

SIMPLIFIED CIRCUIT MODELS FOR FERROMAGNETIC CORES

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Three new models that can be used in the simulation of the ferromagnetic cores behaviour in dynamic regimes are described. They are based on equivalent circuit diagrams, this procedure allowing easy integration of the ferromagnetic core models in the electric diagram of any electromagnetic device. The models have different degrees of complexity, as they consider a greater or a smaller number of phenomena that may occur within them in variable regimes. Their structure based on controlled sources proved easy implementation in computer programs. The obtained results are in accordance with reality and with similar results reported in the literature.

Keywords: ferromagnetic materials, magnetic core model, equivalent circuits

1. Introduction

In the design, analysis and optimization of the electromagnetic devices an important role is represented by the conception and use of some appropriate models of them. These models would be sufficiently precise so as to provide accurate information on their real behaviour. For this purpose, they must take into account as many phenomena that can be produced inside them. Obviously the models become more complex if the devices operate in dynamic regimes.

A basic component of the electromagnetic devices characterized by a complex combination of nonlinear properties and phenomena is represented by the ferromagnetic core. For this component a lot of modelling techniques have been developed through time. Most of them are based on the electromagnetic field modelling using numerical methods. However some researchers propose the use of models based on equivalent circuits due to their facilities of use and implementation in more complex schemes [1], [2], [3]. The idea of developing models based on equivalent electrical circuits is included in the current trends. In this manner we use the same method of analysis for the whole electromagnetic device that sometimes includes dielectric components or moving parts [4], [5]. Using this analysis technique the problems of interconnection between different

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methods of analysis (i.e. for electromagnetic field and for circuit respectively) are avoided.

Our study focuses on the ferromagnetic components. Some alternative models of different complexity for ferromagnetic cores are presented here. They are useful for simulating their behaviour in time-varying regimes. The models are based on equivalent electrical circuits that consider the material properties and the phenomena that may occur within them (hysteresis, eddy currents). Each proposed model was implemented in a program created in MATLAB, and a complete model was analyzed and compared using a commercial program for electrical circuit analysis (SPICE).

2. Ferromagnetic core model based on nonlinear resistance

Let's consider a ferromagnetic piece made of a homogeneous material, without magnetic hysteresis and crossed by a magnetic flux with a slow variation in time so that we can neglect the induced eddy currents. This piece is assumed to have a constant cross section S and the length l . If it is crossed by the magnetic flux $\varphi = B \cdot S$, where B is the magnetic flux density, the appropriate magnetomotive force is $u_m = H \cdot l$, where H is the magnetic field strength. Based on the known analogy between magnetic circuits and electric circuits, this piece can be modeled by a resistor crossed by a current numerically equal to φ , having the electric voltage at its terminals numerically equal to u_m . The link between these two magnetic quantities and the characteristic of magnetization of the material results from the following relations:

$$B = f(H) \Rightarrow \frac{\varphi}{S} = f\left(\frac{u_m}{l}\right) \Rightarrow \varphi = S \cdot f\left(\frac{u_m}{l}\right). \quad (1)$$

Thus, in order to model the ferromagnetic core – considering the above hypotheses – we can use a voltage controlled nonlinear resistance, with a characteristic having the form $i = S \cdot f\left(\frac{u}{l}\right)$.

Based on (1) we can build a model of ferromagnetic core using controlled sources (Fig. 1). It contains three different controlled sources: a voltage controlled voltage source with the transfer factor $\alpha_1 = 1/l$, a current controlled current source with the transfer factor $\beta_1 = 1$, and a voltage controlled current source with the transfer conductance $g_1 = 1$. The nonlinear resistor R_{n1} introduces through its $i_1 - u_1$ characteristic the nonlinear characteristic $B - H$ of the magnetic material. The linear resistor R_2 has the same value as the piece section (S).

So the correspondence between the magnetic quantities of the ferromagnetic core and the electrical quantities of its equivalent model is:

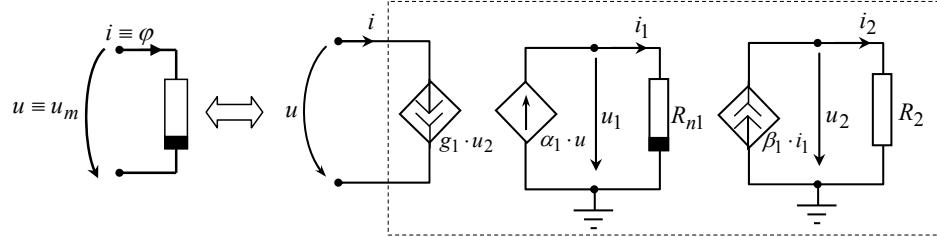


Fig. 1. Equivalent diagram with controlled sources of the ferromagnetic core without hysteresis, for slow variable magnetic fields

$$\begin{array}{lll} u \equiv u_m & u_1 \equiv H & i_2 \equiv B \\ i \equiv \varphi & i_1 \equiv B & u_2 \equiv \varphi \end{array} \quad (2)$$

As example, we consider here a ferromagnetic piece of silicon steel sheet with the dimensions $S = 13.75 \text{ cm}^2$ and $l = 6.50 \text{ cm}$. The sheets have a thickness of 0.35 mm and their characteristic of magnetization is given in fig. 2 [6].

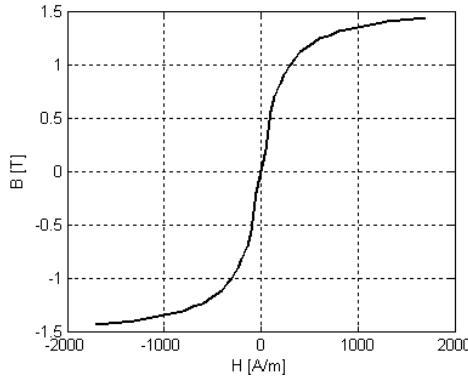


Fig. 2. Characteristic of magnetization of silicon steel sheet

The equivalent circuit of the core model from fig. 1 was introduced in a MATLAB program [7] that, based on the input data (geometric dimensions, material data, external signal as a voltage), can provide the time variation of any quantity from the equivalent circuit. Fig. 3 shows some results, namely: the time variation of the voltage (u_m) and of the current (φ) and the nonlinear reluctance characteristic of the ferromagnetic piece in the form of $i - u$ characteristic. One can notice the more distorted form of the magnetic flux as compared to time variation of the magnetic force u_m , due to the nonlinear character of the ferromagnetic core (fig. 3, a.). The dependence between current and voltage (fig.

3, b.) follows, as expected, the form of the characteristic of magnetization of the material (fig. 2).

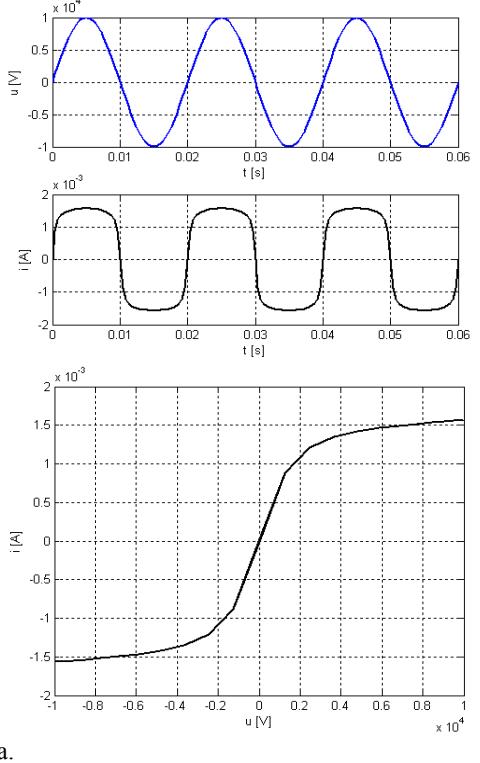


Fig. 3. Simulation results using equivalent circuit model of the ferromagnetic core without hysteresis and eddy currents: a. the time variations of the voltage (u_m) and current (φ); b. $i-u$ characteristic of the ferromagnetic piece.

3. Ferromagnetic core model that considers the eddy current effects

The eddy currents always occur in the ferromagnetic cores of the electromagnetic devices operating in dynamic regimes, even if their effect is more or less visible. Therefore, these currents must be considered in order to create a more exact model of ferromagnetic core. A solution of this is given in [3], where the effect of eddy currents is introduced considering an equivalent electrical resistance R_e that dissipates the same power as in the case of crossing the ferromagnetic piece by an equivalent current i_e equal to the total value of the eddy currents. It is shown that the value of the equivalent resistance R_e [Ω] can be expressed according to the catalog data of the material and geometric dimensions of the ferromagnetic piece ($l[\text{m}]$, $S[\text{m}^2]$) using the formula:

$$R_e = K \cdot \frac{f_0^2 B_0^2 S}{\gamma p_{Fe} l}, \quad (3)$$

where p_{Fe} [W/kg] is the specific power losses, f_0 and B_0 are the frequency and respectively the magnetic flux density that were used to determine the value of p_{Fe} , γ [kg/m³] represents the density of the material, and K is a constant which depends on the cross-section shape of the piece. In [8] some values of K and R_e are specified for different shapes of cross sections.

Therefore, in order to include the eddy currents effect, the following relations are added to the equation (1) of the previously developed model, written based on Faraday's law, Ohm's law and Ampere's law respectively:

$$e_e = -\frac{d\varphi}{dt}, \quad e_e = R_e \cdot i_e, \quad u_{me} = i_e. \quad (4)$$

In the new model of ferromagnetic core, which includes also the induced eddy currents, three additional sources are introduced (Fig. 4), having the following functions: the voltage controlled current source with the transfer conductance $g_2 = 1$ provides a current i_3 numerically equal to the magnetic flux; the voltage controlled voltage source with the transfer factor $\alpha_2 = 1$ has its output voltage, u_4 , numerically equal to the time derivative of the magnetic flux; finally, the current controlled voltage source with the transfer resistance $r_1 = 1$ gives an output voltage numerically equal to the magnetic force due to the eddy currents, u_{me} . As in the previous model, the nonlinear resistor R_{n1} introduces through its $i_1 - u_1$ characteristic the nonlinear characteristic $B - H$ of the magnetic material. The coil of inductance $L_1 = 1$ H, has a voltage at its terminals numerically equal to the derivative with respect to time of the magnetic flux. The resistance R_e , calculated as specified above, is passed by a current numerically equal to the equivalent eddy current, i_e .

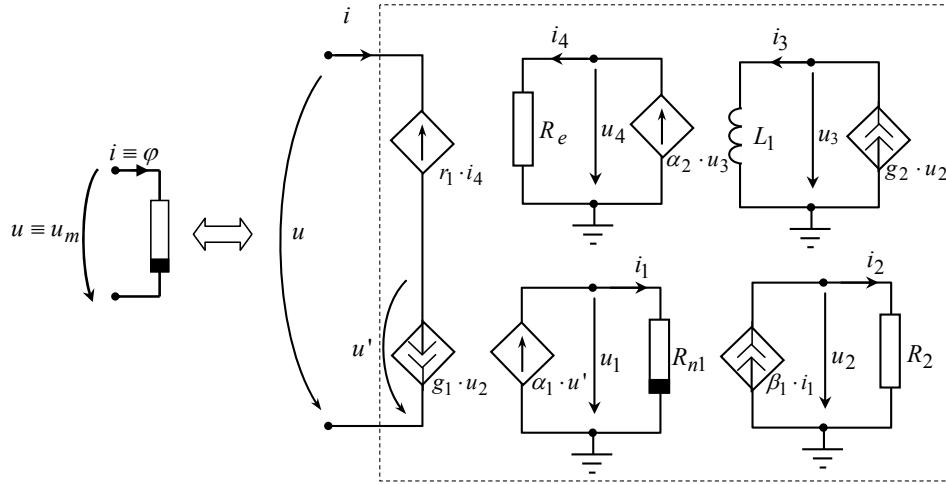


Fig. 4. Equivalent diagram with controlled sources of the nonlinear magnetic core including eddy current effect

Consequently, the correlation between electrical and magnetic quantities is, in this case, the following:

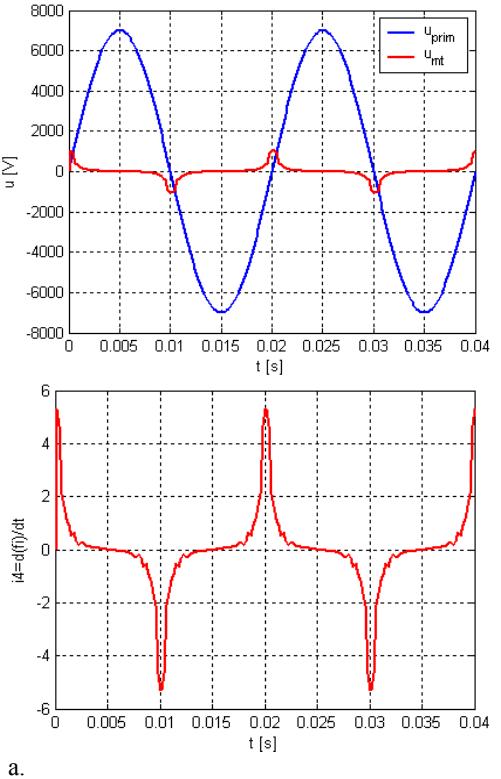
$$\begin{aligned}
 u &\equiv u_m, & u_1 &\equiv H, & i_2 &\equiv B, \\
 i &\equiv \varphi, & i_1 &\equiv B, & u_2 &\equiv \varphi, \\
 i_3 &\equiv \varphi, & u_3 &\equiv \frac{d\varphi}{dt}, & u_4 &\equiv \frac{d\varphi}{dt}, \\
 u' &\equiv u_m - u_{me}, & i_4 &\equiv i_e = u_{me}.
 \end{aligned} \tag{5}$$

For exemplification, we choose the same ferromagnetic core, where we include now the effect of eddy currents. A program created in MATLAB provides, based on the input data (geometric dimensions, material data, external signal as a voltage), the time variation of any quantity of the equivalent electric circuit. The relations were introduced simply, based on the dependence between the quantities of each loop of the circuit. When calculating the magnetic flux derivative using a coil with inductance of 1 H, we used a discrete resistive model of it [9], [10], [11]. The dependency relation between the voltage across the coil, u_3 , and the current through the coil, i_3 , was written as:

$$u_3 = L_1 \cdot \frac{di_3}{dt} = \frac{L_1}{\Delta t} \cdot i_3 - \frac{L_1}{\Delta t} \cdot i_{30} = R_{L1} \cdot i_3 - e_{L1}, \tag{6}$$

where i_3 represents the value of the current at the current time step, i_{30} is the value of the current at the previous time step and Δt is the time step.

Some of the results obtained from MATLAB simulation is shown in fig. 5. Fig. 5, a. depicts the time variations of the two components of the magnetic force across the ferromagnetic core: the sinusoidal signal with frequency of 50 Hz, required as input, and the magnetic force due to the eddy currents. Since the eddy currents depend on the time variation of magnetic flux derivative, this latter was plotted, too (fig. 5, b.), confirming that the waveform of the eddy current magnetic force is similar to the magnetic flux derivative waveform. The magnetic flux through the ferromagnetic core was plotted in fig. 5, c; its waveform is periodical, but nonsinusoidal, as a consequence of the nonlinear characteristic of the magnetic material, but also due to eddy currents.



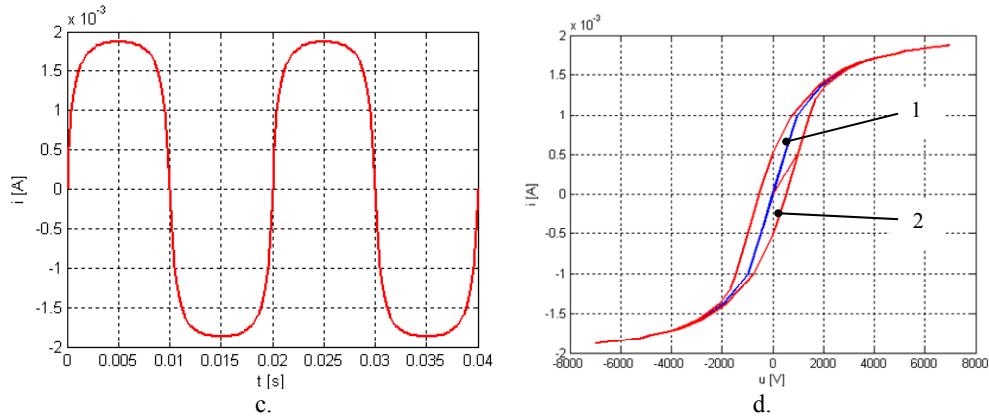


Fig. 5. Simulation results using equivalent circuit model of the ferromagnetic core with eddy currents: a. time variations of the voltage components (u_m): u' and u_m ; b. time variation of the magnetic flux derivative; c. time variation of the current (ϕ); d. comparative representation of the $i - u$ characteristic, with and without considering the eddy current effect.

Last but not least we were interested in plotting the dependence between the current (ϕ) and the voltage (u_m) for the analyzed ferromagnetic core (Fig. 5, d). On the same graph we represented the $i - u$ characteristics without considering the eddy current effect (Fig. 5, d., curve 1), and including their effect (Fig. 5, d., curve 2). In both cases one can notice that these curves are influenced by the shape of the characteristic of magnetization. In case of considering the eddy current effect the curve takes the form of a hysteresis; its area is proportional to the losses.

4. Ferromagnetic core model that considers the hysteresis and the eddy current effects

In order to obtain a complete model of ferromagnetic core, taking into account all the phenomena that occur within it, we must be considered not only the nonlinearity due to the first magnetization curve, but also the magnetic hysteresis and the eddy current effects, which occur when time varying magnetic fields are applied.

Consequently the ferromagnetic core model that considers the effect of eddy currents should be modified and completed in order to include also the magnetic hysteresis. For this purpose the Jiles-Atherton's model of hysteresis was used [12], [13] due to its analytical relations much easier to be implemented in a program that simulates electrical circuit behaviour.

Due to the hysteresis, the magnetization M is not completely reversible, presenting an irreversible component, M_{irr} , together with a reversible one, M_{rev} :

$$M = M_{rev} + M_{irr}. \quad (7)$$

The reversible magnetization can be estimated according to the anhysteretic magnetization, M_{an} , and irreversible component, M_{irr} :

$$M_{rev} = c \cdot (M_{an} - M_{irr}), \quad (8)$$

where the constant c depends on the shape of the hysteresis curve.

The anhysteretic magnetization characteristic is analytically approximated based on the modified Langevin function [14], [15]:

$$M_{an} = \begin{cases} M_s \left(\coth \frac{H_e}{a} - \frac{a}{H_e} \right), & \text{if } H_e \neq 0 \\ M_s \cdot \frac{H_e}{3a}, & \text{if } H_e \rightarrow 0 \end{cases} \quad (9)$$

where $H_e = H + \alpha M$, M_s is the saturation magnetization and α and a depends on the shape of the hysteresis.

The relation of the total magnetic susceptibility, dM/dH , is not useful in a model based on equivalent electrical circuit that will be used in a time domain analysis. Therefore a more useful relation for the hysteresis model is introduced [15]:

$$\frac{dM_{irr}}{dt} = \frac{M_{an} - M_{irr}}{k \operatorname{sgn} \left(\frac{dH}{dt} \right) - \alpha \cdot (M_{an} - M_{irr})} \cdot \frac{dH}{dt}, \quad (10)$$

where k is a constant deduced from the shape of the material hysteresis curve, too.

The relations (7) – (10), together with the law between magnetic flux density, magnetic field strength and magnetization, $B = \mu_0(H + M)$, will be used in the new model to describe both the hysteresis loop and initial magnetization curve, now given by a set coefficients (c, k, α, a, B_s , where B_s is the saturation magnetic flux density). Equivalent circuit model, with controlled sources, of the ferromagnetic core takes the form presented in fig. 6.

In this diagram the conductance G_{n1} imposes the value of the current i_2 , numerically equal to the derivative with respect to time of the magnetization, the law of variation of its current being given by a relation of type (10), written for the magnetization M . The capacitor with $C_1 = 1\text{F}$ acts as an integrator, the voltage across it, u_2 , being numerically equal to the magnetization M .

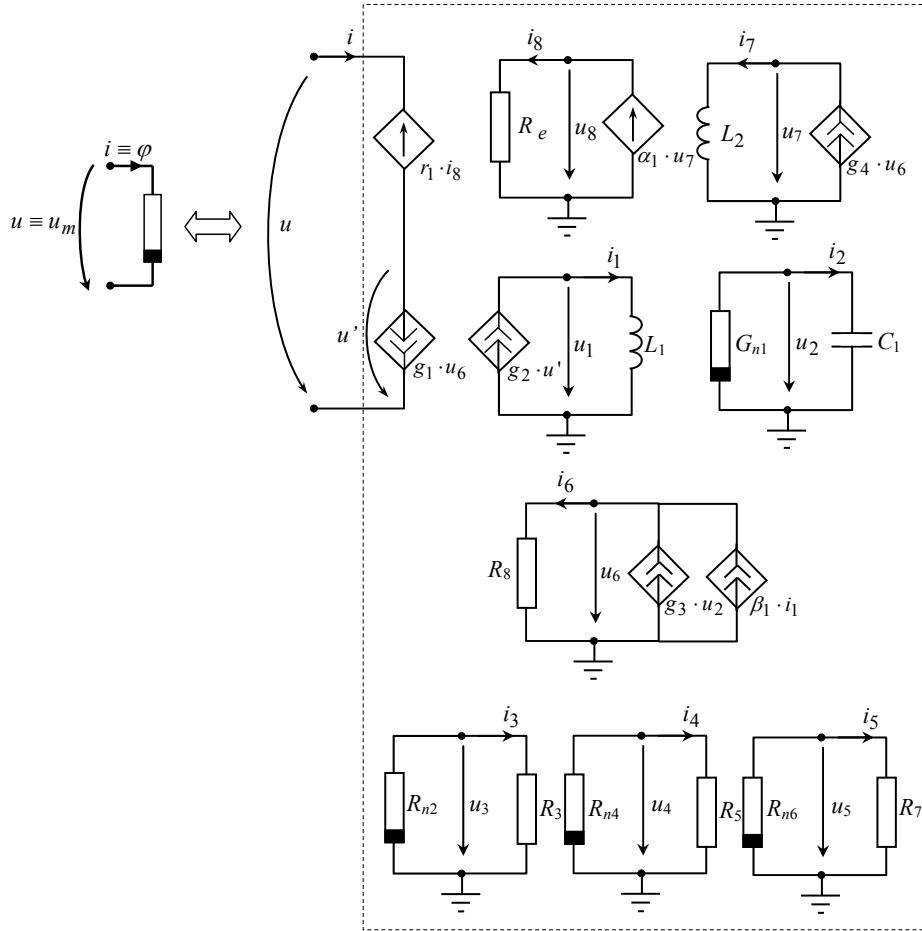


Fig. 6. Equivalent diagram with controlled sources of the nonlinear magnetic core that includes magnetic hysteresis and eddy current effect

The system of two controlled sources connected in parallel, with unity transfer factors, imposes the current through the linear resistor of resistance $R_8 = \mu_0 \cdot S$, whose voltage u_6 has a value equal to the magnetic flux through ferromagnetic piece, φ . This voltage controls the voltage controlled current source having the transfer conductance $g_1 = 1$. The set of circuits with nonlinear resistors, R_{n2}, R_{n4}, R_{n6} , serves to introduce the analytical relations of (H_e/a) , (9) and the expression (9) derived with respect to H [15]. Thus, the tensions u_3, u_4, u_5 are numerically equal to H_e/a , M_{an} and dM_{an}/dH respectively.

The linear resistors R_3, R_5, R_7 are introduced only to close the circuits and they are considered of high values.

As for the eddy current part, the equivalent circuit remains unchanged as compared with the previous model.

The correspondence between the electrical equivalent circuit quantities and the magnetic quantities of the ferromagnetic piece is summed up as follows:

$$\begin{aligned}
 i_1 &\equiv H, & u_2 &\equiv M, & u_4 &\equiv M_{an}, \\
 u_1 &\equiv \frac{dH}{dt}, & i_2 &\equiv \frac{dM}{dt}, & u_3 &\equiv \frac{H_e}{a}, \\
 u_5 &\equiv \frac{dM_{an}}{dH}, & u_6 &\equiv \varphi, & u_7 &\equiv \frac{d\varphi}{dt}, & u_8 &\equiv \frac{d\varphi}{dt}, \\
 u' &\equiv u_m - u_{me}, & i_6 &\equiv H + M, & i_7 &\equiv \varphi, & i_8 &\equiv i_e = u_{me}.
 \end{aligned} \tag{11}$$

To analyze the behaviour of the ferromagnetic core equivalent model with the consideration of hysteresis and eddy currents, a program in MATLAB was conceived. It analyzes the same piece of ferromagnetic core of silicon steel sheet, with section $S = 13.75 \text{ cm}^2$ and length $l = 6.50 \text{ cm}$. The material characteristics are now introduced as material coefficients.

Both for the coils and for the capacitor discrete resistive models were used, of type (6) and respectively [9], [10]:

$$i_2 = C_1 \cdot \frac{du_2}{dt} = \frac{C_1}{\Delta t} \cdot u_2 - \frac{C_1}{\Delta t} \cdot u_{20} \Rightarrow u_2 = \frac{1}{C_1} \cdot i_2 \cdot \Delta t + u_{20}, \tag{12}$$

where u_2 is the voltage at the current time step, u_{20} is the voltage value at the previous time step and Δt is the time step.

Several simulation results are shown in fig. 7-8.

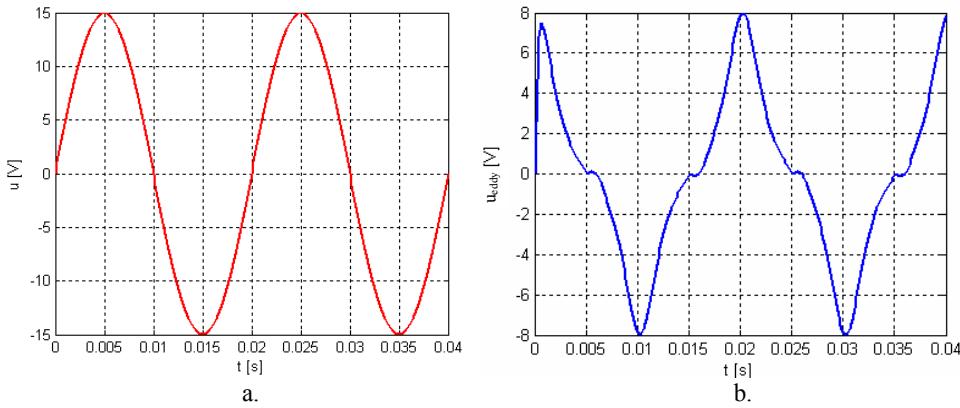


Fig. 7. MATLAB simulation results using equivalent circuit model of the ferromagnetic core, with hysteresis and eddy currents: a. time variation of the applied voltage u' ; b. the time variation of eddy current induced tension, u_{me} .

Fig. 7, a. shows the sinusoidal signal applied to the input, as a voltage of the same value as the magnetic force. Fig. 7, b. presents the contribution of the eddy currents in the form of an equivalent voltage, strongly distorted. The dependence $\varphi - u_m$ in the form of hysteresis cycles with and without considering the effect of eddy currents is also represented (Fig. 8, b.). For comparison reason, these curves were placed alongside the corresponding curves derived from the implementation of the ferromagnetic core model in SPICE, as a subcircuit (Fig. 8, a.). Analyzing the obtained hysteresis curves one can find the influence both of the phenomenon of hysteresis (curve 1, red) and of the eddy currents induced in the core (curve 2, black). For the analyzed case, the eddy current weight is greater. We can remark the saturation of the core and a good behaviour of the model in the extreme points of the curves. By comparison, the curves obtained in MATLAB and SPICE have the same allure and losses weights are the same, the areas of hysteresis curves given by the both programs resulting approximately the same.

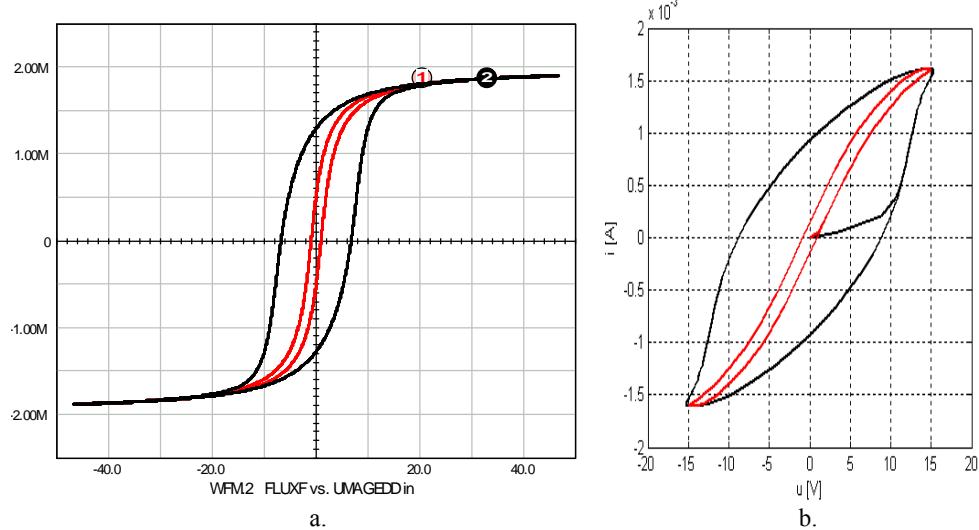


Fig. 8. Characteristics $i - u$ ($\varphi - u_m$), with and without the consideration of the eddy current effect, obtained by implementation of the ferromagnetic core model: a. in SPICE; b. in MATLAB.

5. Conclusions

The advantages of using equivalent electrical circuits that contain controlled sources are primarily related to the simplicity of the component circuits - each containing a single loop, so only one relation between its elements – and to the extremely suggestive form of representation. Among developed models, the ferromagnetic core model which considers both magnetic hysteresis and eddy current effect is most suitable for the analysis of the electromagnetic devices in

dynamic regimes, providing accurate information and a more real image over the phenomena that occur in them. The models associated to the ferromagnetic cores were designed from a new perspective, aiming to obtain complex models as phenomenology, but simple as implementation and solving.

R E F E R E N C E S

- [1] *L. Dixon*, “An Electrical Circuit Model for Magnetic Cores”, online: www.ti.com, Texas Instruments data books, Dallas, Texas, USA, October 1994, pp. 6-1– 6-9.
- [2] *P. R. Wilson, J. Neil Ross and A. D. Brown*, “Simulation of Magnetic Component Models in Electric Circuits Including Dynamic Thermal Effects”, *IEEE Transactions on Power Electronics*, **vol.17**, no.1, January 2002, pp. 55-65.
- [3] *L. Mandache, K. Al-Haddad*, “Accurate Design Tool for Filter Inductors Intended for Adjustable Speed Drives”, 33rd Annual Conference of the IEEE Industrial Electronics Society IECON 2007, Taipei, Taiwan, November 5-8, 2007, pp. 1414-1419.
- [4] *Lisheng Shi, M. L. Crow*, “Comparison of Ultracapacitor Electric Circuit Models”, *IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Engineering in the 21st Century*, Pittsburgh, PA, USA, July 20-24, 2008, pp. 1-6.
- [5] *K. Thorborg, A. D. Unruh and C. J. Struck*, “An Improved Electrical Equivalent Circuit Model for Dynamic Moving Coil Transducers”, 122nd Audio Engineering Society Convention Papers, Vienna, Austria, May 5-8, 2007, pp. 1-13.
- [6] *Gh. Hortopan, Gh. Cosmin, M. Huhulescu, V. Panaite, D. Simulescu, R. Tomoioagă*, Aparate electrice de joasă tensiune (Low Voltage Electrical Apparatus), Editura Tehnică, Bucureşti, 1969.
- [7] *M. Ghinea, V. Fireteanu*, MATLAB - Calcul numeric. Grafică. Aplicații (MATLAB - Numeric Computation. Graphics. Applications), Editura Teora, Bucureşti, 2003.
- [8] *L. Mandache, D. Topan, K. Al-Haddad*, “Modeling of Nonlinear Ferromagnetic Cores”, *Revue Roumaine des Sciences Techniques - Série Electrotechnique et Energétique*, **no.53**, vol.4, 2008, pp. 403-412.
- [9] *M. Iordache, L. Mandache*, Analiza asistată de calculator a circuitelor analogice neliniare (Computer-Aided Analysis of Nonlinear Analog Circuits), Editura Politehnica Press, Bucureşti, 2004.
- [10] *Cătălina-Gabriela Popescu*, “Discrete Resistive Models for Multi-Rate Analysis of Integrated Analog Circuits”, in U.P.B. Scientific Bulletin, Series C, **vol. 72**, issue 3, 2010, pp. 269-276
- [11] *A. G. Gheorghe, F. Constantinescu*, “Solution Computation for Resistive Circuits Containing Companion Models”, in U.P.B. Scientific Bulletin, Series C, **vol. 72**, issue 4, 2010, pp. 193-204
- [12] *D. C. Jiles, X. Fang and W. Zhang*, “Modeling of Hysteresis in Magnetic Materials”, in Handbook of Advanced Magnetic Materials, Publisher Springer US, 2006, pp. 745-779
- [13] *D.C. Jiles, J.B. Thoelke and M.K. Devine*, “Numerical Determination of Hysteresis Parameters for the Modeling of Magnetic Properties Using the Theory of Ferromagnetic Hysteresis”, *IEEE Transactions on Magnetics*, **vol. 28**, no. 1, January 1992, pp. 27-35.
- [14] *J.T. Hsu, K.D.T. Ngo*, “Subcircuit Modeling of Magnetic Cores with Hysteresis in PSpice”, *IEEE Transactions on Aerospace and Electronic Systems*, **vol. 38**, no. 4, October 2002, pp. 1425-1434.

[15] *L. Mandache, D. Topan, Ioana-Gabriela Sîrbu*, “Accurate Time-Domain Simulation of Nonlinear Inductors Including Hysteresis and Eddy-Current Effects”, Proceedings of the World Congress on Engineering - WCE 2011, July 6-8, 2011, London, U.K., Vol. II, pp.1327-1332.