

## RECONSTRUCTION OF A TIME-DEPENDENT POTENTIAL IN A PSEUDO-HYPERBOLIC EQUATION

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*In this paper, an initial boundary value problem for a pseudo-hyperbolic equation is considered. Giving an over-determination condition, a time-dependent potential is determined and existence and uniqueness theorem for small times is proved. Also, theorem of the conventional stability of the solution of the inverse problem is given.*

**Keywords:** Pseudo-hyperbolic equation, Inverse problem, Fourier method.

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### 1. Introduction

Problems on vibrations of continuous media (string, rod, gas, membrane, etc.) and problems on electromagnetic oscillations are reducible to equations of hyperbolic type.

Transverse vibrations of a flexible rod reduce to a pseudo-hyperbolic equation of fourth order, while the longitudinal vibrations reduce to a hyperbolic equation of second order. There are number of important physical problems, reducible to equations of hyperbolic type for functions: for example; in the steady flow around a body of a supersonic stream of gas an equation of hyperbolic type is obtained for the velocity potential.

Consider the problem for the transverse vibration of a rod which ends are fixed by hinges with arbitrary initial conditions;

$$u_{tt} + a^2 u_{xxxx} = p(t)u + f(x, t), \quad 0 \leq x \leq 1, \quad 0 \leq t \leq T, \quad (1)$$

$$u(x, 0) = \varphi(x), \quad u_t(x, 0) = \psi(x), \quad 0 \leq x \leq 1, \quad (2)$$

$$u(0, t) = u(1, t) = u_{xx}(0, t) = u_{xx}(1, t) = 0, \quad 0 \leq t \leq T, \quad (3)$$

where  $a^2 = \frac{EJ}{\rho S}$ ,  $E$  is the modulus of elasticity of the rod,  $\rho$  is the mass density,  $S$  is the cross-sectional area of the rod,  $J$  is the geometric moment of inertia

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of a cross-section with respect to a diameter, perpendicular to the plane of vibration and  $f(x, t)$  is continuously distributed transverse force.

When the potential  $p(t)$  is given, the problem of finding  $u(x, t)$  from the equation (1), initial conditions (2), and boundary conditions (3) is termed as a direct (forward) problem. The well posedness of the direct problem has been established in [1,2] and for more information see [3,4,5].

When  $p(t)$  for  $t \in [0, T]$  is unknown, the inverse problem formulates as that of finding a pair of functions  $\{p(t), u(x, t)\}$  which satisfy the equation (1), initial conditions (2), boundary conditions (3) and the over-determination condition

$$u(x_0, t) = h(t), \quad x_0 \in (0, 1), \quad t \in [0, T]. \quad (4)$$

The inverse problems for pseudo-hyperbolic equations are scarce. Identification problems for pseudo-hyperbolic integro-differential operator equations are studied in [6] and inverse boundary value problem for a Boussinesq type equation of fourth order with non-local time integral conditions of the second kind is investigated in [7].

In this paper, we have an initial-boundary value problem for a pseudo-hyperbolic equation of fourth order. Giving an additional condition, a time-dependent potential multiplying linear term is determined. We prove the existence and uniqueness of the solution of the inverse problem for small times, and characterize the conventional stability of this solution.

The article is organized as following: In Section 2, we first present auxiliary spectral problem of the problem (1)-(4) and its eigenvalues and eigenfunctions. Then the series expansion method in terms of eigenfunctions converts the inverse problem to a fixed point problem in a suitable Banach space. Under some consistency and regularity conditions on the initial and boundary data the existence and uniqueness of the solution of the inverse problem is shown by the way that the fixed point problem has unique solution for small  $T$ . In Section 3, we will give the theorem for continuous dependence upon the data in a certain class of data.

## 2. Existence and Uniqueness of the Solution

In this section, we will examine the existence and uniqueness of the solution of the inverse initial-boundary value problem for the Eq.(1) with time-dependent potential.

**Definition 2.1.** *The pair  $\{p(t), u(x, t)\}$  from the class  $C[0, T] \times (C^4(D_T) \cap C^2(\overline{D}_T))$  for which the conditions (1)-(4) are satisfied is called the classical solution of the inverse problem (1)-(4).*

From this definition, the consistency conditions

$$(A_0): \begin{cases} \varphi(0) = \varphi''(0) = \varphi(1) = \varphi''(1) = 0, \\ \psi(0) = \psi''(0) = \psi(1) = \psi''(1) = 0, \\ h(0) = \varphi(x_0), \quad h'(0) = \psi(x_0), \end{cases}$$

holds for the data  $\varphi(x), \psi(x) \in C^2[0, 1]$  and  $h(t) \in C^1[0, T]$  with  $h(t) \neq 0, \forall t \in [0, T]$ .

Since the function  $p$  is space independent and the boundary conditions are linear and homogeneous, the method of separation of variables is suitable.

The auxiliary spectral problem of the problem (1)-(3) is

$$\begin{cases} X^{(4)}(x) - \lambda X(x) = 0, \quad 0 \leq x \leq 1, \\ X(0) = X(1) = X''(0) = X''(1) = 0. \end{cases} \quad (5)$$

This spectral problem has the eigenvalues  $\lambda_n = \mu_n^2$ , and the eigenfunctions  $X_n(x) = \sin(\mu_n x)$ , where  $\mu_n = n\pi, n = 1, 2, \dots$  (see [1])

Let seek the solution of the problem (1)-(4) in the following form:

$$u(x, t) = \sum_{n=1}^{\infty} u_n(t) X_n(x). \quad (6)$$

From the Eq.(1) and initial conditions (2), we obtain

$$\begin{cases} u_n''(t) + a^2 \mu_n^4 u_n(t) = F_n(t; p, u), \\ u_n(0) = \varphi_n, \quad u_n'(0) = \psi_n, \end{cases}, n = 1, 2, \dots \quad (7)$$

where

$$\begin{aligned} F_n(t; p, u) &= p(t)u_n(t) + f_n(t), \quad f_n(t) = \sqrt{2} \int_0^1 f(x, t) \sin(\mu_n x) dx, \\ \varphi_n &= \sqrt{2} \int_0^1 \varphi(x) \sin(\mu_n x) dx, \\ \psi_n &= \sqrt{2} \int_0^1 \psi(x) \sin(\mu_n x) dx, \quad n = 1, 2, \dots \end{aligned}$$

Solving the Cauchy problems (7), we get

$$\begin{aligned} u_n(t) &= \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \\ &+ \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t - \tau)) d\tau. \end{aligned} \quad (8)$$

Substituting Eq.(8) into Eq.(6), the second component of the pair  $\{p(t), u(x, t)\}$  is

$$u(x, t) = \sum_{n=1}^{\infty} \left[ \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \right. \\ \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t - \tau)) d\tau \right] \sin(\mu_n x). \quad (9)$$

Consider  $x = x_0$  in Eq.(1) and using the over-determination condition (4) and the Eq.(8), we obtain the first component of the pair as

$$p(t) = \frac{1}{h(t)} \left[ h''(t) - f(x_0, t) + \sum_{n=1}^{\infty} \mu_n^4 \left( \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \right. \right. \\ \left. \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t - \tau)) d\tau \right) \sin(\mu_n x_0) \right]. \quad (10)$$

Thus, the solution of problem (1)-(4) is reduced to the solution of system (9)-(10) with respect to the unknown functions  $\{p(t), u(x, t)\}$ .

From the definition of the classical solution of problem (1)-(4), the following lemma is proved.

**Lemma 2.1.** *If  $\{p(t), u(x, t)\}$  is any solution of problem (1)-(4), then the functions*

$$u_n(t) = \sqrt{2} \int_0^1 u(x, t) \sin(\mu_n x) dx, n = 1, 2, \dots$$

*satisfy the Eq.(8) in  $[0, T]$ .*

From Lemma 2.1, it follows that to prove the uniqueness of the solution of the problem (1)-(4) is equivalent to prove the uniqueness of the solution of system (9)-(10).

Let us denote  $z = [p(t), u(x, t)]^T$ . The vector function  $z$  satisfies the system of Eqs.(9) and (10) which can be written as an operator equation

$$z = \Phi(z). \quad (11)$$

The operator  $\Phi$  is determined in the set of functions  $z$  and has the form  $[\phi_1, \phi_2]^T$ , where

$$\phi_1(z) = \frac{1}{h(t)} \left[ h''(t) - f(x_0, t) + \sum_{n=1}^{\infty} \mu_n^4 \left( \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \right. \right. \\ \left. \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t - \tau)) d\tau \right) \sin(\mu_n x_0) \right], \quad (12)$$

$$\begin{aligned}\phi_2(z) = & \sum_{n=1}^{\infty} \left[ \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \right. \\ & \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t-\tau)) d\tau \right] \sin(\mu_n x).\end{aligned}\quad (13)$$

Let us introduce the functional space

$$\begin{aligned}B_{2,T}^5 &= \left\{ u(x, t) = \sum_{n=1}^{\infty} u_n(t) \sin(\mu_n x) : u_n(t) \in C[0, T], \right. \\ J_T(u) &= \left[ \sum_{n=1}^{\infty} \left( \mu_n^5 \|u_n(t)\|_{C[0,T]} \right)^2 \right]^{1/2} < +\infty \left. \right\}\end{aligned}$$

with the norm  $\|u(x, t)\|_{B_{2,T}^5} := J_T(u)$  which relates the Fourier coefficients of the function  $u(x, t)$  by the eigenfunctions  $\sin(\mu_n x)$ ,  $n = 1, 2, \dots$ . It is shown in [8] that  $B_{2,T}^5$  is Banach space. Obviously  $E_T^5 = B_{2,T}^5 \times C[0, T]$  with the norm

$$\|z\|_{E_T^5} = \|u(x, t)\|_{B_{2,T}^5} + \|p(t)\|_{C[0,T]}$$

is also Banach space.

Let us show that  $\Phi$  maps  $E_T^5$  onto itself continuously. In other words, we need to show  $\phi_1(z) \in C[0, T]$  and  $\phi_2(z) \in B_{2,T}^5$  for arbitrary  $z = [p(t), u(x, t)]^T$  with  $p(t) \in C[0, T]$ ,  $u(x, t) \in B_{2,T}^5$ .

We will use the following assumptions on the data of problem (1)-(4):

$$\begin{aligned}(A_1): \varphi(x) &\in C^5[0, 1], \quad \left\{ \begin{array}{l} \varphi(0) = \varphi''(0) = \varphi^{(4)}(0) = 0, \\ \varphi(1) = \varphi''(1) = \varphi^{(4)}(1) = 0, \\ \psi(0) = \psi''(0) = 0, \end{array} \right. \\ (A_2): \psi(x) &\in C^3[0, 1], \quad \left\{ \begin{array}{l} \psi(1) = \psi''(1) = 0, \end{array} \right. \\ (A_3): h(t) &\in C^2[0, T], \quad h(t) \neq 0, \quad \forall t \in [0, T], \quad h(0) = \varphi(x_0), \quad h'(0) = \psi(x_0), \\ (A_4): f(x, t) &\in C(\overline{D}_T), \quad f(\cdot, t) \in C^3[0, 1], \quad \forall t \in [0, T], \quad \left\{ \begin{array}{l} f(0, t) = f_{xx}(0, t) = 0, \\ f(1, t) = f_{xx}(1, t) = 0. \end{array} \right.\end{aligned}$$

By using integration by parts under the assumptions (A<sub>0</sub>)-(A<sub>4</sub>), we have

$$\begin{aligned}\varphi_n &= \frac{-\sqrt{2}}{\mu_n^5} \int_0^1 \varphi^{(5)}(x) \cos(\mu_n x) dx, \\ \psi_n &= \frac{-\sqrt{2}}{\mu_n^3} \int_0^1 \varphi'''(x) \cos(\mu_n x) dx, \\ f_n(t) &= \frac{-\sqrt{2}}{\mu_n^3} \int_0^1 f_{xxx}(x, t) \cos(\mu_n x) dx.\end{aligned}\quad (14)$$

First, let us show that  $\phi_1(z) \in C[0, T]$ . From the Eqs.(12) and (14), we obtain

$$|\phi_1(z)| \leq \frac{1}{|h(t)|} \left[ |h''(t)| + |f(x_0, t)| + \sum_{n=1}^{\infty} \left( \frac{1}{\mu_n} |\alpha_n| + \frac{1}{a\mu_n} |\beta_n| \right. \right. \\ \left. \left. + \frac{1}{a\mu_n} \int_0^T \{|p(t)| (\mu_n^3 |u_n(t)|) + |\gamma_n(t)|\} dt \right) \right]$$

where  $\alpha_n = \mu_n^5 \varphi_n$ ,  $\beta_n = \mu_n^3 \psi_n$ ,  $\gamma_n(t) = \mu_n^3 f_n(t)$ .

By using Cauchy-Schwartz and Bessel inequalities, we get

$$\max_{0 \leq t \leq T} |\phi_1(z)| \leq \frac{1}{\|h(t)\|_{C[0,T]}} \left[ \|h''(t)\|_{C[0,T]} + \|f(x_0, t)\|_{C[0,T]} \right. \\ \left. + \frac{1}{\sqrt{6}} \|\varphi^{(5)}(x)\|_{L_2[0,1]} + \frac{1}{a\sqrt{6}} \|\psi'''(x)\|_{L_2[0,1]} \right. \\ \left. + \frac{1}{a\sqrt{6}} \|f_{xxx}(x, t)\|_{L_2(D_T)} + \frac{T}{a\sqrt{6}} \|p(t)\|_{C[0,T]} \|u(x, t)\|_{B_{2,T}^5} \right]. \quad (15)$$

Since the right hand side of Eq.(15) is bounded, this implies that  $\phi_1(z)$  belongs to  $C[0, T]$ .

Now, let us show that  $\phi_2(z) \in B_{2,T}^5$ , i.e. we need to show that

$$J_T(\phi_2) = \left[ \sum_{n=1}^{\infty} \left( \mu_n^5 \|\phi_{2n}(t)\|_{C[0,T]} \right)^2 \right]^{1/2} < +\infty,$$

where

$$\phi_{2n}(t) = \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; a, u) \sin(a\mu_n^2(t-\tau)) d\tau$$

by (13). After some manipulations under the assumptions (A<sub>0</sub>)-(A<sub>4</sub>), we get

$$\phi_{2n}(t) = \frac{1}{\mu_n^5} \alpha_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^5} \beta_n \sin(a\mu_n^2 t) \\ + \frac{1}{a\mu_n^2} \int_0^t \left( p(\tau) u_n(\tau) + \frac{1}{\mu_n^3} \gamma_n(\tau) \right) \sin(a\mu_n^2(t-\tau)) d\tau.$$

From the last equality, we get

$$\left[ \sum_{n=1}^{\infty} \left( \mu_n^5 \|\phi_{2n}(t)\|_{C[0,T]} \right)^2 \right]^{1/2} \leq \sqrt{2} \|\varphi^{(5)}(x)\|_{L_2[0,1]} + \frac{\sqrt{2}}{a} \|\psi'''(x)\|_{L_2[0,1]} \\ + \frac{T\sqrt{2}}{a} \|f_{xxx}(x, t)\|_{L_2(D_T)} + \frac{TC_1}{a} \|p(t)\|_{C[0,T]} \|u(x, t)\|_{B_{2,T}^5}, \quad (16)$$

where  $C_1 = \left[ \sum_{n=1}^{\infty} \frac{1}{\mu_n^4} \right]^{1/2}$ . The right hand side of Eq.(16) is finite. Thus  $J_T(\phi_2) < +\infty$  and  $\phi_2(z)$  is belongs to the space  $B_{2,T}^5$ .

Let us now show that  $\Phi$  is a contraction mapping on  $E_T^5$ . Let  $z_1$  and  $z_2$  be any two elements of  $E_T^5$ . We know that  $\|\Phi(z_1) - \Phi(z_2)\|_{E_T^5} = \|\phi_1(z_1) - \phi_1(z_2)\|_{C[0,T]} + \|\phi_2(z_1) - \phi_2(z_2)\|_{B_{2,T}^5}$ . Here  $z_i = [p^i(t), u^i(x, t)]^T$ ,  $i = 1, 2$ .

From the Eqs.(12) and (13), we obtain

$$\phi_1(z_1) - \phi_1(z_2) = \frac{1}{ah(t)} \sum_{n=1}^{\infty} \mu_n^2 \int_0^t (p^1(\tau)u_n^1(\tau) - p^2(\tau)u_n^2(\tau)) d\tau \sin(\mu_n x_0), \quad (17)$$

$$\phi_2(z_1) - \phi_2(z_2) = \frac{1}{a} \sum_{n=1}^{\infty} \frac{1}{\mu_n^2} \int_0^t (p^1(\tau)u_n^1(\tau) - p^2(\tau)u_n^2(\tau)) d\tau \sin(\mu_n x). \quad (18)$$

After some manipulations in Eqs.(17) and (18), we get

$$\|\Phi(z_1) - \Phi(z_2)\|_{E_T^5} \leq A(T)C(p^1, u^2) \|z_1 - z_2\|_{E_T^5}$$

where  $A(T) = \frac{T}{a} \left( C_1 + \frac{1}{6\|h(t)\|_{C[0,T]}} \right)$  and  $C(p^1, u^2)$  is the positive constant which is equal to  $\max \left\{ \|p^1(t)\|_{C[0,T]}, \|u^2(x, t)\|_{B_{2,T}^5} \right\}$ .

Since  $A(T)$  has limit zero as  $T$  tends to zero. Thus for sufficiently small  $T$ , the operator  $\Phi$  is contraction mapping which maps  $E_T^5$  onto itself continuously. Then according to Banach fixed point theorem there exists a unique solution of Eq.(11) in  $E_T^5$ .

Thus, we proved the following theorem:

**Theorem 2.1** (Existence and uniqueness). *Let the assumptions  $(A_0)$ - $(A_4)$  be satisfied. Then, the inverse problem (1)-(4) has a unique solution for small  $T$ .*

### 3. Continuous dependence upon the data

In this section, we characterize the estimation of conventional stability of the solution of inverse problem. Such an estimate can be obtained by setting a certain class of data  $\mathfrak{S}(\alpha, M_0, M_1, M_2, M_3)$  for the functions  $\varphi(x)$ ,  $\psi(x)$ ,  $h(t)$ ,  $f(x, t)$  and a class  $\mathfrak{N}(N_0)$  for the function  $p(t)$  if they satisfy

$$\|f\|_{C^3(\overline{D}_T)} \leq M_0, \quad \|\varphi\|_{C^5[0,1]} \leq M_1, \quad \|\psi\|_{C^3[0,1]} \leq M_2,$$

$$\|h\|_{C^2[0,T]} \leq M_3, \quad 0 < \alpha \leq |h(t)|,$$

and

$$\|p(t)\|_{C[0,T]} \leq N_0,$$

respectively.

It is easy to see that, since  $\varphi(x), \psi(x), h(t), f(x, t) \in \mathfrak{F}(\alpha, M_0, M_1, M_2, M_3)$  and  $p(t) \in \mathbb{N}(N_0)$ ,

$$\|u(x, t)\|_{B_{2,T}^5} \leq N_1$$

where  $N_1 = \frac{\sqrt{2}}{1 - \frac{T C_1}{a} N_0} (M_1 + \frac{M_2}{a} + \frac{T M_0}{a})$ .

Let  $\{p(t), u(x, t)\}$  and  $\{\bar{p}(t), \bar{u}(x, t)\}$  be two solutions of the inverse problem (1)-(4) corresponding to the data  $\varphi(x), \psi(x), h(t), f(x, t)$  and  $\bar{\varphi}(x), \bar{\psi}(x), \bar{h}(t), \bar{f}(x, t)$ , respectively. Then, we obtain from Eqs.(9) and (10)

$$\begin{aligned} p(t) - \bar{p}(t) &= \frac{1}{h(t)\bar{h}(t)} \left\{ \bar{h}(t) [h''(t) - f(x_0, t) + \sum_{n=1}^{\infty} \mu_n^4 (\varphi_n \cos(a\mu_n^2 t) \right. \\ &\quad \left. + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; p, u) \sin(a\mu_n^2(t-\tau)) d\tau) \sin(\mu_n x_0)] \right. \\ &\quad \left. - h(t) [\bar{h}''(t) - \bar{f}(x_0, t) + \sum_{n=1}^{\infty} \mu_n^4 (\bar{\varphi}_n \cos(a\mu_n^2 t) \right. \\ &\quad \left. + \frac{1}{a\mu_n^2} \bar{\psi}_n \sin(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; \bar{p}, \bar{u}) \sin(a\mu_n^2(t-\tau)) d\tau) \sin(\mu_n x_0)] \right\} \end{aligned} \quad (19)$$

and

$$\begin{aligned} u(x, t) - \bar{u}(x, t) &= \sum_{n=1}^{\infty} \left[ \varphi_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \psi_n \sin(a\mu_n^2 t) \right. \\ &\quad \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; p, u) \sin(a\mu_n^2(t-\tau)) d\tau \right] \sin(\mu_n x) \\ &\quad - \sum_{n=1}^{\infty} \left[ \bar{\varphi}_n \cos(a\mu_n^2 t) + \frac{1}{a\mu_n^2} \bar{\psi}_n \sin(a\mu_n^2 t) \right. \\ &\quad \left. + \frac{1}{a\mu_n^2} \int_0^t F_n(\tau; \bar{p}, \bar{u}) \sin(a\mu_n^2(t-\tau)) d\tau \right] \sin(\mu_n x) \end{aligned} \quad (20)$$

where

$$\begin{aligned} F_n(t; \bar{p}, \bar{u}) &= \bar{p}(t)\bar{u}_n(t) + \bar{f}_n(t), \quad \bar{f}_n(t) = \sqrt{2} \int_0^1 \bar{f}(x, t) \sin(\mu_n x) dx, \\ \bar{\varphi}_n &= \sqrt{2} \int_0^1 \bar{\varphi}(x) \sin(\mu_n x) dx, \\ \bar{\psi}_n &= \sqrt{2} \int_0^1 \bar{\psi}(x) \sin(\mu_n x) dx, \quad n = 1, 2, \dots \end{aligned}$$

Denote the difference between two functions with the tilde ( $\sim$ ), i.e.  $\tilde{p} = p - \bar{p}$ ,  $\tilde{u} = u - \bar{u}$ , etc. Then, under the conditions (A<sub>0</sub>)-(A<sub>4</sub>) by using the estimates given above we obtain from Eqs.(19) and (20)

$$\|\tilde{p}(t)\|_{C[0,T]} \leq \frac{D_1}{\Delta(T)} \left\{ \|\tilde{h}\|_{C^2[0,T]} + \|\tilde{\varphi}\|_{C^5[0,1]} + \|\tilde{\psi}\|_{C^3[0,1]} + \|\tilde{f}\|_{C^3(\bar{D}_T)} \right\} \quad (21)$$

where  $D_1 = \max\{-r_3\sqrt{2} + r_4\frac{M_3}{\alpha^2\sqrt{6}}, -r_3\frac{\sqrt{2}}{a} + r_4\frac{M_3}{\alpha^2a\sqrt{6}}, -r_3\frac{T\sqrt{2}}{a} + r_4\frac{M_3}{\alpha^2}(1 + \frac{T}{a\sqrt{6}}), \frac{r_4}{\alpha^2}[2M_3 + M_2(1 + \frac{1}{a\sqrt{6}}) + \frac{M_0}{\sqrt{6}} + \frac{1}{a\sqrt{6}}(M_1 + N_0N_1T)]\}$ ,  $\Delta(T) = r_1r_4 - r_2r_3 \neq 0$ ,  $r_1 = 1 - \frac{TM_3N_1}{\alpha^2a\sqrt{6}}$ ,  $r_2 = \frac{TN_1C_1}{a}$ ,  $r_3 = \frac{TM_3N_0}{\alpha^2a\sqrt{6}}$ ,  $r_4 = 1 - \frac{TN_0C_1}{a}$ .

Similarly, we get the estimate

$$\|\tilde{u}(x, t)\|_{B_{2,T}^5} \leq \frac{D_2}{\Delta(T)} \left\{ \|\tilde{h}\|_{C^2[0,T]} + \|\tilde{\varphi}\|_{C^5[0,1]} + \|\tilde{\psi}\|_{C^3[0,1]} + \|\tilde{f}\|_{C^3(\bar{D}_T)} \right\} \quad (22)$$

where  $D_2$  is dependent only the parameters  $\alpha, M_0, M_1, M_2, M_3, N_0$  and  $N_1$ .

Thus, we proved the following theorem:

**Theorem 3.1** (continuous dependence upon the data). *Let  $\{p(t), u(x, t)\}$  and  $\{\bar{p}(t), \bar{u}(x, t)\}$  be two solutions of the inverse problem (1)–(4) with the data  $\varphi(x), \psi(x), h(t), f(x, t)$  and  $\bar{\varphi}(x), \bar{\psi}(x), \bar{h}(t), \bar{f}(x, t)$ , respectively, which satisfy the conditions of the Theorem 2.1. Moreover,  $\varphi(x), \psi(x), h(t), f(x, t), \bar{\varphi}(x), \bar{\psi}(x), \bar{h}(t), \bar{f}(x, t) \in \mathfrak{S}(\alpha, M_0, M_1, M_2, M_3)$  and  $p(t), \bar{p}(t) \in \mathfrak{N}(N_0)$ . Then the estimates (21)–(22) are true for small  $T$ . The constants  $D_1$  and  $D_2$  depend only on the choice of the classes  $\mathfrak{S}(\alpha, M_0, M_1, M_2, M_3)$  and  $\mathfrak{N}(N_0)$ .*

#### 4. Conclusions

The inverse problems for pseudo-hyperbolic equations connected with recovery of the coefficient are scarce. The paper considers the inverse problem of recovering a time-dependent potential in an initial-boundary value problem for a pseudo-hyperbolic equation of fourth order. The series expansion method in terms of eigenfunction of a Sturm-Liouville problem converts the considered inverse problem to a fixed point problem in a suitable Banach space. Under some consistency and regularity conditions on the initial and boundary data, the existence and uniqueness of the solution of the inverse problem is shown by using the Banach fixed point theorem and conventional stability of the solution of the inverse problem is shown in a certain class of data.

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