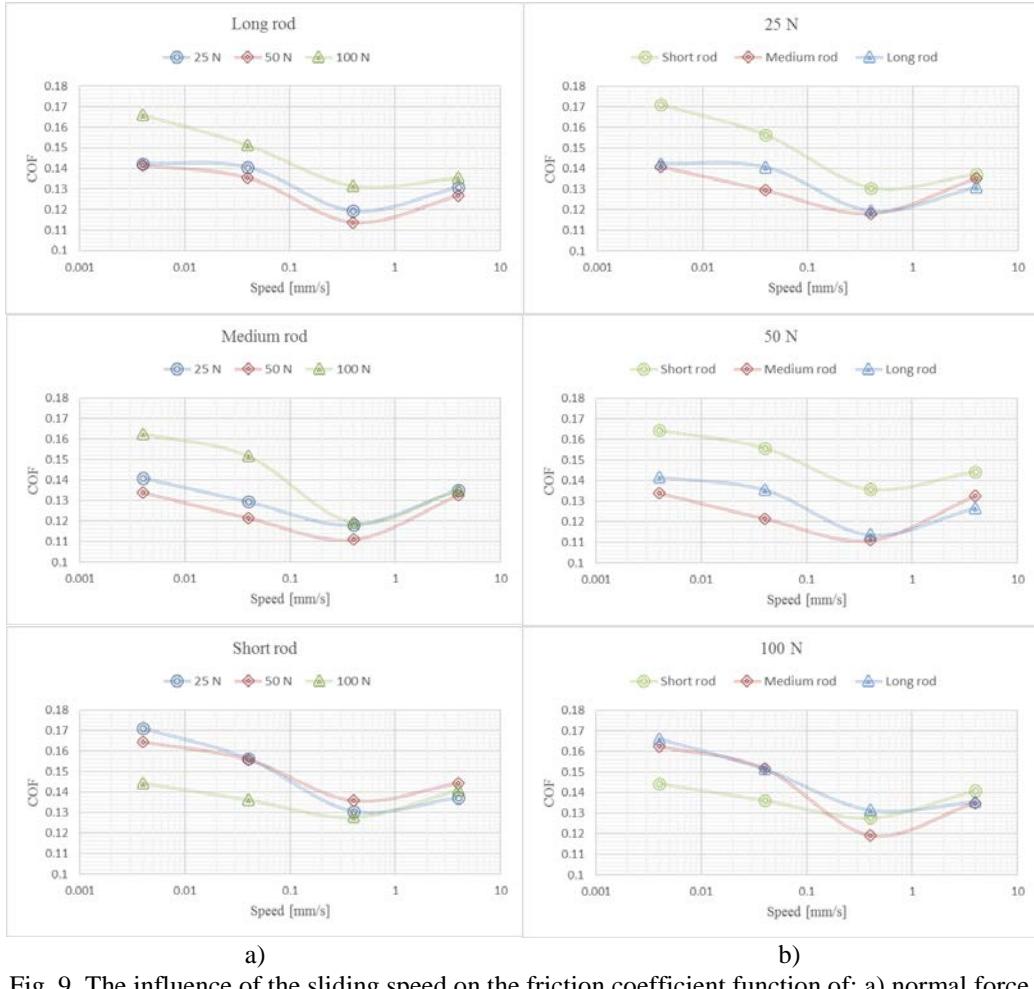


Fig. 9. The influence of the sliding speed on the friction coefficient function of: a) normal force,
b) system rigidity

4. Conclusions

The values of the coefficient of variation for the static friction coefficient (μ_s), for the kinetic friction coefficient (μ_k) and for the stick-slip amplitude (μ_v), indicate that the stick-slip phenomenon that appears during the friction tests for the automotive disc-brake friction materials couple is a stable phenomenon.

The Kolmogorov-Smirnov test indicates that the experimental results for μ_{smax} , μ_{smed} , μ_{kmed} and μ_{vmed} are compliant with the normal (Gauss) distribution law.

The coefficients of variation for the maximum static friction coefficient (μ_{smax}), for the average static friction coefficient (μ_{smed}), for the average kinetic friction coefficient (μ_{kmed}) and for the average stick-slip amplitude (μ_{vmed}) reveal that the linear friction tests for the automotive disc-brake friction materials couple when the stick-slip phenomenon occurs are repeatable.

The relative sliding speed, the contact pressure between the two specimens and the system rigidity influence the evolution of the friction coefficient on the movement type that occurs during the braking process.

During the linear friction tests, different forms of sliding have been revealed: stick-slip motion, driven oscillations mixed with stick-slip motion, harmonic oscillations, damped harmonic oscillations and smooth sliding. These forms of sliding are directly influenced by the relative sliding speed, the contact pressure and the system rigidity. By increasing the contact pressure and reducing the sliding speed and system rigidity, the stick-slip phenomenon appears more often, with an increased amplitude, while decreasing the contact pressure and increasing the sliding speed and system rigidity leads to smooth sliding.

For the current tests, the average value of the friction coefficient decreases with the increase of the relative sliding speed from thousands of mm/s to tenths of mm/s, while the increase from tenths of mm/s to mm/s causes an increase of the average friction coefficient. When the friction coefficient decreases with the increase of the relative sliding speed, the stick-slip phenomenon can occur, fact that is confirmed by the present experimental results, where the stick-slip phenomenon is characteristic to sliding speeds up to tenths of mm/s, after which it is replaced by smooth sliding.

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