

ALGEBRAIC CONSTRUCTION OF ENTANGLED COHERENT STATES IN THE PRESENCE OF SINGLE-MODE NOISE

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We propose a scheme for constructing entangled coherent states among two atoms with the presence of single-mode bosonic field mode, considered as noise mode, through non-localizing the respective Lie algebras of the atoms and field. It is observed that, due to the non-localized nature of the associated raising and lowering operators of the related Lie algebras, the constructed coherent state becomes entangled on the one hand, and on the other hand, it is figured out that the existence of the noise mode causes the degradation of constructed atomic entanglement. Also, our proposed dynamical approach for generating the constructed coherent states is discussed.

Keywords: Entanglement, Lie algebra, Non-locality, Coherent states.

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1. Introduction

Entanglement, as a pure quantum mechanical phenomenon, is one of the main differences between the classical and quantum physics which appears in composite systems. The description and quantification of the entanglement in the complex systems are very difficult tasks [1]. A state is said to be entangled if it cannot be factorized into tensor product of the associated states of the subsystems. Entanglement is now recognized to be of fundamental importance for developing new technologies notably those related to quantum information processing, such as quantum teleportation [2], quantum key distribution [3], superdense coding [4], and quantum computation [5]. Entanglement between the bosonic fields (such as photons or phonon) and atoms or ion, for its fundamental importance in quantum non-locality [6, 7] and quantum information [8], has been extensively studied both in theoretical and experimental efforts [9-19].

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In this paper, we study algebraic construction of entangled coherent among two identical $(2j + 1)$ -dimensional atoms in the presence of a single-mode bosonic field considered as a noise. To this aim, we consider that the atoms have angular momentum symmetry structure and the noise mode has the same structure as the harmonic oscillator.

In the next step, we introduce a scheme for non-localizing the related Lie algebras of atoms and noise mode. It is revealed that by this approach, the constructed coherent state for the atom-field (or atom-noise) system becomes entangled. Consequently, the degree atomic entanglement is evaluated by the concurrence. It is observed that the amount of atomic entanglement is strongly influenced by the power of the noise mode.

2. Non-localized model for the atom-field system

We consider a system composed of two identical atoms with the same structure along with a bosonic field mode called noise mode. It is supposed that the internal symmetrical structure of the atoms is characterized by the well-known angular momentum Lie algebra whose generators satisfy the following commutation relations

$$[\hat{J}_{i+}, \hat{J}_{i-}] = 2\hat{J}_{iz}, \quad [\hat{J}_{iz}, \hat{J}_{i\pm}] = \pm\hat{J}_{i\pm}, \quad (1)$$

where $i = 1, 2$, and the corresponding $(2j + 1)$ -dimensional representation spaces are

$$\mathcal{H}_i := \text{span}\{ | j, m_i \rangle\}_{m_i=0}^{2j}, \quad (2)$$

with the following realizations

$$\hat{J}_{i+} | j, m_i \rangle = \sqrt{(m_i + 1)(2j - m_i)} | j, m_i + 1 \rangle, \quad (3)$$

$$\hat{J}_{i-} | j, m_i \rangle = \sqrt{m_i(2j - m_i + 1)} | j, m_i - 1 \rangle, \quad (4)$$

$$\hat{J}_{iz} | j, m_i \rangle = (-j + m_i) | j, m_i \rangle. \quad (5)$$

The orthonormality relation for the base kets is

$$\langle j, m_i | j, m'_i \rangle = \delta_{m_i, m'_i}, \quad (6)$$

and these states are also eigenstates of the self-adjoint Casimir operator, i.e.

$$\hat{J}_i^2 = \hat{J}_{i+}\hat{J}_{i-} + \hat{J}_{iz}^2 - \hat{J}_{iz}, \quad (7)$$

as

$$\hat{J}_i^2 | j, m_i \rangle = j(j + 1) | j, m_i \rangle. \quad (8)$$

It should be noted that in quantum theory a quantum state is represented by a ket, i.e. $| \alpha \rangle$, and it dual by the bra, i.e. $\langle \beta |$. As a further illustration, for two quantum states α and β the inner product is written as $\langle \alpha | \beta \rangle$.

In quantum theory of radiation fields or photons, the electric and magnetic fields are Hermitian operators. The simplest method of constructing these operators is to decompose the fields into modes and associate to each mode a quantum harmonic oscillator as it can be done for the vibrational

modes or phonons. It is suitable to describe the observables associated to such an oscillator in terms of the annihilation and creation operators \hat{a} and \hat{a}^\dagger . The Hamiltonian for this oscillator, or field mode, is

$$\hat{H} = \hbar\omega(\hat{a}\hat{a}^\dagger + \frac{1}{2}) = \hbar(\hat{N} + \frac{1}{2}), \quad (9)$$

where \hat{N} is the Hermitian number operator defined as $\hat{N} = \hat{a}^\dagger\hat{a}$. The energy eigenstates are also eigenstates of \hat{N} and are denoted as number states, i.e. $|n\rangle$, where $n = 0, 1, 2, \dots$ and $\langle n | \hat{n} \rangle = \delta_{n,\hat{n}}$. The orthonormal number states $|n\rangle$ form a complete basis states for the Hilbert space of the harmonic oscillator, i.e. $\mathcal{H}_b := \text{span}\{|n\rangle\}_{n=0}^\infty$. The non-Hermitian operators \hat{a} and \hat{a}^\dagger and the Hermitian one \hat{N} satisfy the following commutation relations

$$[\hat{a}, \hat{a}^\dagger] = 1, \quad [\hat{N}, \hat{a}^\dagger] = \hat{a}, \quad [\hat{N}, \hat{a}] = -\hat{a} \quad (10)$$

and operate on the respective Hilbert space through the following forms:

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle, \quad (11)$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle, \quad (12)$$

$$\hat{N} |n\rangle = n |n\rangle. \quad (13)$$

Let us define the Hermitian number operator for each atom on the $(2j+1)$ -dimensional Hilbert space as

$$\hat{M}_i := j + \hat{J}_{iz}, \quad (14)$$

where $\hat{M}_i |j, m_i\rangle = m_i |j, m_i\rangle$. Also consider the following unitary operators corresponding to the number operators \hat{M}_i and \hat{N} as

$$\hat{\Pi}_i = e^{i\hat{M}_i\varphi}, \quad \hat{\Pi}_b = e^{i\hat{N}\varphi}, \quad (15)$$

where the subscripts b stands for the bosonic field and $\varphi \in [0, 2\pi]$. The operators $\hat{\Pi}_i$ and $\hat{\Pi}_b$ are unitary since $\hat{\Pi}_i^\dagger \hat{\Pi}_i = 1_i$ and $\hat{\Pi}_b^\dagger \hat{\Pi}_b = 1_b$ in which 1_i and 1_b are identity operators associated to Hilbert spaces of the atoms and the harmonic oscillator respectively. It is clear that the actions of $\hat{\Pi}_i$ and $\hat{\Pi}_b$ on the respective Hilbert spaces are as

$$\hat{\Pi}_i |j, m_i\rangle = e^{i\hat{m}_i\varphi} |j, m_i\rangle, \quad (16)$$

and

$$\hat{\Pi}_b |n\rangle = e^{i\hat{n}\varphi} |n\rangle. \quad (17)$$

Now we define the following operators for the atoms as

$$\hat{J}_{1\Pi+} := (\hat{J}_{1+} \otimes 1_2 \otimes 1_b) \hat{\Pi}, \quad \hat{J}_{1\Pi-} := \hat{\Pi}^\dagger (\hat{J}_{1-} \otimes 1_2 \otimes 1_b), \quad \hat{J}_{1\Pi z} := \hat{J}_{1z} \otimes 1_2 \otimes 1_b, \quad (18)$$

and

$$\hat{J}_{2\Pi+} := (1_1 \otimes \hat{J}_{2+} \otimes 1_b) \hat{\Pi}, \quad \hat{J}_{2\Pi-} := \hat{\Pi}^\dagger (1_1 \otimes \hat{J}_{2-} \otimes 1_b), \quad \hat{J}_{2\Pi z} := 1_1 \otimes \hat{J}_{21z} \otimes 1_b, \quad (19)$$

where $\hat{\Pi} = \hat{\Pi}_1 \otimes \hat{\Pi}_2 \otimes \hat{\Pi}_b$. It is easy to show that the above operators satisfy the angular momentum algebra commutation relations for $i = 1, 2$, i.e.

$$[\hat{J}_{i\Pi+}, \hat{J}_{i\Pi-}] = 2\hat{J}_{i\Pi z}, [\hat{J}_{i\Pi z}, \hat{J}_{i\Pi\pm}] = \pm\hat{J}_{i\Pi\pm}. \quad (20)$$

The operators in (19) are defined on a Hilbert space composed of the tensor product of the atomic Hilbert spaces and the bosonic one, i.e., $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_b$. Their actions on the basis states of the mentioned Hilbert space are realized as

$$\begin{aligned} & \hat{J}_{1\Pi+} | j, m_1, m_2 \rangle \otimes | n \rangle \\ &= e^{im_1\varphi} \sqrt{(m_1 + 1)(2j - m_1)} | j, m_1 + 1, m_2 \rangle \otimes | n \rangle, \end{aligned} \quad (21)$$

$$\begin{aligned} & \hat{J}_{1\Pi-} | j, m_1, m_2 \rangle \otimes | n \rangle \\ &= e^{-i(m_1-1)\varphi} \sqrt{m_1(2j - m_1 + 1)} | j, m_1 - 1, m_2 \rangle \otimes | n \rangle, \end{aligned} \quad (22)$$

$$\hat{J}_{1\Pi z} | j, m_1, m_2 \rangle \otimes | n \rangle = (-j + m_1) | j, m_1, m_2 \rangle \otimes | n \rangle. \quad (23)$$

And similarly,

$$\begin{aligned} & \hat{J}_{2\Pi+} | j, m_1, m_2 \rangle \otimes | n \rangle \\ &= e^{im_2\varphi} \sqrt{(m_2 + 1)(2j - m_2)} | j, m_1, m_2 + 1 \rangle \otimes | n \rangle, \end{aligned} \quad (24)$$

$$\begin{aligned} & \hat{J}_{2\Pi-} | j, m_1, m_2 \rangle \otimes | n \rangle \\ &= e^{-i(m_2-1)\varphi} \sqrt{m_2(2j - m_2 + 1)} | j, m_1, m_2 - 1 \rangle \otimes | n \rangle, \end{aligned} \quad (25)$$

$$\hat{J}_{2\Pi z} | j, m_1, m_2 \rangle \otimes | n \rangle = (-j + m_2) | j, m_1, m_2 \rangle \otimes | n \rangle. \quad (26)$$

Evidently, the ladder operators for example $\hat{J}_{1\Pi+}$ and $\hat{J}_{1\Pi-}$, along with the $\hat{J}_{1\Pi z}$, not only describe the symmetrical structure of the 1th atom but also give out information about the 2th atom and the bosonic field through the unitary operator $\hat{\Pi}$. Hence it is concluded that the action of $\hat{J}_{i\Pi+}$ and $\hat{J}_{i\Pi-}$ are non-local.

In the next step, we define the following operators as

$$\hat{a}_\Pi := (1_1 \otimes 1_2 \otimes \hat{a}^\dagger) \hat{\Pi}, \hat{a}_\Pi := \hat{\Pi}^\dagger (1_1 \otimes 1_2 \otimes \hat{a}), \hat{N}_\Pi := 1_1 \otimes 1_2 \otimes \hat{N}. \quad (27)$$

The commutation relations of these operators are the same as the commutation relations of harmonic oscillator (10) as it can be seen from

$$[\hat{a}_\Pi, \hat{a}_\Pi^\dagger] = 1, [\hat{N}_\Pi, \hat{a}_\Pi^\dagger] = \hat{a}_\Pi^\dagger, [\hat{N}_\Pi, \hat{a}_\Pi] = -\hat{a}_\Pi, \quad (28)$$

where $1 = 1_1 \otimes 1_2 \otimes 1_b$. The operators (27) with the commutation relations (28) operate on the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_b$ as follows:

$$\hat{a}_\Pi^\dagger | j, m_1, m_2 \rangle \otimes | n \rangle = e^{in\varphi} \sqrt{n+1} | j, m_1, m_2 \rangle \otimes | n+1 \rangle, \quad (29)$$

$$\hat{a}_\Pi | j, m_1, m_2 \rangle \otimes | n \rangle = e^{-i(n-1)\varphi} \sqrt{n} | j, m_1, m_2 \rangle \otimes | n-1 \rangle, \quad (30)$$

$$\hat{N}_\Pi | j, m_1, m_2 \rangle \otimes | n \rangle = n | j, m_1, m_2 \rangle \otimes | n \rangle. \quad (31)$$

As in the atomic case, the ladder field operators \hat{a}_Π and \hat{a}_Π^\dagger along with \hat{N}_Π in (27) show, on one hand, the symmetric properties of the harmonic oscillator, and, on the other hand, figure out information about the atomic system by the

unitary operator $\hat{\Pi}$. Here, we have developed a model for non-locality of the atom-field system throughout the above scheme. So, we expect to deal with entangled states for this compound quantum system.

3. Atomic entanglement and the noise effect

In this section, the construction of the standard coherent state for the atom-field system is obtained. Due to the non-locality of the atom-field system discussed in the previous section, the coherent state of the whole system is expected to be entangled. To this aim, the standard coherent states of the Lie algebra $su(2)$ for the atoms and harmonic oscillator algebras for the noise mode are constructed. The coherent state corresponding to the $su(2)$ Lie algebra, for each of the atoms, is obtained as

$$|\alpha_i\rangle := \hat{D}(\alpha_i) |j, 0\rangle = e^{\eta_i \hat{J}_{i+}} (1 + |\eta_i|^2)^{-\hat{J}_{iz}} e^{-\eta_i^* \hat{J}_{i-}} |j, 0\rangle \quad (32)$$

$$= \frac{1}{(1 + |\eta_i|^2)^j} e^{\eta_i \hat{J}_{i+}} |j, 0\rangle = \frac{1}{(1 + |\eta_i|^2)^j} \sum_{m_i=0}^{2j} \eta_i^{m_i} \sqrt{C_{m_i}^{2j}} |j, m_i\rangle,$$

where $\hat{D}(\alpha_i) = e^{\alpha_i \hat{J}_{i+} - \alpha_i^* \hat{J}_{i-}}$ is the displacement operator and $\eta_i = \frac{\alpha_i \tan(|\alpha_i|)}{|\alpha_i|}$. Similarly, the coherent state of the harmonic oscillator algebra (10), corresponding to the noise mode is presented as below

$$|\beta\rangle := \hat{D}(\beta) |0\rangle = e^{\frac{-|\beta|^2}{2}} e^{\beta \hat{a}^\dagger} |0\rangle = e^{\frac{-|\beta|^2}{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{n!} |n\rangle, \quad (33)$$

where $\hat{D}(\beta) = e^{\beta \hat{a}^\dagger - \beta^* \hat{a}}$ is the respective displacement operator. Now, the coherent state of the whole system is calculated as follow

$$|\alpha_1, \alpha_2, \beta\rangle = \hat{D}(\alpha_1, \alpha_2, \beta) |j, 0, 0\rangle \otimes |0\rangle \quad (34)$$

$$= \frac{e^{\frac{-|\beta|^2}{2}}}{(1 + |\eta_1|^2)^j (1 + |\eta_2|^2)^j} \sum_{m_1, m_2=0}^{2j} \sum_{n=0}^{\infty} \eta_1^{m_1} \eta_2^{m_2} \frac{\beta^n}{n!} \sqrt{C_{m_1}^{2j} C_{m_2}^{2j}} |j, m_1, m_2\rangle \otimes |n\rangle,$$

where $\hat{D}(\alpha_1, \alpha_2, \beta) = \hat{D}(\alpha_1) \otimes \hat{D}(\alpha_2) \otimes \hat{D}(\beta)$. Obviously, the coherent state in (34) is separable. In this step, we are interested in deriving the coherent state of the atom-field system with the introduced non-local operators (18), (19) and (27). By this considerations, the coherent state of the atom-field system corresponding to the commutation relations (20) and (28) in the associated Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_b$ leads to

$$|\alpha_1, \alpha_2, \beta\rangle_{\Pi} = \hat{D}_{\Pi}(\alpha_1, \alpha_2, \beta) |j, 0, 0\rangle \otimes |0\rangle \quad (35)$$

$$= \frac{e^{\frac{-|\beta|^2}{2}}}{(1 + |\eta_1|^2)^j (1 + |\eta_2|^2)^j} \sum_{m_1, m_2=0}^{2j} \sum_{n=0}^{\infty} e^T \eta_1^{m_1} \eta_2^{m_2} \frac{\beta^n}{n!} \sqrt{C_{m_1}^{2j} C_{m_2}^{2j}} |j, m_1, m_2\rangle \otimes |n\rangle,$$

where

$$T = \frac{i(m_1(m_1-1) + m_2(m_2-1) + n(n-1))\varphi}{2} + \frac{2m_1m_2 + 2m_1n + 2m_2n - 4m_1m_2n}{2}, \quad (36)$$

and $\hat{D}_\Pi(\alpha_1, \alpha_2, \beta) = \hat{D}_\Pi(\alpha_1) \otimes \hat{D}_\Pi(\alpha_2) \otimes \hat{D}_\Pi(\beta)$ with $\hat{D}_\Pi(\alpha_i) = e^{\alpha_i \hat{J}_{i\Pi+} - \alpha_i^* \hat{J}_{i\Pi-}}$ ($i = 1, 2$) and $\hat{D}_\Pi(\beta) = e^{\beta \hat{a}_\Pi^\dagger - \beta^* \hat{a}_\Pi}$. Consequently the state in (35) can not be factorized except for $\varphi = 0$ and $\varphi = 2\pi$. There is a remarkable situation for $\varphi = \pi$, where the state (35) takes the following form

$$|\alpha_1, \alpha_2, \beta\rangle_\Pi := \frac{1}{\sqrt{2}} (e^{\frac{i\pi}{4}} | -i\alpha_1\rangle \otimes | -i\alpha_2\rangle \otimes | -i\beta\rangle + e^{-\frac{i\pi}{4}} | i\alpha_1\rangle \otimes | i\alpha_2\rangle \otimes | i\beta\rangle), \quad (37)$$

where $|\pm i\alpha_i\rangle$, ($i = 1, 2$) and $|\pm i\beta\rangle$ are the same as (32) and (33) respectively. It should be noted that in deriving the equation (37), we used the following identity

$$(-1)^{\frac{m_1(m_1-1) + m_2(m_2-1) + n(n-1) + 2m_1m_2 + 2m_1n + 2m_2n - 4m_1m_2n}{2}} = \frac{1}{\sqrt{2}} e^{\frac{i\pi}{4}} (-i)^{m_1+m_2+n} + \frac{1}{\sqrt{2}} e^{-\frac{i\pi}{4}} i^{m_1+m_2+n}. \quad (38)$$

Clearly, the obtained coherent state in (37) is not generally separable so it can be entangled. It is observed that, by this construction, the noise mode is entangled with the atoms and therefore it can play a destructive role on the entanglement of atoms. To see this explicitly, let us write-down the reduced density operator of the two atoms for the case that $\alpha_1 = \alpha_2 = \alpha$, as follows

$$\begin{aligned} \rho_a &= Tr_b(|\alpha, \alpha, \beta\rangle_\Pi \langle \alpha, \alpha, \beta|) \\ &= \frac{1}{2} \begin{bmatrix} 1 + p^4 & (p^2 - iq)p\sqrt{1 - p^2} & 0 & 0 \\ (p^2 + iq)p\sqrt{1 - p^2} & p^2(1 - p^2) & 0 & 0 \\ (p^2 + iq)p\sqrt{1 - p^2} & p^2(1 - p^2) & 0 & 0 \\ (p^2 + iq)(1 - p^2) & p(1 - p^2)\sqrt{(1 - p^2)} & 0 & 0 \end{bmatrix} \\ &\quad + \frac{1}{2} \begin{bmatrix} 0 & 0 & (p^2 - iq)p\sqrt{1 - p^2} & (p^2 - iq)(1 - p^2) \\ 0 & 0 & p^2(1 - p^2) & p(1 - p^2)\sqrt{(1 - p^2)} \\ 0 & 0 & p^2(1 - p^2) & p(1 - p^2)\sqrt{(1 - p^2)} \\ 0 & 0 & p(1 - p^2)\sqrt{(1 - p^2)} & (1 - p^2)^2 \end{bmatrix}, \end{aligned} \quad (39)$$

where ρ_a stands for the reduced density matrix of the atomic system and

$$p = \left(\frac{1 - |\eta|^2}{1 + |\eta|^2} \right)^{2j}, \quad q = e^{-2|\beta|^2}.$$

As shown in Fig. 1, when the noise mode is in the vacuum state ($\beta = 0$), the concurrence [26] of the atomic system can reach to its maximum value. So, by this construction, two atoms can be maximally entangled for some values of α . However, by increasing the noise power (for example, $\beta = 0.5$ and $\beta = 1.5$) the atomic entanglement will be effectively decreased, i.e. the presence of noise

mode has a destructive role on the atomic entanglement.

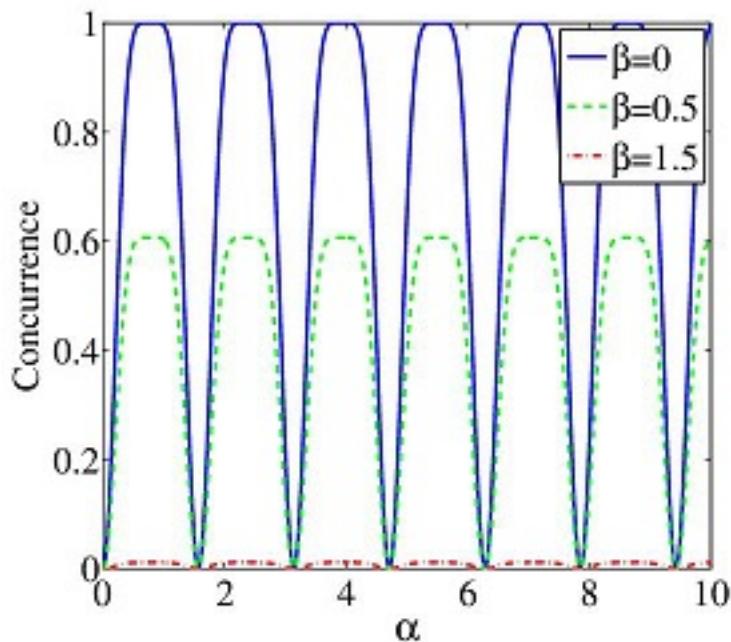


Figure 1. Degree of bipartite atomic entanglement in terms of concurrence as a function of parameter α for some values of parameter β ($j = \frac{3}{2}$).

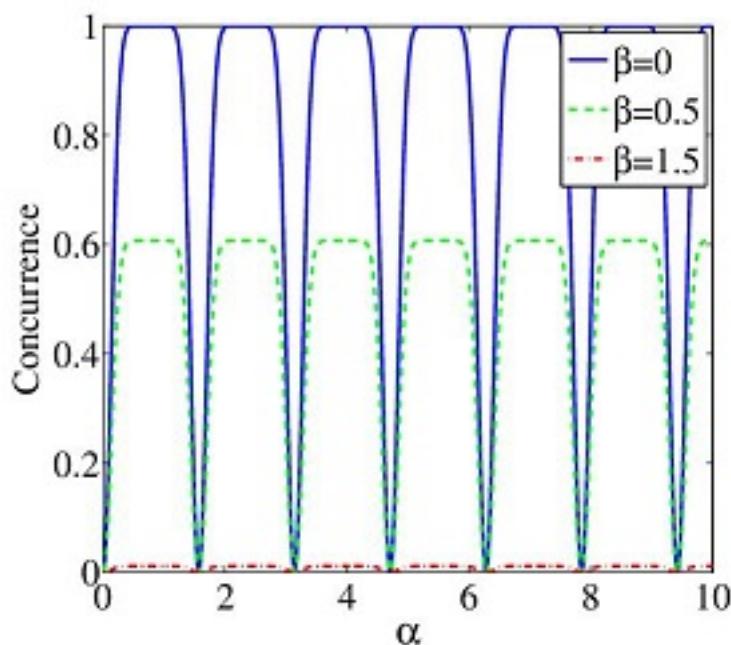


Figure 2. The concurrence in terms of parameter α for some values of parameter β ($j = \frac{7}{2}$).

4. Hamiltonian dynamical generation

In the previous section, we derived algebraically a generally entangled coherent state for two atoms and the noise mode such as equation (37). Now, it is worthwhile to generate the constructed entangled coherent state dynamically. To this aim, let us introduce the following Hamiltonian as

$$\hat{H} = \xi(\hat{M}_1^2 + \hat{M}_2^2 + \hat{N}^2 + 2\hat{M}_1\hat{M}_2 + 2\hat{M}_1\hat{N} + 2\hat{M}_2\hat{N} - 4\hat{M}_1\hat{M}_2\hat{N}). \quad (40)$$

The parameter ξ in (40) has energy dimension. The operators \hat{M}_1 , \hat{M}_2 and \hat{N} introduced in (13) and (14) respectively, are number operators. The fourth, fifth and sixth terms in (40) refer to the two-term interactions (atom-atom and atom-noise mode) while the latest term refers to the three-term interaction between two atoms and noise mode. It should be noted that the atom-photon entanglement generated by the Faraday rotation in a cavity can be analyzed by the Hamiltonian similar to the interaction term of (40) [26-28]. Now we assume that the initial state of the atom-field system is a coherent state such as $|e^{\frac{-i\varphi}{2}}\alpha_1, e^{\frac{-i\varphi}{2}}\alpha_2, e^{\frac{-i\varphi}{2}}\beta\rangle$ which is separable as the state (34). It is easily obtained that the time evolution of the state $|e^{\frac{-i\varphi}{2}}\alpha_1, e^{\frac{-i\varphi}{2}}\alpha_2, e^{\frac{-i\varphi}{2}}\beta\rangle$ under Hamiltonian (40), after $t = \frac{\pi}{2\xi}$, gives the coherent state (37).

5. Conclusions

In conclusion, by this approach, we analyzed an algebraic method for construction of bipartite atomic entanglement in the presence of a single-mode noise. It was shown that the existence of noise mode causes a decrement of the degree of entanglement. Also, we reproduced the algebraically constructed entangled state (37) by dynamical generation through introducing the Hamiltonian (40). It is worthwhile to note that the noise mode can be replaced by a large number of noise modes with a particular spectral distribution function such as Lorentzian or Ohmic which in turn shows the effect of a dissipative environment on the constructed atomic entanglement in the real world, which can be considered as a subject of future research.

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