

## ENTROPY GENERATION ANALYSIS AND COP EVALUATION FOR A REVERSED QUASI-CARNOT CYCLE (Refrigeration Machine) BY USING THE DIRECT METHOD FROM FINITE SPEED THERMODYNAMICS

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*Lucrarea prezintă o abordare complet nouă și originală a analizei ireversibilităților generate în timpul funcționării mașinilor frigorifice cvasi-Carnot cu vapor. În acest articol sunt dezvoltate expresii pentru calculul sursei interne de generare a entropiei și a coeficientului de performanță, în funcție de viteza finită a procesului și de alți parametri, precum temperatura vaporilor, raportul de comprimare/destindere, debitul masic și proprietățile vaporilor pentru fiecare proces ce se desfășoară cu viteză finită în mașinile frigorifice ireversibile cvasi-Carnot. În cadrul acestor calcule sunt luate în considerare proprietățile gazului real. Această abordare, bazată pe Metoda Directă, propune o alternativă viabilă pentru proiectarea și optimizarea acestui tip de mașini.*

*The paper presents a completely new and original approach of quasi-Carnot refrigeration machines working with vapor. It provides expressions for calculating the internal entropy generation source and COP, as a function of the finite speed of the process and other parameters such as temperature of the vapor, compression/expansion ratio, mass flow rate and vapor properties for each finite speed process of the reversed irreversible quasi-Carnot cycle refrigeration machines. The real gas properties are considered in these calculations. This approach, based on the Direct Method, becomes an important issue in design and optimization of such machines.*

**Keywords:** entropy generation, irreversible quasi-Carnot cycle, Direct Method, refrigeration machines with finite speed, Thermodynamics with Finite Speed

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**Nomenclature**

$a$	coefficient ( $= \sqrt{3\gamma}$ )	$w$	specific work, $J\ kg^{-1}$
$c$	average molecular speed, $m\ s^{-1}$	$w_p$	piston speed, $m\ s^{-1}$
$c_p, c_v$	specific heats, $J\ kg^{-1}\ K^{-1}$	<b>Greek symbols</b>	
$COP$	coefficient of performance	$\alpha$	convection heat transfer coefficient, $W\ m^{-2}\ K$
$D$	diameter, $m$	$\lambda$	thermal conductivity, $W\ m^{-1}\ K^{-1}$
$f$	coefficient related to the friction contribution $\in (0, 1)$	$\lambda_p$	compression ratio in the compressor
$h$	specific enthalpy, $J\ kg^{-1}$	$\theta$	the temperature ratio
$K$	the overall heat transfer coefficient, $W\ m^{-2}\ K^{-1}$	$k$	ratio of the specific heats
$L$	length, $m$	$\rho$	density of vapor, $kg\ m^{-3}$
$m$	mass, $kg$	<b>Subscripts</b>	
$N$	number of pipes,	$ad$	adiabatic
$p$	pressure, $Pa$	$C_p$	compressor
$\Delta p$	pressure loss, $Pa$	$exp$	expansion
$Q$	heat, $J$	$f$	friction
$q$	heat lost, $J\ kg^{-1}$	$H$	the hot-end of the machine
$R$	gas constant, $J\ kg^{-1}\ K^{-1}$	$i$	internal
$s$	specific entropy, $J\ kg^{-1}\ K^{-1}$	$ins$	insulation
$S$	entropy generation, $J\ K^{-1}$	$irr$	irreversible
$\dot{S}$	entropy generation rate, $W\ K^{-1}$	$L$	the cold-end of the engine
$T$	temperature, $K$	$med$	average
$U$	internal energy, $J$	$r$	reversible
$V$	volume, $m^3$	$thr$	throttling
$v$	specific volume, $m^3\ kg^{-1}$	$w, f$	with finite speed and friction
$\Delta V$	volume variation, $m^3$		

**1. Introduction**

The **entropy generation** estimation becomes an important issue when internal and external irreversibilities of a thermodynamic cycle are considered and an analytical approach of them is sought.

The **optimization of Carnot cycle** and the calculation of the **entropy generation** is a topic previously developed by the authors for Carnot and Stirling cycle engines [1-13]. The papers [1-12] present analysis models based on the **Direct Method and the First Law of Thermodynamics for processes with finite speed** [14-18]. In these papers, the study was further developed for an irreversible Carnot cycle with perfect gas as working fluid, achieved in four separate machine components (an isothermal expansion component at  $T_H$ , an adiabatic expansion component, an isothermal compression component at  $T_L$  and an adiabatic compression component) that are connected through tubes and

valves, keeping the expansion ratio during the isothermal expansion at the high temperature constant.

The results were obtained for a particular set of engine parameters, namely the optimum piston speed for maximum power and optimum speed for maximum first law efficiency have been found and it has been shown that minimum entropy generation per cycle occurs at maximum efficiency.

Recently a similar model was developed for a Carnot cycle refrigeration machine [11,12]. The objective of those works was to find an analytical expression for the internal entropy generation per cycle and entropy generation rate [11], and to compare it [12] with proposed equations in the literature [14]. Also, an attempt of validation was made by using experimental data [15] available for a real operating refrigeration machine [12].

This paper analyses the irreversibilities generation in a reversed irreversible quasi-Carnot cycle machine (Mechanical Vapor Compression Refrigeration Machine) starting from previous work [12,13] in which a reversed irreversible Carnot cycle with perfect gas was studied, functioning with finite speed.

We are going to define a *quasi-Carnot cycle* as any cycle reversible or irreversible, direct or reversed, which departs a “little bit” from a Carnot cycle [16]. In this sense, Rankine and refrigeration cycles (and corresponding heat pump cycles) are quasi-Carnot cycles, because they depart from a Carnot cycle (2 adiabatic and 2 isothermal processes) just on the small portion which is only isobaric (at high pressure) and not isothermal (in the domain of saturated vapor) and all the others 3 processes are exactly like in the Carnot cycle, respectively one entirely isothermal (at low pressure) and two adiabatic processes [16].

The objective of this approach, in comparison with others [4] is to take into account the essential differences between the behavior of perfect gases and vapors, and to analyze the changes necessary to develop a methodology for calculating fully analytical the irreversibilities (*entropy generation*) and performances evaluation (*efficiency* and *power*) of such a cycle.

The Direct Method takes into consideration any irreversible cycle step by step, by using the mathematical expression of the First Law of Thermodynamic for processes with finite speed on each process and integrating them throughout the cycle. This leads to finding the equations of irreversible processes in the cycle depending on finite speed of the processes (and other characteristic parameters for the cycle: pressure ratio, temperature ratio at the sources etc., finally the analytical assessment of the sources of entropy and of the performances (*COP* and *power* [16])).

In the previous papers mentioned above [12,13] the expressions of the internal generation of entropy depending on the finite speed of the process were obtained and also depending on other parameters such as gas temperature,

volumetric ratio, mass flow rate and gas properties in the adiabatic processes with finite speed of reversed Carnot cycle machines, with perfect gas.

In this paper the pressure ratio  $p_2/p_1$  (exit-entry of the compressor) replaces the volumetric ratio, while the deviation of vapors behavior from perfect gas is considered like in [16].

An example of a comparative study (four cycles in which irreversibilities are introduced gradually) is presented in order to illustrate the use of these expressions to assess the internal entropy generation and also the evaluation of the machine performance, *COP*.

## 2. Fundamental equations from Finite Speed Thermodynamics [16]

The *entropy generation per cycle*, *entropy generation rate* and *COP* for a quasi-Carnot refrigeration cycle with irreversible processes are computed based on the **First Law of Thermodynamics for processes with Finite Speed** [8, 17, 18]:

$$dU = \delta Q_{irr} - p \left( 1 \pm \frac{aw_p}{c} \pm \frac{f \Delta p_f}{p} \pm \frac{\Delta p_{th}}{p} \right) dV \quad (1)$$

where the **work in irreversible processes** is expressed as:

$$\delta W_{irr} = p \left( 1 \pm \frac{aw_p}{c} \pm \frac{\Delta p_f}{p} \pm \frac{\Delta p_{th}}{p} \right) dV \quad (2)$$

The two terms from eq. (1 and 2) look similar, but actually they have different significance, despite of some correlations between them. Namely, in eq. (2) the term multiplied with  $p$  and  $dV$  is the friction work between piston and cylinder. But the term from eq. (1), which contains  $f$ , is only the amount of heat generated by friction which remains in the system (gas). The difference between the two terms, namely  $\Delta p_f (1 - f) dV$  represents the heat passing through the cylinder and given to the surroundings. Because of that this portion from the friction work transformed into heat does not remain in the system, and must not appear in the eq. (1).

In equations (1) and (2), the sign (+) is for compression and the sign (−) is for expansion, and each term in parenthesis takes into account one type of internal irreversibility, as follows:

$aw_p/c$  = contribution of **finite speed** of the piston, with:  $c = \sqrt{k R T}$ ;  $a = \sqrt{3 \gamma}$ ;

$\Delta p_f/p$  = contribution of **mechanical friction** between mechanical parts;

$\Delta p_{th}/p$  = contribution of **throttling process** through the valves;

where:  $p$  is a new concept in comparison with Reversible Thermodynamics, namely: *the instantaneously average pressure in the system* [17].

The mechanical friction and throttling losses are expressed in a similar manner to the case of internal combustion engines from [19], adapted by the authors to be included in the mathematical expression of the First Law for Processes with Finite Speed ( $w_p$ ) for the study of irreversible cycles with external and internal irreversibilities [8, 9]:

$$\Delta p_f = A + B' w_p \quad \Delta p_{thr} = C w_p^2 \quad (3)-(4)$$

where  $A = 0.94$ ,  $B' = 0.045$  and  $C = 0.0045$  [19].

*Observation:* When  $A = 0$  the friction at the limit  $w_p \rightarrow 0$  is not taken into account, but of the friction with velocity  $w_p$  is taken into account the variation and in the case  $A = 0.94$  it takes into account the existing friction even at the limit  $w_p \rightarrow 0$ .

The First Law expression is combined with the *internal entropy generation definition* given by:

$$\frac{Q_H}{T_H} + \frac{Q_L}{T_L} + S_i = 0 \quad (5)$$

respectively, with the *total entropy generation per cycle*, given by:

$$\frac{Q_H}{T_{H,S}} + \frac{Q_L}{T_{L,S}} + S_T = 0 \quad (6)$$

The *internal and total entropy generation* ( $S_i$  and  $S_T$ ) as well as *the internal and total entropy generation rates*:  $\dot{S}_i$ ,  $\dot{S}_T$  will be computed from equations (5) –(6) combined with (1) and (2).

## 2. Application of the Direct Method to the irreversible quasi-Carnot cycle refrigeration machine

The Direct Method from Finite Speed Thermodynamics consists in analyzing any irreversible cycle, step by step, on each process, by writing the corresponding equation of the First Law of Thermodynamics for finite speed processes and integrating it on the whole cycle. The First Law expression for finite speed processes includes all three of the principal sources of internal irreversibility, namely: (1) *finite speed* interaction between the piston and the gas/vapor, (2) *friction* due to the finite speed of the piston within the cylinder, (3) *throttling processes* in valves. These throttling losses are directly due to finite gas/vapor velocity and ultimately due to the finite speed of the piston.

The paper presents a comparison of the reversible cycle 1-2r-3-4r-1 with the irreversible cycle with finite speed 1-2irr-3-4irr-1, from the point of view of **COP** and **entropy generation**. Equations (1) and (2) can be integrated

(analytically) for any processes in an irreversible cycle with finite speed, in order to obtain the process equations and also the expressions for the irreversible work and heat. This is the “power” of the Direct Method, in comparison with Thermodynamics in Finite Time developed after Curzon – Ahlborn seminal paper [20].

For the cycle illustrated in *Fig. 1*, equation (1) is integrated only for the irreversible adiabatic process 1-2<sub>irr</sub>, in the compressor. Thus, one gets the **equation of irreversible adiabatic compression in the compressor**. This equation contains the origin of the internal irreversibilities, namely: (1) *finite speed of the piston* and (2) *friction between piston and cylinder*.

Based on this equation, it will be possible to get the temperature in the state 2<sub>irr</sub>, namely  $T_{2irr}$ . With this temperature one can get from the table of superheated vapor the specific properties  $h_{2irr}$  and  $s_{2irr}$ , which are necessary for computation of the work consumed in the compressor and the increase of entropy (entropy generation) in the irreversible adiabatic process 1-2<sub>irr</sub>, namely:

$$\Delta s_{Cp} = s_{2irr} - s_1 \quad (7)$$

The tables with thermodynamic properties of vapor will not be used, because the aim is to obtain a **total analytic scheme of computation** of the irreversible cycle, in a similar way as it is done in the Reversible Classical Thermodynamics, and it was done by us for Stirling Machines [5].

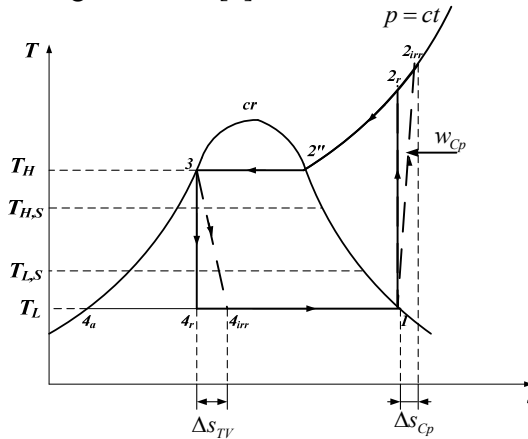


Fig. 1. Irreversible quasi-Carnot Refrigeration cycle on  $T$ - $S$  coordinates.

Each process in the irreversible quasi-Carnot cycle shown in *Fig. 1* occurs in a separate component: compressor (1-2<sub>irr</sub>), condensate (2<sub>irr</sub>-3), detentor (3-4) or throttling valve (3-4<sub>irr</sub>). The resulting four components are assumed to be connected by tubes. In order to derive an expression easy to apply (in an analytical way) for getting the *internal entropy generation*, and  $COP_{irr}$ , only the irreversibility on the adiabatic processes in the compressor (1-2<sub>irr</sub>) have been

considered here, as shown in *Fig. 1*. In the total irreversible process 3-4<sub>irr</sub> (in the throttling valve) enthalpy is constant:  $h_3 = h_{4irr}$ , and thus there is no need of another equation for this process.

### 2.1. Internal Entropy Generation calculations

For the *internal entropy generation per cycle*, one gets from (5):

$$\Delta s_{i,cycle} = -\frac{q_H}{T_H} - \frac{q_L}{T_L} = -\frac{q_{2irr3}}{T_H} - \frac{q_{4irr1}}{T_L} \quad (8)$$

which leads to:

$$\Delta s_{i,cycle} = -(s_3 - s_{2irr}) - (s_1 - s_{4irr}) = (s_{4irr} - s_3) + (s_{2irr} - s_1) = \Delta s_{3-4irr} + \Delta s_{1-2irr} \quad (9)$$

Regarding the first term:  $\Delta s_{3-4irr} = \Delta s_{i,ad.exp}$ , it is easy to compute as in the Classical Thermodynamics, based on the fact that  $h_3 = h_{4irr}$  and using tables. But because a total analytic computation is sought, the properties  $h$  and  $s$  on the limiting curves as a function of pressure and temperatures are expressed as in *Table 1*.

Regarding the second term  $\Delta s_{1-2irr} = \Delta s_{i,ad.Cp}$ , the situation is completely different. Here the Direct Method is used in order to get an equation of this irreversible process 1-2<sub>irr</sub>.

### 2.2. Entropy generation during an adiabatic irreversible process with finite speed

In order to determine the entropy generation during an adiabatic irreversible process,  $\Delta s_{ad,irr}$ , one needs to find the irreversible adiabatic process equation, with finite speed, friction and throttling, which in our cycle (*Fig. 1*) is the equations for the irreversible adiabatic processes 1-2<sub>irr</sub>.

The First Law for processes with finite speed, eq. (1), is used and integrated by applying the Direct Method.

Assuming the hypothesis that the working fluid is a perfect gas, and imposing the adiabatic process condition,  $\delta Q_{irr} = 0$ , from eq. (1) it results:

$$mc_v dT = -p \left( 1 \pm \frac{aw_p}{\sqrt{3RT}} \pm \frac{f \Delta p_f}{p} \pm \frac{\Delta p_{th}}{p} \right) dV \quad (10)$$

where the factor  $f$  shows the part of friction heat that remains inside the system,  $0 \leq f \leq 1$ . The case  $f = 0$  corresponds to the case when all the friction heat is „lost” towards the surroundings (at the cold source); the case  $f = 1$  is the other

extreme, when all friction heat remains inside the system. The influence of this factor  $f$  on  $COP$  and *entropy generation* will be presented further in this work.

Equation (10) could be integrated in different assumptions in order to avoid combersome calculations. The simplest method of integration is described below: one denotes the parenthesis that collects the irreversibility causes with  $B = const. = f(T_{med,1-2}, p_{med,1-2})$ :

$$B = 1 \pm \frac{aw_p}{\sqrt{3RT_{med,1-2}}} \pm \frac{f \cdot \Delta p_f}{p_{med,1-2}} \pm \frac{\Delta p_{thr}}{p_{med,1-2}} \quad (11)$$

where the mean temperature is expressed as:

$$T_{med,1-2} = \frac{T_1 + T_2}{2} \quad (12)$$

In order to estimate  $T_2$  from (12) we assume, in a first approximation, that  $T_2 \cong T_{2r}$ . The equation for the reversible adiabatic process 1-2<sub>r</sub> yields:

$$\theta = \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k'-1}{k'}} = \lambda_p^{\frac{k'-1}{k'}} \quad (13)$$

where:  $\lambda_p = p_2/p_1$  and  $k'$  is a *corrected adiabatic exponent*, which takes into account the difference between perfect gas and vapor of R134a (Table 1). This corrected  $k'$  exponent was obtained comparing  $T_2$  computed with eq. (13), and  $T_{2r}$  computed based on constant entropy in the reversible adiabatic process 1-2<sub>r</sub>, and using tables data for vapor in state 2<sub>r</sub>. As result of this computation Fig. 2 was plotted and the corresponding analytical formula for  $k'$  was derived.

For evaluation of  $p_{med,1-2}$  one uses the arithmetic average between initial and final pressures.

$$p_{med,1-2} = \frac{p_1 + p_2}{2} = \frac{p_1}{2} \left( 1 + \frac{p_2}{p_1} \right) = \frac{p_1}{2} (1 + \lambda_p). \quad (14)$$

Upon substitution of (13) and (14) in (11), it results:

$$B = 1 \pm \frac{aw_p}{\sqrt{1.5RT_1 \left( 1 + \lambda_p^{\frac{k'-1}{k'}} \right)}} \pm \frac{2f(A+B'w_p)}{p_1(1+\lambda_p)}. \quad (15)$$



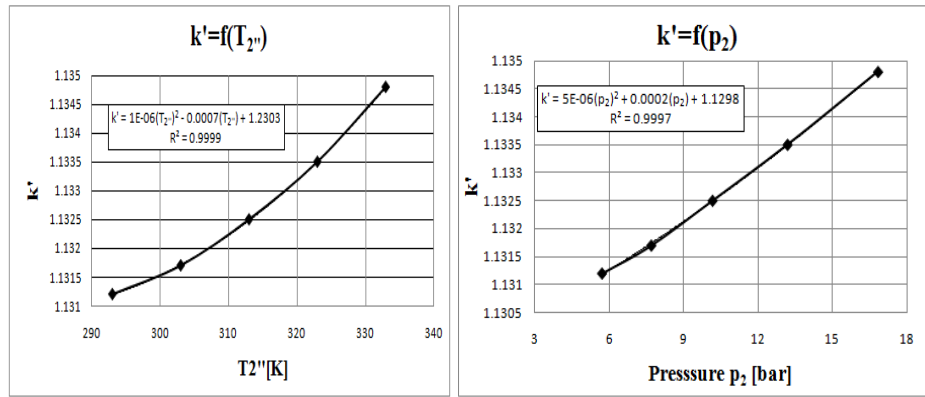


Fig. 2. Corrected adiabatic exponent  $k'$  variation versus the saturation vapor temperature  $T_{2''}$ , respectively pressure. [16]

Here we take into account only the contribution of the finite speed and friction. The throttling into the valves of the compressor will be taken into account separately.

Once the coefficient  $B$  is expressed as a function of the piston speed and the other gas parameters, one proceeds with a variable separation in eq. (10):

$$\frac{mc'_v dT}{pB} = -dV, \quad (16)$$

where the pressure is expressed from the state equation:

$$p = \frac{mRT}{V}. \quad (17)$$

By taking into account that a corrected specific heat  $c'_v$  which depends on  $k'$  (the *corrected adiabatic exponent*) is used in eq. (16):

$$c'_v = \frac{R}{k'-1}, \quad (18)$$

equation (16) becomes:

$$\frac{1}{B(k'-1)} \frac{dT}{T} = -\frac{dV}{V}. \quad (19)$$

This equation is different in comparison with the differential equation of adiabatic processes from Classical Thermodynamics, because of two terms:  $B$  and  $k'$ . Term  $B$  takes into account the internal irreversibilities as function of the speed,  $w_p$  and term  $k'$  takes into account the departure of the superheated vapor in the compressor exit from the perfect gas behaviour.

Table 1

Refrigerant property expressions in the two cycles main states						
STATE 1	STATE 2r	STATE 2irr	STATE 3	STATE 4a	STATE 4r	STATE 4irr
$T_1 = 74.582(p_1)^{0.1035}$	$T_{2r} = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k'-1}{k'}}$	$T_{2irr} = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{B(k'-1)}{B(k'-1)+1}}$	$T_3 = -2 \cdot 10^{-11}(p_3)^2 + 8 \cdot 10^{-5}(p_3) + 253.87$	$T_{4a} = T_1$	$T_{4r} = T_1 = 74.582(p_1)^{0.1035}$	$T_{4irr} = T_1 = 74.582(p_1)^{0.1035}$
$p_1 - \text{given}$	$p_{2r} - \text{given}$	$p_{2irr} = p_{2r}$	$p_3 = p_{2irr} = p_{2r}$	$p_{4a} = p_1$	$p_{4r} = p_1$	$p_{4irr} = p_1$
$s_1 = -15.51 \ln(p_1) + 1128.8 \neq 0.0045(T_1)^2 - 3.0762(T_1) + 1436.2$	$s_{2r} = -0.0032(T_{2r})^2 + 5.3361(T_{2r}) - 428.19$	$s_{2irr} = -0.0032(T_{2irr})^2 + 5.3361(T_{2irr}) - 428.19$	$s_3 = -10^{-10}(p_3)^2 + 0.0004(p_3) + 113.3 = 4.725(T_3) - 1084.8$	$s_{4a} = -10^{-10}(p_1)^2 + 0.0004(p_1) + 113.3 = 4.725(T_1) - 1084.8$	$s_{4r} = s_4 = -10^{-10}(p_3)^2 + 0.0004(p_3) + 113.3 = 4.725(T_3) - 1084.8$	$s_{4irr} = s_{4a} + (s_1 - s_4) \left( \frac{h_3 - h_{4a}}{h_1 - h_{4a}} \right)$
$h_1 = -1.3367(T_1)^2 + 1305.8(T_1) - 15846 \ln(p_1) + 51610$	$h_{2r} = 1085(T_{2r}) - 68451$	$h_{2irr} = 1085(T_{2irr}) - 68451$	$h_3 = -3 \cdot 10^{-8}(p_3)^2 + 0.1086(p_3) + 25166 = 2.1886(T_3)^2 + 124.47(T_3) - 145250$	$h_{4a} = -3 \cdot 10^{-8}(p_1)^2 + 0.1086(p_1) + 25166 = 1450.6(T_1) - 1084.8$	$h_{4r} = h_{4a} + x_4(h_1 - h_{4a})$	$h_{4irr} = h_3 = -3 \cdot 10^{-8}(p_3)^2 + 0.1086(p_3) + 25166 = 1450.6(T_3) - 1084.8$
$v_1 = 8 \cdot 10^{-21}(T_1)^{-9.46} = 15715(p_1)^{0.981}$	$v_{2r} = v_1 \left( \frac{T_1}{T_{2r}} \right)^{\frac{1}{k'-1}}$	$v_{2irr} = v_1 \theta^{B(k'-1)}$	$v_3 = -2 \cdot 10^{-17}(p_3)^2 + 2 \cdot 10^{-10}(p_3) + 0.0007 = 2 \cdot 10^{-8}(T_3)^2 - 9 \cdot 10^{-6}(T_3) + 0.0018$	$v_{4a} = -2 \cdot 10^{-17}(p_1)^2 + 2 \cdot 10^{-10}(p_1) + 0.0007$	$v_{4r} = v_3 \left( \frac{p_3}{p_4} \right)^{k'}$	$v_{4irr} = v_3 \left( \frac{T_1}{T_1} \right)^{\frac{1}{B(k'-1)}}$
$x_1 = 1$			$x_3 = 0$	$x_{4a} = 0$	$x_{4r} = \frac{s_{4r} - s_{4a}}{s_1 - s_{4a}}$	$x_{4irr} = \frac{h_3 - h_{4a}}{h_1 - h_{4a}}$
$B = 1 \pm \frac{aw_p}{\sqrt{1.5RT_1 \left( 1 + \lambda_p k' \right)}} \pm \frac{2f \left( \lambda' + B' w_p \right)}{p_1 \left( 1 + \lambda_p \right)}$				$k' = 10^{-6}(T_{2r})^2 - 0.0007(T_{2r}) + 1.2303 = 5 \cdot 10^{-6}(p_{2r})^2 + 0.0002(p_{2r}) + 1$		

All the equations which derives from this one (14) will contain these two „corrections”, and are an important results of the Direct Method which consist in integrating this equation (14) and using the results for computation of the performances of the irreversible cycle that are studied.

By integrating eq. (19) for the irreversible adiabatic process 1 – 2<sub>irr</sub>, one gets:

$$\ln \frac{T_{2irr}}{T_1} = -B(k'-1) \ln \frac{V_{2irr}}{V_1} = \ln \left( \frac{V_1}{V_{2irr}} \right)^{B(k'-1)}, \quad (21)$$

which leads to the following equations for the irreversible *adiabatic process with finite speed and friction*:

- in coordinates  $T$ - $V$ :  $T_1 V_1^{B(k'-1)} = T_{2irr} V_{2irr}^{B(k'-1)}, \quad (22)$

- in coordinates  $p$ - $V$ :  $p_1 V_1^{B(k'-1)+1} = p_{2irr} V_{2irr}^{B(k'-1)+1}, \quad (23)$

- in coordinates  $T$ - $p$ :  $\frac{T_1}{T_{2irr}} = \left( \frac{p_1}{p_{2irr}} \right)^{\frac{B(k'-1)}{1+B(k'-1)}}, \quad (24)$

With equation (24), one computes  $T_{2irr}$ . After getting the correlations between  $h$  and  $s$  as function of  $T$  on the isobaric process 2''-2<sub>irr</sub>, one can get

immediately  $h_{2irr}$  and  $s_{2irr}$ , necessary for computation of entropy generation  $\Delta s_{1-2irr}$  and irreversible work in the compressor ( $h_{2irr} - h_1$ ). Since 1-2<sub>irr</sub> is a compression process, in the analytical expression of  $B$ , the (+) sign appears.

$$\begin{aligned}\Delta s_{1-2irr} &= c'_v \ln \frac{T_{2irr}}{T_1} + R \ln \frac{V_{2irr}}{V_1} = \\ &= \frac{R}{k' - 1} \ln \frac{T_{2irr}}{T_1} + R \ln \left( \frac{T_{2irr}}{T_1} \right)^{-\frac{1}{B(k' - 1)}} = \\ &= \frac{R}{k' - 1} \ln \frac{T_{2irr}}{T_1} \left[ 1 - \frac{1}{B} \right].\end{aligned}\quad (25)$$

Finally the **entropy generation per cycle** yields:

$$\Delta s_{i,cycle} = \Delta s_{1-2irr} + \Delta s_{3-4irr}, \quad (26)$$

with  $\Delta s_{3-4irr} = s_{4irr} - s_3$ .

The **internal entropy generation rate**, in the case in which the internal heat losses between the condenser and evaporator are not taken into account, is given by the expression:

$$\Delta \dot{S}_i = \dot{m} \Delta s_{i,cycle}. \quad (27)$$

The **internal entropy generation rate**, in the case in which the internal heat losses between the condenser and evaporator are taken into account, is given by the expression:

$$\Delta \dot{S}_{i,losses} = \dot{m} \Delta s_{i,cycle} + KA \frac{(T_H - T_L)^2}{T_H T_L}. \quad (28)$$

Equation (26) will represent *the internal generation of entropy rate*, which accounts for three causes of irreversibility generation: irreversibility in the compressor (caused by friction and finite speed), also the internal entropy generation due to heat losses between  $T_H$  and  $T_L$  (between condenser and evaporator), and the entropy generation throughout the cycle. The second term from eq. (28) is the classical expression of *entropy generation* between two bodies having different temperatures, exchanging the heat flux  $KA(T_H - T_L)$ , namely:  $KA(T_H - T_L)/T_L - KA(T_H - T_L)/T_H$ .

In this way the analytical expressions for *coefficient of performance, COP* are obtained, analyzing the successive influence of all the five internal losses. (Table 2)

Table 2

Formulas for COP [16]

	COP
Finite speed of the piston in the compressor	$COP_I = \frac{h_1 - h_{4r}}{(h_{2irr})_w - h_1}$
Finite speed of the piston and the friction in the compressor	$COP_{II} = \frac{h_1 - h_{4r}}{(h_{2irr})_{w,f} - h_1}$
Finite speed of the piston, the friction in the compressor and the throttling in the throttling valve	$COP_{III} = \frac{h_1 - h_{4irr}}{(h_{2irr})_{w,f} - h_1}$
Finite speed of the piston, the friction in the compressor, the throttling in the throttling valve and the throttling in the compressor	$COP_{IV} = \frac{h_1 - h_{4irr}}{(w_{cpr})_{w,f} + w_{thcp}}$ <p>with: <math>(w_{cpr})_{w,f} = (h_{2irr})_{w,f} - h_1</math></p> $w_{thcp} = \Delta p_{thrR} \cdot v_{2irr} + \Delta p_{thrA} \cdot v_1$
Finite speed of the piston, the friction in the compressor, the throttling in the throttling valve, the throttling in the compressor, but also the heat losses between sources	$COP_V = \frac{q_{ref} - q_{lost}}{ w_{cpr} } = \frac{\dot{m}(h_1 - h_{4h}) - \dot{Q}_{lost}}{(h_{2irr} - h_1)\dot{m}}$ <p>where: <math>\dot{m} = \rho_1 \frac{\pi D^2}{4} w_p</math>, <math>\rho_1 = \frac{1}{v_1}</math>;</p> $\dot{Q}_{lost} = kA(T_H - T_L)$

### 3. Results

The calculations are done considering the following data for dimensions and properties:  $L=1m$ ,  $D=0.05m$ ,  $N_{pipes}=8$ ,  $\alpha_e = 7W/m^2K$ ,  $\alpha_i = 5W/m^2K$ ,  $\lambda_{ins} = 0.044W/mK$ ,  $\delta_{ins} = 0.1m$  and  $A_{Ev} = A_{Cd} = 0.176m^2$ . The heat transfer coefficient  $\alpha$  of natural convection in air, at temperatures close to ambient temperature is usually between  $5-7W/m^2K$ . Taking into account the fact that the space between condenser and evaporator is limited, and also taking into account the insulation we have chosen for the numerical example calculation the lower value. For more precise calculation of course, the correlation equations must be used, and also the geometry of the heat exchanger must be taken into account. The given numerical example is not pretending to be realistic. The objective of this study was to develop a methodology for analytical calculation of entropy generation and performances of the refrigeration systems with mechanical vapor compression, and to numerically exemplify it. The usage of this methodology for specific optimization and redesign requires the adoption of appropriate values of

the convection coefficient and thermal conductivity of the wall and of the heat exchange surfaces, resulting either from experiments or calculated using the heat transfer correlations.

Using the above (completely analytical) formula for *entropy generation* and for *COP* the curves in Figures 3 – 5 were plotted giving the chance to see the influence of different types of internal irreversibilities on the COP and corresponding entropy generation.

The analytical model was taken into account the finite speed of the friction process, the irreversibilities produced during adiabatic compression (1-2<sub>irr</sub>) and adiabatic irreversible expansion (3-4<sub>irr</sub>) in the throttling valve (see Table 3).

Table 3

Notations used for  $\Delta s_{i,cycle}$ 

Types of irreversibilities	$\Delta s_{i,cycle}$
Irreversibilities occurred during the adiabatic irreversible expansion in the throttling valve	$\Delta s_{i,cycle.I}$
Irreversibilities due to the finite speed, and the friction (when $A = 0$ is not taken into account the friction at the limit $w_p \rightarrow 0$ , but is taken into account the variation of the friction with velocity $w_p$ )	$\Delta s_{i,cycle.II}$
Irreversibilities occurred during the adiabatic irreversible expansion in the throttling valve, and due to the finite speed, and the friction (when $A = 0$ (when the friction is not taken into account the friction at the limit $w_p \rightarrow 0$ , but is taken into account the variation of the friction with velocity $w_p$ )	$\Delta s_{i,cycle.III}$
Irreversibilities due to the finite speed, and the friction (when $A = 0.94$ that takes into account the existing friction even at the limit $w_p \rightarrow 0$ .	$\Delta s_{i,cycle.IV}$
Irreversibilities occurred during the adiabatic irreversible expansion in the throttling valve, and due to the finite speed, and the friction (when $A = 0.94$ that takes into account the existing friction even at the limit $w_p \rightarrow 0$ ).	$\Delta s_{i,cycle.V}$

To highlight the influence of all internal irreversibilities on the functioning of a real refrigerator, we analyzed the results obtained by varying the temperature  $T_H = T_2$ . Figures 3 and 4 show that, with the decrease in the temperature difference between the machine heat sources, *COP* increases and the internal entropy generation per cycle decreases.

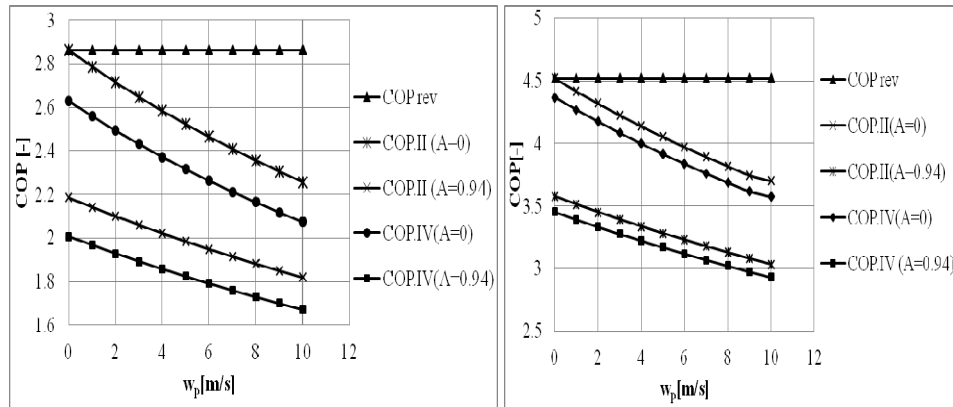


Fig. 3. The effect of piston speed  $w_p$  on  $COP$ , for  $T_2'' = 313K$ , respectively  $T_2'' = 293K$

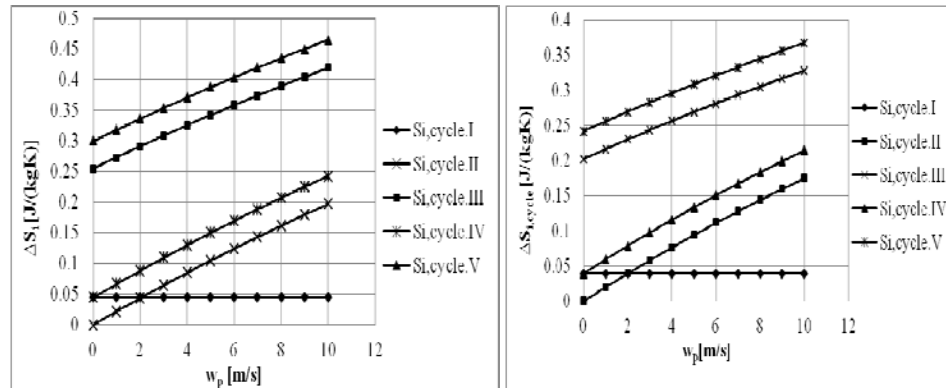


Fig. 4. The effect of piston speed  $w_p$  on  $COP$ , entropy generation per cycle  $\Delta S_{i,cycle}$ , for  $T_2'' = 313K$ , respectively  $T_2'' = 293K$

In these figures appears the effect of the successive introduction of irreversibilities produced by finite speed of the processes, generated during adiabatic expansion in the throttling valve (3-4irr), respectively during the irreversible adiabatic compression (1-2irr). It also examines the influence of the factor  $A$  in calculating friction losses, and by default upon the  $COP$  and internal entropy generation per cycle.

In the real reversible cycle 1-2r-3-4r, the work is minimal and  $COP$  is maximal, compared to any other cycle that could reach the same cooling power, because for any other cycle losses due to external and internal irreversibility would appear. It can be noticed that for a constant temperature of the cooled space  $T_L$ , and as the superheated saturated vapor temperature decreases, the work needed for the reversed quasi – Carnot cycle and the refrigeration efficiency  $COP$  of the cycle decrease, so seemingly the cold cycle is more efficient with the

decrease in the temperature difference between sources. Analyzing the curves of Fig. 5 it is found that  $COP_{IV}$  decreases with the increase of  $T_2''$  and with the increase of the speed  $w_p$ . It seems good to decrease  $T_2''$  to enlarge the  $COP$ . This conclusion is false because it was not yet taken into account the effect of external irreversibilities.

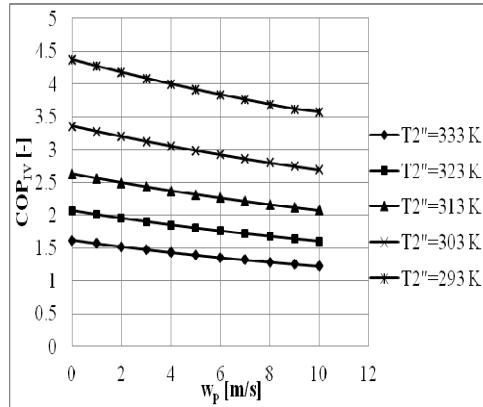


Fig. 5. The effect of piston speed  $w_p$  on  $COP_{IV}$  in the quasi-Carnot cycle for various temperatures of saturation  $T_2''$ .

For example, with the decrease of  $T_2''$  the temperature differences increasingly lowers (at the hot source where heat is evacuated). This would entail the need to increase the surface of the heat exchange of the condenser, which would involve additional costs for the construction of the machine and also larger storage space.

Figures 6 and 7 illustrate the influence of the piston speed, the factor  $f$ , and the coefficient  $A$ , upon  $COP_{III}$  of the irreversible quasi-Carnot cycle, but also upon the variation of the specific entropy per cycle  $\Delta s_{i,cycle}$ . It can be noticed that, with the increase in influence of the friction and of the speed of process, the  $COP$  decrease. It is found that when the friction at the limit  $w_p \rightarrow 0$  ( $A=0$ ) is not taken into account, the  $COP_{III}$ , respectively the entropy generation per cycle, have the same value regardless of the part of friction heat that remains inside the system,  $f$ .

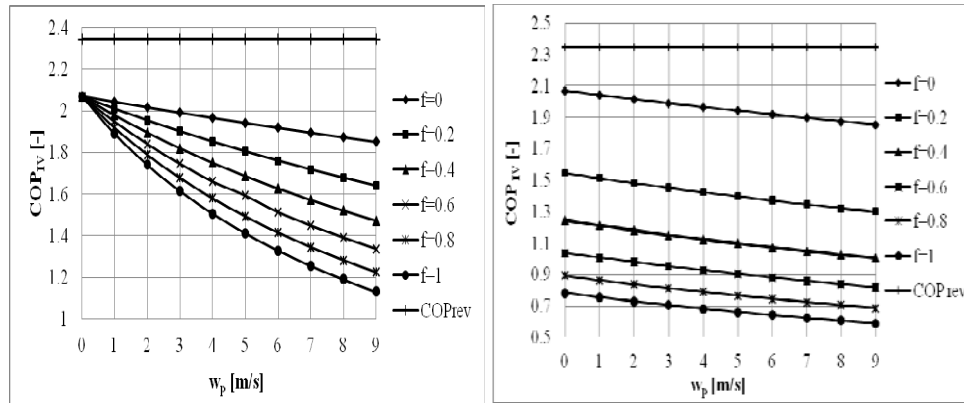


Fig. 6. The effect of piston speed  $w_p$  upon the  $COP_{IV}$  for various values of the factor  $f$ . Case  $A=0$ , respectively case  $A=0.94$ .

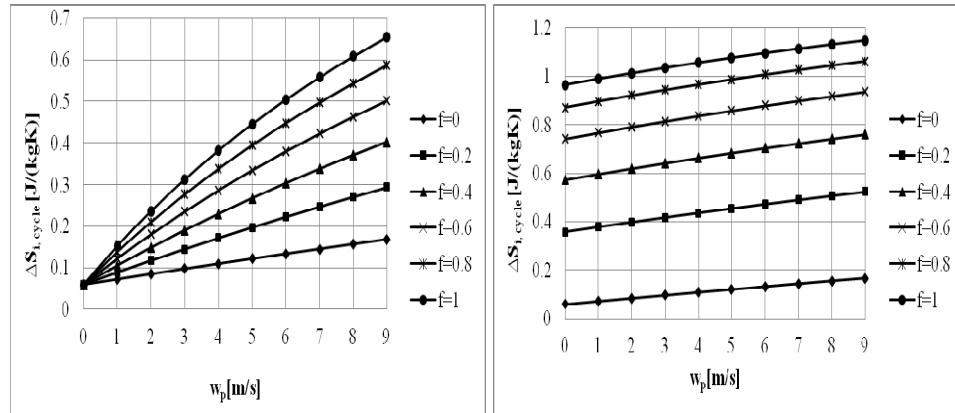


Fig. 7. The effect of piston speed  $w_p$  on the entropy generation in quasi-Carnot cycle for various values of the factor  $f$ . Case  $A=0$ , respectively case  $A=0.94$

With the increase of the piston speed, factor  $f$  produces a decrease in  $COP_{III}$ , and therefore an increase in entropy generation. In the case of real operating systems ( $w_p \approx 2 \text{ m/s}$ ,  $f \approx 0.6$ ) the  $COP$  that takes into account the existing friction even at the limit  $w_p \rightarrow 0$  ( $A=0.94$ ) witnessed a decrease of about 19% compared to the  $COP$  calculated where the friction even at the limit  $w_p \rightarrow 0$  is not taken into account, but is taken into account the variation of the friction with the velocity  $w_p$  ( $A=0$ ). The same comparison of the corresponding entropy generation values leads to a growth per cycle with about 26.5%.



Increase of the piston speed  $w_p$  and  $f$  (part of the friction heat that remains inside the system) results in an increase of the entropy generation rate. By comparing the values of generated entropy rate (Fig. 8), it is found that with the increase of temperature  $T_2''$ , the values of the analyzed rates increase as well. Figure 9 reveals that the maximum value of the COP corresponds to a minimum value for the entropy generation per cycle.

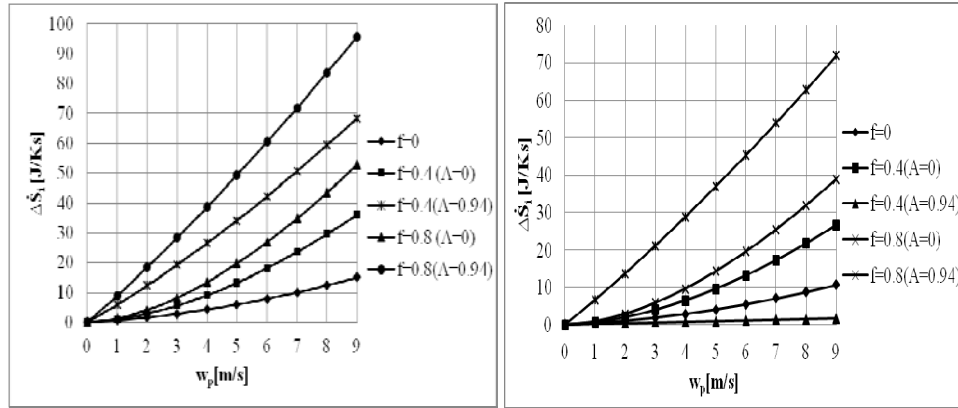


Fig. 8. Influence of piston speed  $w_p$ , upon the entropy generation rate in the cycle  $\Delta\dot{S}_i$ , for different values of factor  $f$ , for two different temperatures  $T_2'' = 313K$ , respectively  $T_2'' = 293K$ .

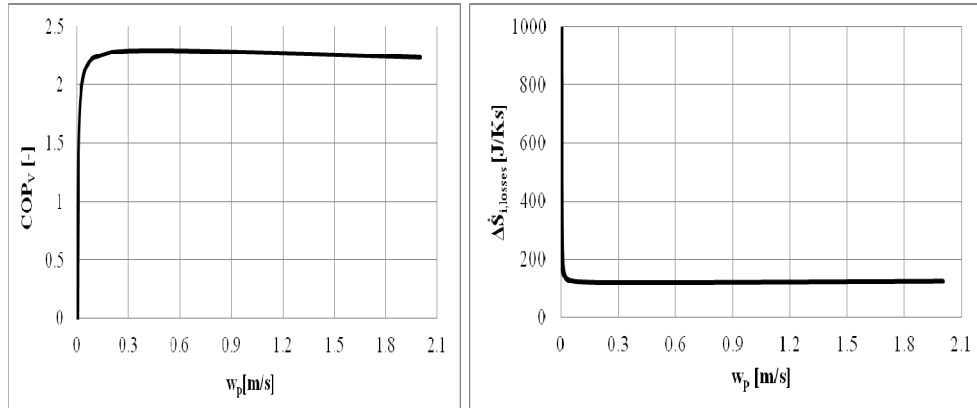


Fig. 9. The effect of piston speed  $w_p$ , on  $COP_V$ , respectively on the internal entropy generation rate  $\Delta\dot{S}_{i,losses}$ .

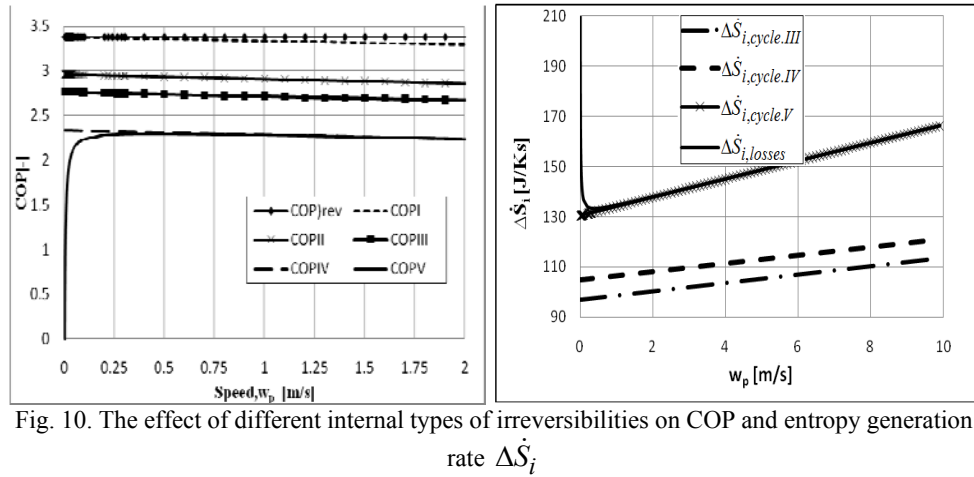


Fig. 10. The effect of different internal types of irreversibilities on COP and entropy generation rate  $\Delta \dot{S}_i$

By using the above derived (completely analytical) formulae for  $COP$  and *internal entropy generation*, the effect of internal irreversibilities progressively introduced on the cycle performances are illustrated in Figure 10. Thus, the major reductions of COP are registered when the friction losses are considered ( $COP_{II}$ ), respectively the throttling in the compressor valves ( $COP_{IV}$ ).

By comparing the variation of the two performances with the piston speed, clearly appears that small values of the piston speed provide economical operational regime, mainly from the power consumption reason.

This study emphasizes the entropy generation in the cycle, and not in the machine. It should be noted that the variation of the analyzed parameters is strictly influenced by the speed of the process and friction coefficient.

#### 4. Conclusion and perspectives

**Internal Entropy Generation** and **COP** calculation for a reversed irreversible cycle quasi-Carnot machine (Refrigeration Machine with mechanical compression of vapor) are presented in this paper. Irreversibility due to the finite speed of the piston, friction and the throttling during the adiabatic compression and expansion have been taken into account.

A thermodynamic approach based on the Direct Method and The First Law of Thermodynamic for processes with finite speed is shown to be especially effective for analytical estimation of the entropy generation rate. The treatment is done here completely analytical, which means that formulae for  $COP$ ,  $\Delta s_i$  and  $\dot{S}_i$  as function of piston speed  $w_p$  in the compressor, and other parameters (compression ratio, temperature ratios, etc) have been got. Such a treatment allowed insight of the factors causing internal irreversibilities by a sensibility

study in which they are taking into account step by step. In this way one can see which one influence in what way, and also how much is the influence of each one. Based on such calculation the designer has a chance to “see” where to intervene in order to improve the performances of the whole system.

Taking into account other internal irreversibilities, it is possible to develop this scheme of computation to the final goal of validation. Such a validated scheme could help the optimization and design of refrigeration machines and heat pumps.

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