

# ROBUST FAULT-TOLERANT CONTROL BASED ON T-S FUZZY MODEL FOR INSPECTION UAV WITH EVENT-TRIGGERED MECHANISM

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*In this paper, considering uncertainties, external disturbances and faults, a T-S fuzzy model for the power inspection quadrotor UAVs is established. The event-triggered mechanism is introduced to determine the data transmission by comparing whether the state error of the UAV meets the event-triggered condition. Firstly, the robust observer is given to detect the system states in real time. Secondly, based on the event-triggered mechanism, the robust fault-tolerant controller is provided to make the system stable and have anti-disturbance performance when the actuator fails. Finally, the algorithm application simulation was carried out on the inspection UAV, which verifies the reliability and validity of the designed control method.*

**Keywords:** T-S fuzzy model; inspection UAV; event-triggered mechanism; observer; robust fault-tolerant control

## 1. Introduction

The inspection of transmission lines has an important impact on the safe and reliable operation of power system equipment. It can reduce the burden of the staff, improve work efficiency, and ensure the personal safety by using UAV for power transmission line inspection [1-2]. In the process of inspection, UAV is required to be as close as possible to the transmission line for the accurate fault diagnosis. If the speed and flight attitude of the UAV are not controlled, the transmission line or other transmission facilities may be hit. That is, the UAV will be disturbed during the inspection. This disturbance is divided into external disturbances and internal disturbances. The external disturbances are mainly harsh natural environments such as strong winds and heavy rains, and electromagnetic environments around transmission lines. The internal disturbances are actuator failures of UAV, including actuator interruption failures, partial failure failures, and drift failures [3-4]. When UAV is performing tasks, due to the complexity of the surrounding environment and its own reliability, it is inevitable that sensors or actuators will fail. During the inspection process of the UAV, the flight control system must ensure stability and good anti-disturbance performance against

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different interferences [5-6]. Therefore, it is required to have good fault tolerance for actuator interruption, partial failure, etc. Then, it is necessary to probe the robust fault-tolerant control method for the inspection UAV. Robust fault-tolerant control is to provide a suitable control mechanism to meet the expected performance index when there are faults or uncertain factors in the system. In recent years, fault-tolerant control technology has developed rapidly [7-9].

The idea of the event-triggered mechanism is to set a trigger condition in advance. When the sampled states or measurement outputs meet the event-triggered condition, the controller or state estimator updates their input. Otherwise, due to the role of the zero-order holder, the input signal remains unchanged. The purpose of the event-triggered mechanism is to reduce network transmission time-delay and packet loss, which can effectively improve network efficiency. In recent years, the use of event-triggered mechanisms to study the fault tolerance of network control systems has attracted the attention of scholars [10-12].

In 1985, Takagi and Sugeno proposed the famous T-S fuzzy model [13]. The idea is to divide the input space into several fuzzy subspaces, and establish a simple linear relationship model about input-output in each fuzzy subspace. The antecedent of a fuzzy rule is used to represent the fuzzy subspace, and the latter represents the linear relationship between input and output in this fuzzy subspace. And the research result has shown that the T-S fuzzy model can be used to approximate the non-linear system [14]. Due to the characteristics of the T-S fuzzy model, many theories and methods of linear system control can be applied to design related controllers and analyze nonlinear systems. And compared with other fuzzy methods, its computational efficiency is high. The T-S fuzzy system research is of great significance in the field of the non-linear system control, and has attracted the interest of many scholars [15-16].

At present, although there are many papers on the stability of the quadrotor UAVs, there are relatively few papers on the fault-tolerant control of UAV systems by introducing event-triggered mechanisms. In addition, most research results in the event-triggered scheme assume that the states for the controlled object are measurable. However, in actual engineering, when technology and other conditions are restricted, it is very difficult to obtain all the states information for the controlled object, so it is necessary to consider the observer-based control problem. Therefore, the robust fault-tolerant control based on T-S fuzzy model for inspection UAV with event-triggered mechanism is studied in this paper. The main contributions are as follows. Firstly, the factors such as parameter uncertainties, time-delay, external disturbance and system failures are considered in the UAV system, and the UAV system model is established based on the T-S fuzzy method. Based on this, the robust observer is designed and the condition for the asymptotic stability of the closed-loop system

is given. Secondly, the robust fault-tolerant control based on event-triggered mechanism is designed. The robust  $H_\infty$  stability condition of the system in the case of actuator failure is provided. The control strategy satisfies fuzzy rules and event-triggered constraints. Finally, the application simulation study of the quadrotor UAV system is done. The effectiveness of the observer design and fault-tolerant control strategy is verified. The event-triggered mechanism adopted can reduce the number of data transmissions and the amount of data transmission.

## 2. Problem Statement

### 2.1 Quadrotor UAV Modelling Based on the T-S Fuzzy Theory

The structure for the inspection quadrotor UAV is shown in Fig.1 [17].

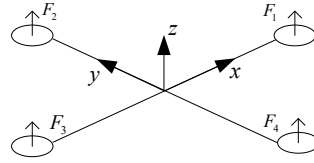


Fig.1 The structure of the quadrotor UAV

Let  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [\psi \ \phi \ \theta \ \dot{\psi} \ \dot{\phi} \ \dot{\theta}]^T$  is a state variable. The angle  $\psi$  is the yaw angle. The angle  $\phi$  is the pitch angle. The angle  $\theta$  is the roll angle.  $u = [\omega_1^2 \ \omega_2^2 \ \omega_3^2 \ \omega_4^2]^T$  is control. The  $\omega_i (i=1,2,3,4)$  is the motor speed.  $y = [\psi \ \phi \ \theta]$  is the output angle. During the operation for the UAV system, due to changes in the external or internal environment, the UAV is bound to be affected by disturbances. When designing the controller, the output disturbance is set to  $\omega = [\tilde{\omega}_1 \ \tilde{\omega}_2] \in L_2[0, \infty]$ . The following state space expression can be obtained [17]

$$\begin{cases} \dot{x} = Ax + Bu + D\omega \\ y = Cx \end{cases} \quad (1)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{d}{I_z} & -\frac{d}{I_z} & \frac{d}{I_z} & \frac{d}{I_z} \\ \frac{bl}{I_x} & -\frac{bl}{I_x} & 0 & 0 \\ 0 & 0 & \frac{bl}{I_y} & \frac{bl}{I_y} \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.71 & 0.84 \\ 0.37 & 0.06 \\ 2.1 & 0.96 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$I_x$  is the moment of inertia of the roll.  $I_y$  is the moment of inertia for the pitch.  $I_z$  is the moment of inertia of the yaw axis.  $b$  is the tensile force coefficient.  $d$  is the drag coefficient.  $l$  is the length from the center of the rotor to the center of gravity.

For the actual quadrotor UAVs, the complex UAV nonlinear system is described based on the T-S fuzzy model. In the T-S fuzzy system, the local dynamic characteristics of the modeled object are described by a local linear model. Then, these local linear models are used to form a global T-S fuzzy system model by smoothly linking membership functions.

In the UAV modelling process, the impact of the uncertainties and the failures on the system is added. Considering the nonlinear time-delay UAV network system with parameter uncertainties approximately described by the T-S fuzzy model, the  $i$ th fuzzy rule can be described as follows

Plant Rule  $i$ : IF  $\theta_1(t)$  is  $N_{i1}$  and... and  $\theta_g(t)$  is  $N_{ig}$ , Then

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + A_{di}x(t - \tau_1) + (B_i + \Delta B_i)u(t) \\ \quad + B_{di}u(t - \tau_2) - (B_i + \Delta B_i)f(t) + D_i\omega(t) \\ y(t) = C_i x(t) \quad i = 1, 2, \dots, r \end{cases} \quad (2)$$

where  $r$  is the fuzzy rule number.  $N_{is}$  ( $s = 1, 2, \dots, g$ ) is the fuzzy set.  $\theta(t) = [\theta_1(t), \theta_2(t), \dots, \theta_g(t)]$  is the known antecedent variable.  $x(t) \in R^n$  is the state vector.  $u(t) \in R^m$  is the input vector.  $y(t) \in R^p$  is the output vector.  $\omega(t) \in R^q$  is the external disturbance, and  $\omega(t) \in L_2[0, \infty)$ .  $A_i, A_{di}, B_i, B_{di}, C_i$  and  $D_i$  ( $i = 1, \dots, r$ ) are the corresponding constant matrices in the  $i$ -th subsystem of appropriate dimensions.  $\tau_1$  and  $\tau_2$  are the network induced time-delay from the input controller to the actuator respectively, the total time-delay for the network can be regarded as  $\tau_t = \tau_1 + \tau_2$ , which satisfies  $0 < \tau_m \leq \tau_t \leq \tau_M$ , where  $\tau_m$  and  $\tau_M$  are the minimum and maximum values of the total time-delay, respectively.  $f(t)$  is an unknown actuator failure.  $\Delta A_i$  and  $\Delta B_i$  are the parameter uncertainties matrix and satisfy

$$[\Delta A_i \quad \Delta B_i] = MF(t)[N_{ai} \quad N_{bi}] \quad (3)$$

$M$ ,  $N_{ai}$  and  $N_{bi}$  are the known constant matrices with appropriate dimensions,  $F(t)$  is an unknown time-varying matrix, and satisfies  $F^T(t)F(t) \leq I$ .

By applying single point fuzzification, product inference and weighted average defuzzification methods, the corresponding UAV system fuzzy model can be expressed as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t))((A_i + \Delta A_i)x(t) + A_{di}x(t - \tau_1) \\ \quad + (B_i + \Delta B_i)u(t) + B_{di}u(t - \tau_2) - (B_i + \Delta B_i)f(t) + D_i\omega(t)) \\ y(t) = \sum_{i=1}^r h_i(\theta(t))C_i x(t). \end{cases} \quad (4)$$

Where  $\mu_i(\theta(t)) = \prod_{j=1}^g N_{is}(\theta_s(t))$ ,  $h_i(\theta(t)) = \mu_i(\theta(t)) / \sum_{i=1}^r \mu_i(\theta(t))$ .  $N_{is}(\theta_s(t))$  is the membership function of the antecedent variable  $\theta_s(t)$  on the fuzzy set  $N_{is}$ ,  $h_i(\theta(t)) \geq 0$ ,  $\sum_{i=1}^r h_i(\theta(t)) = 1$ . The expression of membership function refers to [18].

The actuator failures are considered. The switch matrix  $L = \text{diag}\{l_1, l_2, \dots, l_m\}$  indicates the failure.  $l_m = 0$  is the complete failure of the  $m$ -th actuator.  $l_m = 1$  is the normal of the  $m$ -th actuator.  $l_m = (0, 1)$  is the partial failure of the  $m$ -th actuator. Let  $Lu_f(t) = u(t) - f(t)$ , then the closed-loop fault network control system is

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t))((A_i + \Delta A_i)x(t) + A_{di}x(t - \tau_1) \\ \quad + (B_i + \Delta B_i)Lu(t) + B_{di}u(t - \tau_2) + D_i\omega(t)) \\ y(t) = \sum_{i=1}^r h_i(\theta(t))C_i x(t). \end{cases} \quad (5)$$

## 2. 2 Event-triggered Mechanism

The method of event-triggered mechanism has more advantages than the method of periodic sampling. By constructing a function related to the state at the sampling time, the state or state error of the system is compared in real time, and the data is transmitted if the event trigger condition is met. The purpose is to reduce the time wasted in UAV system data transmission, as well as the impact of network time-delay and packet loss.

**Assumption 1** The system is completely observable. Controllers and actuators are driven by events. The sampling period is  $h$ . The sampling time is  $i_k h$ ,  $i_k h = t_k h + 1h, l = 0, \dots, t_{k+1} - t_k - 1$ , where  $t_k h$  is the data transmission time.

**Assumption 2** The real-time detection of states information is not affected by event-triggered conditions. That is, the information at the each sampling time can be sent to the observer.

The event-triggered mechanism is created as follows

$$t_{k+1}h = t_k h + \min\{lh | e_k^T(i_k h)\Sigma e_k(i_k h) \geq \beta x_k^T(t_k h)\Sigma x_k(t_k h)\} \quad (6)$$

$\Sigma$  is the event-triggered matrix, and  $\beta \in [0, 1)$  is the event-triggered parameter.

The error is

$$e_x(i_k h) = x(i_k h) - e_x(t_k h) \quad (7)$$

**Definition 1** At the moment of transmission, the maximum time-delay and state time-delay from the controller to the actuator are both  $\tau_1(t)$ , and  $\tau_1(t) = t - i_k h$ , satisfying  $0 < \tau_m \leq \tau_1(t) \leq \tau_M$ , where  $\tau_m$  and  $\tau_M$  are the minimum and maximum total time-delays respectively. Let  $\tau_s = \tau_M - \tau_m$ .

### 2.3 Several Lemmas

The following theoretical derivation will use the several lemmas.

**Lemma 1** [19] For any positive definite symmetric matrix  $U, W$ . Parameter  $0 \leq h(t) \leq h_M$  and vector  $\dot{x}: [-h_M \ 0] \rightarrow R^n$ , the following integral inequality holds

$$-h(t) \int_{-h(t)}^0 \dot{x}^T(t+s) W \dot{x}(t+s) ds \leq \begin{bmatrix} x^T(t) & x^T(t-h(t)) \end{bmatrix} \begin{bmatrix} -W & U \\ * & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \quad (8)$$

**Lemma 2** [20] Assuming that  $f_1, f_2, \dots, f_N: R^m \rightarrow R$  has a positive value in a subset of the open set D, the convex combination for  $f_i$  in the set D satisfies

$$\begin{bmatrix} f_i(t) & g_{i,j(t)} \\ g_{i,j(t)} & f_i(t) \end{bmatrix} \geq 0 \quad (9)$$

where  $\{g_{i,j}: R^m \rightarrow R, g_{j,i(t)} = g_{i,j(t)}\}$ .

**Lemma 3** [21] Given matrix Y, M and E with the appropriate dimensions, where  $Y = Y^T$ , then  $Y + MF(t)E + E^T F^T(t)M^T < 0, \forall F: FF^T \leq I$ , if and only if there is a constant  $\varepsilon > 0$ , such that  $Y + \varepsilon MM^T + \varepsilon^{-1} E^T E < 0$ .

### 3. Robust Observer Design

The states of the quad-rotor UAV based on the T-S fuzzy theory is observable. Let  $\hat{x}(t) \in R^n$  be the estimated state of  $x(t)$  and  $\hat{y}(t)$  be the output of the system observation. The following robust state observer is constructed

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^r h_i(\theta(t)) ((A_i + \Delta A_i) \hat{x}(t) + A_{di} \hat{x}(t - \tau_1) + (B_i + \Delta B_i) u(t) \\ \quad + B_{di} u(t - \tau_2) + G_i (y(t) - \hat{y}(t))) \\ \hat{y}(t) = \sum_{i=1}^r h_i(\theta(t)) C_i \hat{x}(t). \end{cases} \quad (10)$$

where  $G_i$  is the gain matrix of the state observer.

The system residual is defined as  $r(t) = W(y(t) - \hat{y}(t))$ . The estimation error is defined as  $e(t) = x(t) - \hat{x}(t)$ . The residual estimation error is defined as  $r_e(t) = r(t) - f(t)$ . Where  $W$  is the residual gain matrix. Then the fuzzy model is

$$r(t) = \sum_{i=1}^r h_i(\theta(t)) W C_i e(t) \quad (11)$$

$$\dot{e}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) h_j(\theta(t)) ((A_i + \Delta A_i) e(t) + (A_{di} - G_i C_j) e(t - \tau_i) - (B_i + \Delta B_i) f(t) + D_i \omega(t)) \quad (12)$$

$$r_e(t) = \sum_{i=1}^r h_i(\theta(t)) (W C_i e(t) - f(t)) \quad (13)$$

The observer closed-loop fault system meets the  $H_\infty$  performance  $\gamma$ . Defining the  $H_\infty$  performance index as

$$J_1 = \int_0^t (r_e^T(t) r_e(t) - \gamma_1^2 f^T(t) f(t) - \gamma_2^2 \omega^T(t) \omega(t)) dt \quad (14)$$

**Theorem 1** Given normal numbers  $\tau_m, \tau_M, \tau_s, \gamma_1, \gamma_2, a, b, c$  and  $\alpha, \beta$ , if there are positive definite symmetric matrices  $P, V_i (i=1, \dots, r), U, W, Q_1, Q_2, R_1, R_2$  and  $R_3$  satisfy the following linear matrix inequalities

$$\Phi = \begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} \\ * & \bar{\Phi}_{22} \end{bmatrix} < 0 \quad (15)$$

where

$$\bar{\Phi}_{11} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}, \bar{\Phi}_{12} = \begin{bmatrix} \Phi_{17} & \Phi_{18} & \Phi_{19} & \Phi_{110} & \Phi_{111} & \Phi_{112} \\ \Phi_{27} & \Phi_{28} & \Phi_{29} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Phi_{57} & \Phi_{58} & \Phi_{59} & \Phi_{510} & 0 & \Phi_{512} \\ \Phi_{67} & \Phi_{68} & \Phi_{69} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Phi}_{22} = \begin{bmatrix} \Phi_{77} & 0 & 0 & 0 & \Phi_{711} & 0 \\ * & \Phi_{88} & 0 & 0 & \Phi_{811} & 0 \\ * & * & \Phi_{99} & 0 & \Phi_{911} & 0 \\ * & * & * & \Phi_{1010} & 0 & 0 \\ * & * & * & * & \Phi_{1111} & 0 \\ * & * & * & * & * & \Phi_{1212} \end{bmatrix},$$

$$\begin{aligned} \Phi_{11} &= A_i^T P + P A_i + Q_1 - R_1 - R_2, \\ \Phi_{12} &= P A_{di} - V_i C_j, \\ \Phi_{13} &= R_1, \Phi_{14} = R_2, \Phi_{15} = -P B_i, \Phi_{16} = P D_i, \\ \Phi_{17} &= \tau_m A_i^T P, \Phi_{18} = \tau_M A_i^T P, \Phi_{19} = \tau_s A_i^T P, \\ \Phi_{110} &= C_j^T W^T, \Phi_{111} = P M, \Phi_{112} = N_{di}^T, \\ \Phi_{22} &= -2R_3 + U^T + U, \\ \Phi_{23} &= R_3 - U, \Phi_{24} = R_3 - U^T, \Phi_{27} = \tau_m (P A_{di} - V_i C_j)^T, \Phi_{28} = \tau_M (P A_{di} - V_i C_j)^T, \Phi_{69} = \tau_s D_i^T P, \\ \Phi_{29} &= \tau_s (P A_{di} - V_i C_j)^T, \Phi_{33} = -Q_1 + Q_2 - R_1 - R_3, \Phi_{34} = U^T, \Phi_{44} = -Q_2 - R_2 - R_3, \Phi_{55} = -\gamma_1^2 I, \\ \Phi_{57} &= -\tau_m B_i^T P, \Phi_{58} = -\tau_M B_i^T P, \Phi_{59} = -\tau_s B_i^T P, \Phi_{510} = -I, \Phi_{67} = \tau_m D_i^T P, \Phi_{68} = \tau_M D_i^T P, \\ \Phi_{512} &= -N_{bi}^T, \Phi_{66} = -\gamma_2^2 I, \Phi_{77} = -2aP + a^2 R_1, \Phi_{711} = \tau_m P M, \Phi_{88} = -2bP + b^2 R_2, \Phi_{811} = \tau_M P M, \\ \Phi_{99} &= -2cP + c^2 R_3, \Phi_{911} = \tau_s P M, \Phi_{1010} = -I, \Phi_{1111} = -\alpha^{-1} I, \Phi_{1212} = -\alpha I. \end{aligned}$$

Then the error system is asymptotically stable, and the observer matrix is  $G_i = P^{-1} V_i$ . \* is the symmetric transpose part of the matrix.

*Proof.* The Lyapunov-Krasovskii function is selected as follows

$$V(t) = e^T(t)Pe(t) + \int_{t-\tau_m}^t e^T(s)Q_1e(s)ds + \int_{t-\tau_M}^{t-\tau_m} e^T(s)Q_2e(s)ds \\ + \int_{-\tau_m}^0 \int_{t+\theta}^t \tau_m \dot{e}^T(s)R_1\dot{e}(s)dsd\theta + \int_{-\tau_M}^0 \int_{t+\theta}^t \tau_M \dot{e}^T(s)R_2\dot{e}(s)dsd\theta + \int_{-\tau_M}^{-\tau_m} \int_{t+\theta}^t \tau_s \dot{e}^T(s)R_3\dot{e}(s)dsd\theta \quad (16)$$

The derivative of  $V(t)$  is

$$\dot{V}(t) = 2e^T P \dot{e} + e^T Q_1 \dot{e} - e_{\tau_m}^T Q_1 e_{\tau_m} + e_{\tau_m}^T Q_2 e_{\tau_m} - e_{\tau_M}^T Q_2 e_{\tau_M} + \tau_m^2 \dot{e}^T R_1 \dot{e} + \tau_M^2 \dot{e}^T R_2 \dot{e} \\ + \tau_s^2 \dot{e}^T R_3 \dot{e} - \tau_m \int_{t-\tau_m}^t \dot{e}^T R_1 \dot{e} ds - \tau_M \int_{t-\tau_M}^t \dot{e}^T R_2 \dot{e} ds - \tau_s \int_{t-\tau_M}^{t-\tau_m} \dot{e}^T R_3 \dot{e} ds + r_e^T(t)r_e(t) \\ - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 \omega^T(t)\omega(t) - r_e^T(t)r_e(t) + \gamma_1^2 f^T(t)f(t) + \gamma_2^2 \omega^T(t)\omega(t) \quad (17)$$

From Lemma 1 and Lemma 2, we have

$$-\tau_m \int_{t-\tau_m}^t \dot{e}^T R_1 \dot{e} ds \leq \begin{bmatrix} e \\ e_{\tau_m} \end{bmatrix}^T \begin{bmatrix} -R_1 & U \\ * & -R_1 \end{bmatrix} \begin{bmatrix} e \\ e_{\tau_m} \end{bmatrix} \quad (18)$$

$$-\tau_M \int_{t-\tau_M}^t \dot{e}^T R_2 \dot{e} ds \leq \begin{bmatrix} e \\ e_{\tau_M} \end{bmatrix}^T \begin{bmatrix} -R_2 & U \\ * & -R_2 \end{bmatrix} \begin{bmatrix} e \\ e_{\tau_M} \end{bmatrix} \quad (19)$$

$$-\tau_s \int_{t-\tau_M}^{t-\tau_m} \dot{e}^T R_3 \dot{e} ds \leq \begin{bmatrix} e_\tau - e_{\tau_M} \\ e_{\tau_m} - e_\tau \end{bmatrix}^T \begin{bmatrix} -R_3 & U \\ * & -R_3 \end{bmatrix} \begin{bmatrix} e_\tau - e_{\tau_M} \\ e_{\tau_m} - e_\tau \end{bmatrix} \quad (20)$$

Let  $\dot{e} = \varepsilon \xi(t)$ , here  $\varepsilon = \begin{bmatrix} (A_i + \Delta A_i) & A_{di} - G_i C_j & 0 & 0 & -(B_i + \Delta B_i) & D_i \end{bmatrix}$ ,

$\xi^T(t) = \begin{bmatrix} e^T & e_{\tau_1}^T & e_{\tau_m}^T & e_{\tau_M}^T & f^T(t) & \omega^T(t) \end{bmatrix}$ . then

$$\tau_m^2 \dot{e}^T R_1 \dot{e} = \tau_m^2 \xi^T(t) \varepsilon^T R_1 \varepsilon \xi(t), \quad \tau_M^2 \dot{e}^T R_2 \dot{e} = \tau_M^2 \xi^T(t) \varepsilon^T R_2 \varepsilon \xi(t) \\ \tau_s^2 \dot{e}^T R_3 \dot{e} = \tau_s^2 \xi^T(t) \varepsilon^T R_3 \varepsilon \xi(t) \quad (21)$$

Let  $r_e(t) = \sigma \xi(t)$ . Where  $\sigma = \begin{bmatrix} WC_i & 0 & 0 & 0 & -I & 0 \end{bmatrix}$ .

Substituting the above equations (18) to (21) into the equation (17), we get

$$\dot{V}(t) + r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 \omega^T(t)\omega(t) \\ \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left( \xi^T(t) \left[ \Phi' + \tau_m^2 \varepsilon^T R_1 \varepsilon + \tau_M^2 \varepsilon^T R_2 \varepsilon + \tau_s^2 \varepsilon^T R_3 \varepsilon + \sigma^T \sigma \right] \xi(t) \right)$$

where

$$\Phi' = \begin{bmatrix} \Phi'_{11} & \Phi'_{12} & \Phi'_{13} & \Phi'_{14} & \Phi'_{15} & \Phi'_{16} \\ * & \Phi'_{22} & \Phi'_{23} & \Phi'_{24} & 0 & 0 \\ * & * & \Phi'_{33} & \Phi'_{34} & 0 & 0 \\ * & * & * & \Phi'_{44} & 0 & 0 \\ * & * & * & * & \Phi'_{55} & 0 \\ * & * & * & * & * & \Phi'_{66} \end{bmatrix} < 0, \quad \begin{aligned} \Phi'_{11} &= (A_i + \Delta A_i)^T P + P(A_i + \Delta A_i) + Q_1 - R_1 - R_2, \\ \Phi'_{12} &= P(A_{di} - G_i C_j), \Phi'_{13} = R_1, \Phi'_{14} = R_2, \\ \Phi'_{15} &= -P(B_i + \Delta B_i), \Phi'_{16} = PD_i, \\ \Phi'_{23} &= R_3 - U, \Phi'_{24} = R_3 - U^T, \\ \Phi'_{33} &= -Q_1 + Q_2 - R_1 - R_3, \Phi'_{34} = U^T, \end{aligned}$$

$$\Phi'_{22} = -2R_3 + U^T + U, \quad \Phi'_{44} = -Q_2 - R_2 - R_3, \quad \Phi'_{55} = -\gamma_1^2 I, \quad \Phi'_{66} = -\gamma_2^2 I.$$

let

$$\Phi'' = \Phi' + \tau_m^2 \varepsilon^T R_1 \varepsilon + \tau_M^2 \varepsilon^T R_2 \varepsilon + \tau_s^2 \varepsilon^T R_3 \varepsilon + \sigma^T \sigma \quad (22)$$



By Schur's Complementary lemma, Lemma 1, Lemma 2 and Lemma 3, Theorem 1 can be obtained. That is, the error system has disturbance suppression performance and robust stability for the unknown time-delays and uncertainties. The detailed proof process is omitted here.

#### 4. Design of Robust Fault Tolerant Controller

The actuators of the UAV are prone to failure. A robust fault-tolerant controller is provided for the fault model. And when the UAV system is subject to the external disturbances, the system can meet the robust performance. The feedback control satisfies fuzzy rules and event-triggered constraints. We have

$$u(t) = \sum_{i=1}^r h_i(\theta(t)) K_i x(t_k h) \quad (23)$$

Considering the time-delay  $\tau_2$  in the input, from the equation (12), and  $\tau_1 = t - i_k h$ , the controller is obtained as

$$u(t) = \sum_{i=1}^r h_i(\theta(t)) K_i (x(t - \tau_1) - e_x(i_k h)), u(t - \tau_2) = \sum_{i=1}^r h_i(\theta(t)) K_i (x(t - \tau_i) - e_x(i_k h)) \quad (24)$$

Where  $\tau_i = \tau_1 + \tau_2$ .  $K_i$  is the controller matrix, which can ensure that the system can still be stable under the actuator failure and meet the robust  $H_\infty$  performance. The  $H_\infty$  index is defined as

$$J_2 = \int_0^t (y^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t)) dt \quad (25)$$

The closed-loop control system is

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) h_j(\theta(t)) \left( (A_i + \Delta A_i) x(t) + (A_{di} + (B_i + \Delta B_i) L K_j) x(t - \tau_1) \right. \\ \quad \left. + B_{di} K_j x(t - \tau_i) - ((B_i + \Delta B_i) L + B_{di}) K_j e_x(i_k h) + D_i \omega(t) \right) \\ y(t) = \sum_{i=1}^r h_i(\theta(t)) C_i x(t). \end{cases} \quad (26)$$

**Theorem 2** Given the positive scalar  $\tau_m, \tau_M, \tau_s, \gamma, \alpha$  and  $\beta \in [0, 1)$ , if there are positive definite symmetric matrices  $X, U, Y_j (j=1, \dots, r), Q_1, Q_2, R_1, R_2$  and  $R_3$  for any actuator failure, the following linear matrix inequality is satisfied

$$\Psi = \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} \\ * & \bar{\Psi}_{22} \end{bmatrix} < 0 \quad (27)$$

where

$$\bar{\Psi}_{11} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} \\ * & \Psi_{22} & 0 & \Psi_{24} & \Psi_{25} & \Psi_{26} & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & \Psi_{45} & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 \\ * & * & * & * & * & * & \Psi_{77} \end{bmatrix}, \bar{\Psi}_{12} = \begin{bmatrix} \Psi_{18} & \Psi_{19} & \Psi_{110} & \Psi_{111} & \Psi_{112} & \Psi_{113} \\ \Psi_{28} & \Psi_{29} & \Psi_{210} & 0 & \Psi_{212} & 0 \\ \Psi_{38} & \Psi_{39} & \Psi_{310} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Psi_{68} & \Psi_{69} & \Psi_{610} & 0 & \Psi_{612} & 0 \\ \Psi_{78} & \Psi_{79} & \Psi_{710} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Psi}_{12} = \begin{bmatrix} \Psi_{88} & 0 & 0 & \Psi_{811} & 0 & 0 \\ * & \Psi_{99} & 0 & \Psi_{911} & 0 & 0 \\ * & * & \Psi_{1010} & \Psi_{1011} & 0 & 0 \\ * & * & * & \Psi_{1111} & 0 & 0 \\ * & * & * & * & \Psi_{1212} & 0 \\ * & * & * & * & * & \Psi_{1313} \end{bmatrix},$$

$$\begin{aligned} \Psi_{11} &= XA_i^T + A_iX + 2X - Q_1, \\ \Psi_{12} &= A_{di}X + B_iLY_j, \Psi_{13} = B_{di}Y_j, \\ \Psi_{14} &= 2X - WC_j, \Psi_{15} = 2X - W_iC_j, \\ \Psi_{16} &= -(B_iL + B_{di})Y_j, \Psi_{17} = PD_i, \\ \Psi_{18} &= \tau_m XA_i^T, \Psi_{19} = \tau_m XA_i^T, \\ \Psi_{110} &= \tau_s XA_i^T, \Psi_{111} = M, \Psi_{112} = XN_{ai}^T, \Psi_{113} = XC_i^T, \Psi_{22} = \beta(2X - S) + U^T + U + \beta\Sigma, \\ \Psi_{24} &= -WC_j + U, \Psi_{25} = WC_j + U^T, \Psi_{26} = -\beta(2X - S) - \beta\Sigma, \Psi_{28} = \tau_m (A_{di}X + B_iLY_j)^T, \\ \Psi_{38} &= \tau_m (B_{di}Y_j)^T, \Psi_{29} = \tau_m (A_{di}X + B_iLY_j)^T, \Psi_{39} = \tau_m (B_{di}Y_j)^T, \Psi_{1313} = -I \\ \Psi_{210} &= \tau_s (A_{di}X + B_iLY_j)^T, \Psi_{310} = \tau_s (B_{di}Y_j)^T, \Psi_{212} = (N_{bi}LY_j)^T, \Psi_{45} = 2X - U^T, \\ \Psi_{44} &= -4X + Q_1 - Q_2, \Psi_{55} = -6X + Q_2, \Psi_{612} = -(N_{bi}LY_j)^T, \Psi_{66} = (\beta - 1)(2X - S), \\ \Psi_{77} &= -\gamma^2 I, \Psi_{68} = -\tau_m Y_j^T (B_iL + B_{di})^T, \Psi_{78} = \tau_m D_i^T, \Psi_{69} = -\tau_m Y_j^T (B_iL + B_{di})^T, \\ \Psi_{79} &= \tau_m D_i^T, \Psi_{610} = -\tau_s Y_j^T (B_iL + B_{di})^T, \Psi_{710} = \tau_s D_i^T, \Psi_{88} = -R_1^{-1}, \Psi_{811} = \tau_m M, \\ \Psi_{99} &= -R_2^{-1}, \Psi_{911} = \tau_m M, \Psi_{1010} = -R_3^{-1}, \Psi_{1011} = \tau_s M, \Psi_{1111} = -\alpha^{-1}I, \Psi_{1212} = -\alpha I. \end{aligned}$$

Then the robust fault-tolerant controller can ensure that the system is stable under any actuator failure, and meet the robust performance index, and  $K_j = Y_j P$ . \* is the symmetric transpose part of the matrix.

*Proof.* The Lyapunov-Krasovskii functional is chosen as follows

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\tau_m}^t x^T(s)Q_1x(s)ds + \int_{t-\tau_M}^{t-\tau_m} x^T(s)Q_2x(s)ds \\ &+ \int_{-\tau_m}^0 \int_{t+\theta}^t \tau_m \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-\tau_M}^0 \int_{t+\theta}^t \tau_M \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\ &+ \int_{-\tau_M}^{-\tau_m} \int_{t+\theta}^t \tau_s \dot{x}^T(s)R_3\dot{x}(s)dsd\theta \end{aligned} \quad (28)$$

Taking the derivative of  $V(t)$ , we get

$$\begin{aligned}
\dot{V}(t) = & 2x^T P \dot{x} + x^T Q_1 x - x_{\tau_m}^T Q_1 x_{\tau_m} + x_{\tau_m}^T Q_2 x_{\tau_m} - x_{\tau_M}^T Q_2 x_{\tau_M} + \tau_m^2 \dot{x}^T R_1 \dot{x} + \tau_M^2 \dot{x}^T R_2 \dot{x} \\
& + \tau_s^2 \dot{x}^T R_3 \dot{x} - \tau_m \int_{t-\tau_m}^t \dot{x}^T R_1 \dot{x} ds - \tau_M \int_{t-\tau_M}^t \dot{x}^T R_2 \dot{x} ds - \tau_s \int_{t-\tau_s}^t \dot{x}^T R_3 \dot{x} ds + y^T(t) y(t) \\
& - \gamma^2 \omega^T(t) \omega(t) - y^T(t) y(t) + \gamma^2 \omega^T(t) \omega(t) + e_x^T(i_k h) \Sigma e_x(i_k h) - e_x^T(i_k h) \Sigma e_x(i_k h)
\end{aligned} \quad (29)$$

The event-triggered mechanism is as follows

$$e_k^T(i_k h) \Sigma e_k(i_k h) \geq \beta x_k^T(t_k h) \Sigma x_k(t_k h) \quad (30)$$

By Schur's Complementary lemma, Lemma 1, Lemma 2 and Lemma 3, Theorem 2 can be obtained. The detailed proof process is omitted here.

## 5. Simulation study

Select the fuzzy rules number  $r=2$ . Other parameters are as follows.  $A, B, C$  and  $D$  are the parameters in the equation (1).

$$\begin{aligned}
A_1=A_2=A, B_1=B_2=B, D_1=D, C_1=C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \\
A_{d1} &= \begin{bmatrix} 0.1 & 0.1 & 0.2 & 0 & 0.1 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & -0.1 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0.1 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.5 & 0.6 \\ 0.3 & 0.1 \\ 2 & 0.9 \end{bmatrix}, M = \begin{bmatrix} 0.01 & -0.2 & 0.02 & 0.01 \\ 0.02 & 0 & 0.1 & 0.1 \\ 0.03 & 0.3 & 0 & 0.02 \\ 0 & -0.2 & 0.1 & 0 \\ 0.02 & -0.1 & 0.3 & 0.1 \\ 0.12 & 0 & 0.3 & 0.02 \end{bmatrix} \\
A_{d2} &= \begin{bmatrix} 0.02 & 0.1 & 0.2 & 0 & 0.1 & 0 \\ 0.15 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0.1 \\ 0.1 & -0.1 & 0 & 0.1 & 0.2 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & 0 & 0.1 & 0.1 \end{bmatrix}, B_{d1} = \begin{bmatrix} 0.1 & -0.02 & 0 & 0.1 \\ 0 & 0 & 0.5 & 0 \\ 0.23 & 0.11 & 0 & 0.1 \\ 0 & -0.02 & 0.1 & 0 \\ 0 & -0.1 & 0 & 0.1 \\ 0.02 & 0 & 0.1 & 0 \end{bmatrix}, B_{d2} = \begin{bmatrix} -0.2 & 0.2 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ 0.2 & 0.1 & 0 & 0.2 \\ 0.1 & 0.2 & -0.1 & 0 \\ -0.1 & 0 & 0 & 0.2 \\ -0.02 & 0 & -0.1 & 0.1 \end{bmatrix}, \\
N_{a1} &= \begin{bmatrix} 0.01 & -0.05 & 0.02 & -0.01 \\ 0.02 & 0 & 0.02 & 0.08 \\ 0.03 & 0.02 & 0 & -0.02 \\ 0 & -0.02 & 0.1 & 0 \\ 0.01 & -0.1 & 0.07 & -0.1 \\ 0.11 & 0 & 0.03 & 0.02 \end{bmatrix}, N_{a2} = \begin{bmatrix} 0.01 & -0.05 & 0.02 & 0.01 \\ 0.02 & 0 & -0.01 & 0.05 \\ -0.03 & 0.03 & 0.01 & 0.02 \\ 0 & -0.03 & 0.1 & 0 \\ 0.07 & -0.1 & 0.03 & 0.1 \\ 0.1 & 0 & 0.03 & 0.04 \end{bmatrix},
\end{aligned}$$

$$N_{b1} = \begin{bmatrix} 0.01 & -0.05 & -0.02 & -0.01 \\ 0.04 & 0 & -0.05 & 0.07 \\ 0.01 & -0.06 & 0 & -0.02 \\ 0.04 & -0.02 & 0.1 & 0 \\ 0.02 & -0.1 & 0.03 & -0.05 \\ 0.12 & 0 & 0.03 & 0.06 \end{bmatrix}, N_{b2} = \begin{bmatrix} 0.01 & -0.05 & 0.02 & 0.04 \\ 0.02 & 0.01 & -0.01 & 0.01 \\ 0.03 & 0.02 & 0 & 0.02 \\ 0 & -0.02 & 0.1 & 0 \\ 0.02 & -0.2 & 0.01 & 0.1 \\ 0.12 & 0 & 0.03 & 0.01 \end{bmatrix},$$

$Q_1 = \text{diag}\{10 \ 10 \ 10 \ 10 \ 10 \ 10\}$ ,  $Q_2 = \text{diag}\{1 \ 1 \ 1 \ 1 \ 1 \ 1\}$ ,  $R_1 = R_2 = 1$ ,  $R_3 = 10$ ,  $F = \sin t$ .

The membership function is chosen as  $\mu_1(\theta(t)) = 1/(1+e^{-\theta(t)})$ ,  $\mu_2(\theta(t)) = 1 - \mu_1(\theta(t))$ , where  $\theta(t) = x_1(t)$ . Let  $\tau_m = 0.2$ ,  $\tau_M = 0.4$ ,  $\gamma_1 = 1.7$ ,  $\gamma_2 = 2.5$ ,  $\gamma = 1$ ,  $\alpha = 1$ ,  $h = 0.1$ ,  $a = 0.1$ ,  $b = 0.2$ ,  $c = 0.3$ . The disturbance is  $\omega(t) = 0.02 \cos(2t)$ . UAV parameter values are  $I_x = I_y = 0.5 \text{ kg} \cdot \text{m}^2$ ,  $I_z = 0.2 \text{ kg} \cdot \text{m}^2$ ,  $b = 0.6 \text{ N} \cdot \text{m/r}$ ,  $d = 0.2 \text{ N} \cdot \text{m/r}$ ,  $l = 0.45 \text{ m}$ .

According to Theorem 1, the observer gain matrices can be obtained as

$$G_1 = \begin{bmatrix} 0.1259 & 1.4962 & -7.3268 \\ 0.1397 & -8.9965 & 1.0054 \\ 10.0044 & 1.3549 & 12.1376 \\ -0.4487 & -0.4521 & -2.0723 \\ 8.3312 & 2.0319 & 9.4579 \\ 7.3460 & 0.5581 & 0.6282 \end{bmatrix}, G_2 = \begin{bmatrix} 1.1457 & -1.7862 & -0.3568 \\ -2.1447 & 3.0246 & 1.0345 \\ -0.9943 & 1.6652 & 0.1458 \\ 2.4789 & -0.3427 & -1.2276 \\ 3.3789 & 12.3475 & -6.4573 \\ 2.2365 & 0.0324 & -0.8732 \end{bmatrix}$$

The observation curves of UAV attitude angles are shown in Fig.2 - Fig.4. The results show that the observer designed in this paper has strong robustness to the external disturbances, which verifies the effectiveness of the observer.

Based on the event-triggered mechanism, let  $\beta = 0.2$ . And the actuator is normal, there is no fault, that is,  $L = \text{diag}\{1, 1, 1, 1\}$ . According to Theorem 2, the event trigger matrix and controllers can be solved as

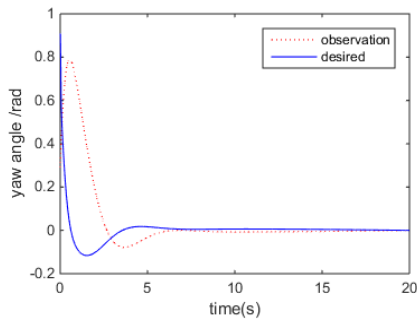


Fig.2 observation curves of yaw angle

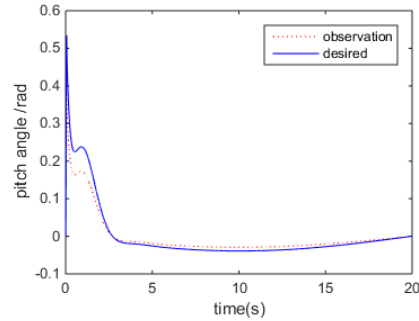


Fig.3 observation curves of pitch angle

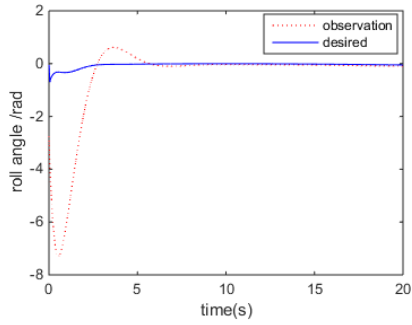


Fig.4 observation curves of roll angle

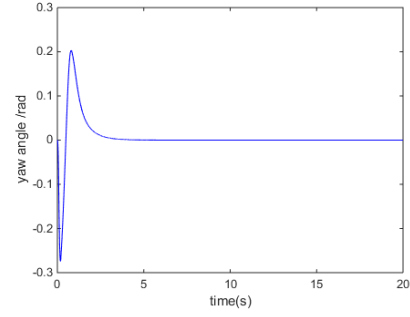


Fig.5 yaw angle curve without failure

$$\Sigma = \begin{bmatrix} 2.8687 & -0.5410 & 0.1271 & 0.4307 & -0.5479 & 0.4288 \\ -0.5410 & 2.8272 & 0.1820 & 0.6097 & -0.5698 & 0.2587 \\ 0.1271 & 0.1820 & 3.3589 & -0.2381 & 0.2716 & 0.0982 \\ 0.4307 & 0.6097 & -0.2381 & 2.8149 & 0.6381 & -0.2113 \\ -0.5479 & -0.5698 & 0.2716 & 0.6381 & 2.7174 & 0.2441 \\ 0.4288 & 0.2587 & 0.0982 & -0.2113 & 0.2441 & 2.6597 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 4.0678 & -7.4530 & -0.5463 & 0.3003 & 8.2343 & 9.1232 \\ 0.6057 & 8.2340 & 0.3404 & 10.8082 & 0.3445 & 1.5644 \\ 3.3456 & -0.4564 & 8.2534 & -7.2434 & 1.5455 & -0.3454 \\ 0.3453 & 3.4054 & -0.2343 & 0.5688 & -1.7898 & 4.5654 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 3.4565 & 6.3064 & -1.6788 & 0.5554 & -39.2034 & -5.3023 \\ 13.5676 & -7.7432 & 2.6754 & 1.6789 & -3.6085 & -8.2353 \\ 9.3454 & 0.6780 & -4.4645 & 0.3053 & 0.3033 & -0.3403 \\ 0.3352 & 14.6504 & 0.7605 & 3.6878 & 0.3453 & 6.4506 \end{bmatrix}$$

The UAV attitude angle curves without failure are shown in Fig.5 to Fig.7. The attitude can quickly reach stability.

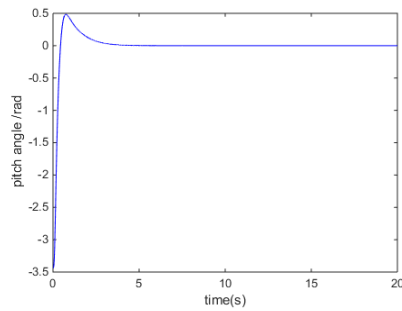


Fig.6 pitch angle curve without failure

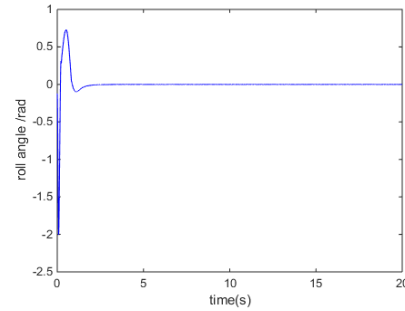


Fig.7 roll angle curve without failure

Based on the event trigger mechanism, let  $\beta = 0.4$ . The partial failure of the actuator occurred, that is,  $L = \text{diag}\{0.8, 0.9, 1, 0.8\}$ . According to Theorem 2, the event trigger matrix and controllers can be solved as

$$\Sigma = \begin{bmatrix} 2.0766 & -0.4246 & 1.2003 & -0.1544 & -0.0534 & 0.3410 \\ -0.4246 & 1.1283 & -0.0999 & 0.3136 & 0.4626 & 0.2823 \\ 1.2003 & -0.0999 & 4.2908 & -0.3918 & -0.0621 & -0.0194 \\ -0.1544 & 0.3136 & -0.3918 & 1.2740 & -0.0901 & 0.0489 \\ -0.0534 & 0.4626 & -0.0621 & -0.0901 & 1.4681 & -0.0853 \\ 0.3410 & 0.2823 & -0.0194 & 0.0489 & -0.0853 & 1.8293 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 4.6765 & -7.3243 & 0.2343 & 0.2323 & -0.4556 & 11.3535 \\ -4.4545 & -0.6766 & 3.4340 & 3.5656 & 4.4546 & -9.3345 \\ 3.6577 & -5.7878 & -0.5656 & 1.454 & -9.2344 & 3.5656 \\ 6.5656 & 0.4545 & 8.3422 & 9.5654 & 7.2320 & 8.4540 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1.5465 & -8.3434 & 6.5056 & 8.2440 & -4.5667 & 0.7656 \\ 6.4540 & 3.5606 & 5.3404 & -0.3434 & 7.2323 & 7.5606 \\ 0.6078 & 4.0044 & -0.4545 & 5.6046 & 8.978 & -8.2300 \\ 2.8009 & -6.6064 & 7.3434 & -0.7997 & 0.7089 & 0.1123 \end{bmatrix}$$

At this time, the UAV attitude angle curves with failure are shown in Fig.8 to Fig.10.

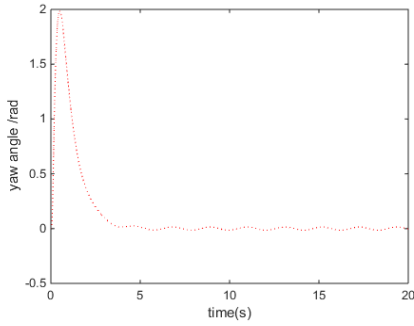


Fig.8 yaw angle with failure

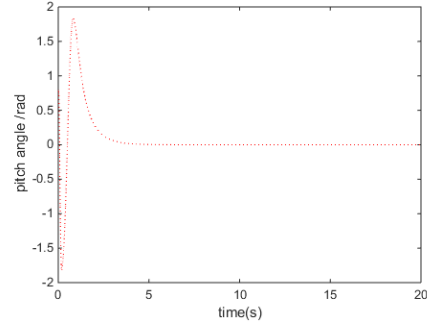


Fig.9 pitch angle with failure

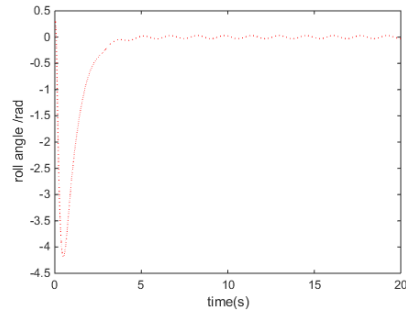


Fig.10 roll angle with failure

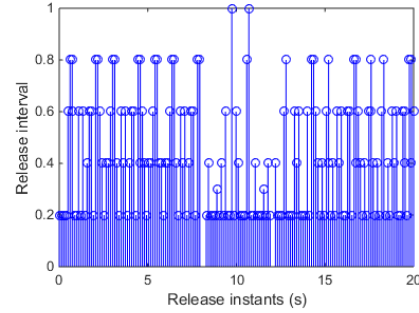


Fig.11 Release instants and release interval

The simulation results show that for UAV systems with actuator failures, time-delays and nonlinear disturbances, the designed controller can stabilize the system.

The signal release instants and release interval are shown in Fig.11. If the time-triggered mechanism is adopted, the control task needs to be executed 200

times. By using the event-triggered mechanism, the control task is executed 164 times, saving 36 times of communication. It can be seen that on the basis of ensuring the asymptotic stability of the UAV attitude, the event-triggered mechanism is adopted to reduce the number of data transmissions in the channel. This saves network resources.

In summary, the simulation research shows that the controller based on the event-triggered mechanism designed in this paper can not only make the UAV attitude system robust, but also reduce the communication network pressure. The validity of the event-triggered mechanism and controller designed is verified.

## 6. Conclusions

In this paper, for the quadrotor inspection UAV, taking into account the UAV system parameter uncertainties, time-delays, external disturbance and system failures, etc., the UAV nonlinear system model based on the T-S fuzzy method is established. In order to reduce the transmission time-delay and packet loss problems in UAV, an event-triggered mechanism is introduced. After judging whether the state error of the UAV system can meet the event-triggered condition in real time, it is determined whether the data is to be transmitted. Based on the event-triggered mechanism, the design for the observer and the robust fault-tolerant controller are given respectively. Through the observer, the states of the UAV system can be estimated. The use of robust fault-tolerant control can make the UAV system stable even when the actuator fails, and has the ability to anti-disturbance. Finally, the algorithm application simulation of the quadrotor inspection UAV is carried out. From the simulation results, the state observer can better estimate the UAV attitudes. In the event of actuator failures and disturbance signals, the pitch, roll, and yaw attitude of the UAV can be stabilized. The simulations verify the reliability and effectiveness of the designed control strategy.

In this paper, the effects of the uncertain factors such as actuator failure, external disturbance and time-delay on the performance for the UAV system are mainly considered. However, in the actual system, there are many other factors, such as data disorder, quantization distortion, etc. These uncertain factors will also affect the performance of the UAV system and should also be considered. In addition, the control strategy designed in this paper is still in the theoretical research stage, and it is only verified by the numerical simulation of the UAV system, and it has not been actually applied to the UAV experimental device. How to apply the theoretical results to actual UAV control system is also a worthwhile study in the future.

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