

## A COMPARISON BETWEEN DIFFERENT DIGITAL FILTERS USED IN DIGITAL RELAYS

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*The relay algorithm, which is implemented through microprocessor, is considered as the Mastermind of a numerical relay. This paper analyzes the characteristics and mathematical formula of some digital filters which are used in numerical relays, the performance of these filters was investigated by applying different current waves to these filters. Furthermore, this paper includes a comparison of responses between these filters. The Simulation study was done by using MATLAB software, and the used current waves for this study were downloaded from a disturbance recorder used in Banias thermal power plant.*

**Keywords:** Digital Filters, Fourier filter, Numerical Relays, Relay Algorithm

### 1. Introduction

Protection relays represent a particular importance of electrical power systems, they detect any malfunction or defect may occur, after that they issue orders to the circuit breakers to isolate the specific defect element selectively for maintaining the continuity of feeding in the rest of the system and the safety of equipment and personnel. Numerical relay is the last generation of protective relays; the digital filter is the principal element which determines the response and speed of protection functions. Each digital filter has a special method to reconstruct the sampled signal received from an A/D Converter after eliminating all unwanted components such as DC offset harmonics, sub-harmonics and noise to a feasible extent, and extracting all fundamental components which are necessary for the relay operation. The values of the reconstructed signal were compared with the pre-set threshold values, and a critical decision (alarm or trip) was issued by the digital filter.

### 2. Digital filters categories

There are two main categories of digital filters according to the number of input signals; digital filters for one-element relay and two-elements relay [1], [2], [3], [4], [5].

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## 2.1. Digital filters for one-element relay

One-element relay uses only a single input signal either current or voltage, i.e. over current relay uses only current signal for its calculation, and the same thing for under voltage relay that uses voltage signal for its operation, etc..., [1].

Many well-known digital filters use a single input signal for its calculation such as Fourier, Walsh, and Kalman filters.

### 2.1.1. Fourier Filters

Fourier Filters are the most used filters in digital relays. Fourier Filters satisfy Dirichlet conditions; any signal can be considered as a set of periodic components if it satisfies the following conditions:

- The Periodic signal repeats itself after each period T.
- The Periodic signal is defined within the period T.
- Limited extremes (maximum and minimum) within any period.

The fundamental component values depending on the Discrete Fourier Transform can be calculated by the following equations [1], [2], [3], [6], [7], [8], [9]:

$$\hat{Y}_s = \frac{2}{K} \sum_{k=1}^K y_k \sin(k\theta) \quad (1)$$

$$\hat{Y}_c = \frac{2}{K} \sum_{k=1}^K y_k \cos(k\theta) \quad (2)$$

$$\theta = \frac{2\pi}{k} = 2\pi \frac{f_0}{f_s} \quad (3)$$

Where:

$\hat{Y}_s$ : the peak value of the imaginary component of the fundamental frequency phasor of the input signal  $y_k$ ;

$\hat{Y}_c$ : the peak value of the real component of the fundamental frequency phasor of the input signal  $y_k$ ;

$\theta$ : the fundamental frequency angle between samples [8];

$k$ : the number of samples per cycle;

$f_0$ : the fundamental frequency of the electrical power system (50 or 60 Hz);

$f_s$ : the sampling frequency.

The magnitude of Fourier filter is calculated by the equation:

$$|Y| = \sqrt{\left(\hat{Y}_S\right)^2 + \left(\hat{Y}_C\right)^2} \quad (4)$$

The main aim of the Fourier filter (DFT) is to get an accurate phasor for the output signal equal to the fundamental frequency phasor. The window length has an important effect on the filter response, a window of Full-Cycle DFT ( $k = N = f_s / f_0$ ) and Half-Cycle DFT ( $K = N/2 = f_s / (2*f_0)$ ) are selected for that purpose, which are corresponding to the fundamental frequency (50 or 60 Hz) of the electrical power system, where  $N$  is the number of samples per cycle[8], [10]. To analyze the performance of this filter, we test number of input signals, as in the following cases.

Case 1: the input signal  $y_i$  contains the fundamental frequency only.

$$y_i = 22 * \sin(2\pi ft) \quad (5)$$

Fig. 1 displays the input signal and the output of the Fourier filter (DFT).

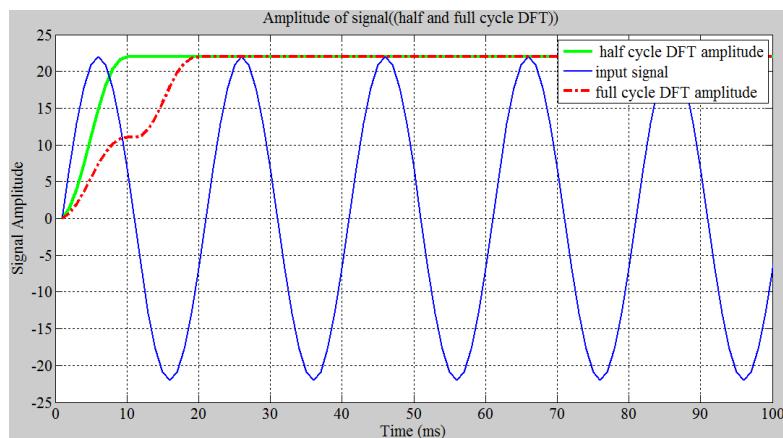


Fig. 1. Input signal and its resultant amplitude by means of Fourier filter

From the previous figure, the filter half cycle needs less time than full cycle to calculate the amplitude of the input signal at the fundamental frequency, but the accuracy is identical for both of them.

Case 2: the input signal contains only odd harmonics as shown in table 1.

Table 1

The Harmonics content of the input signal

Harmonics Order "n"	% of true RMS
1'fundemental'	22
3	11
5	6
7	9
9	8
11	7

Fig. 2 shows the input signal which contains odd harmonics and the amplitude of input signal (r.m.s value) after calculating it by half / full-cycle DFT.

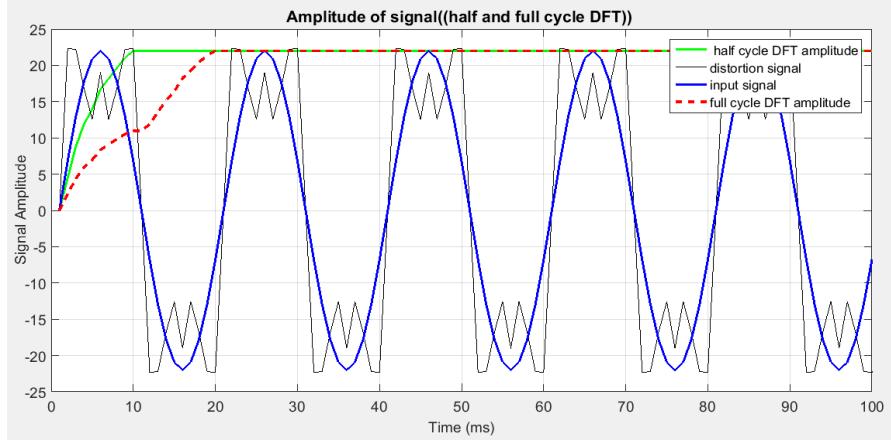


Fig. 2. Input signal and its resultant amplitude by means of Fourier filters in case 2

From the previous figure, it can be noticed that DFT eliminates all odd harmonics, so that the amplitude of that signal equals to the amplitude of the fundamental component, and the filter performance is not affected by the presence of odd harmonics.

Case 3: Sometimes the occurred slight shift of the fundamental frequency compared with the rated frequency ( $f=50$  Hz).

$$y_i = 22 * \sin(2\pi ft) \quad (6)$$

$f=49.5$  Hz

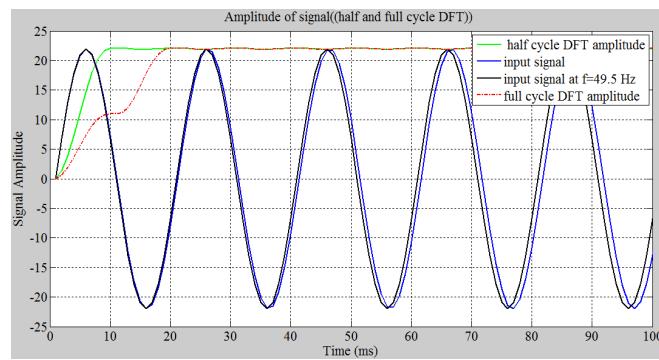


Fig. 3. Input signal and its resultant amplitude by means of Fourier filters

From Fig. 3, it can be noticed clearly that in the case of the shift of the fundamental frequency, slight changes will happen in the output of the DFT (full-cycle & half-cycle), and these changes are increased by the increasing of the frequency shift.

Case 4: Saturation of current transformers

Fig. 4 shows current waves downloaded from disturbances recorder in Banias Thermal Power Plant; the recorded wave shows severe saturation of current transformer.

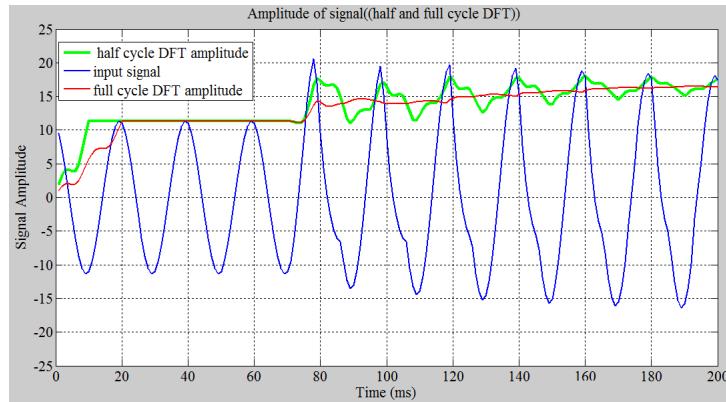


Fig. 4. Saturated wave current and its amplitude by using DFT

As it was shown, the response of DFT full-cycle is better than DFT half-cycle, but the accuracy of DFT full-cycle is better than DFT half -cycle.

In both cases, the measured value is less than the actual value because the saturation of the current transformers.

Case 5: The input signal contains odd and even harmonics as shown in Fig. 4. All of nonlinear loads, power transformer (saturation case) and power electronics elements are considered as main sources of harmonics distortion.

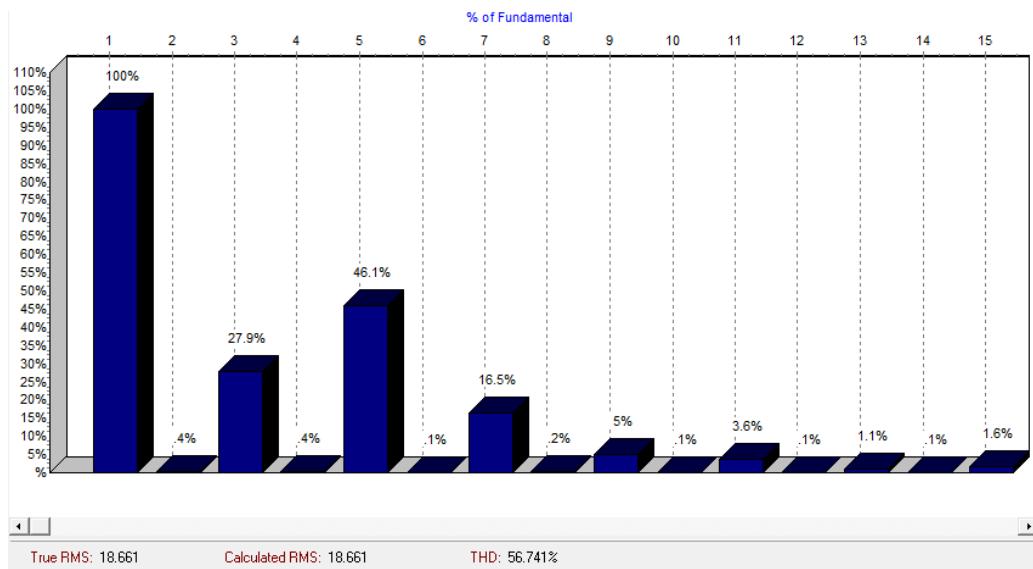


Fig. 4. Harmonics analysis of input signal by using Micom S1 Agile

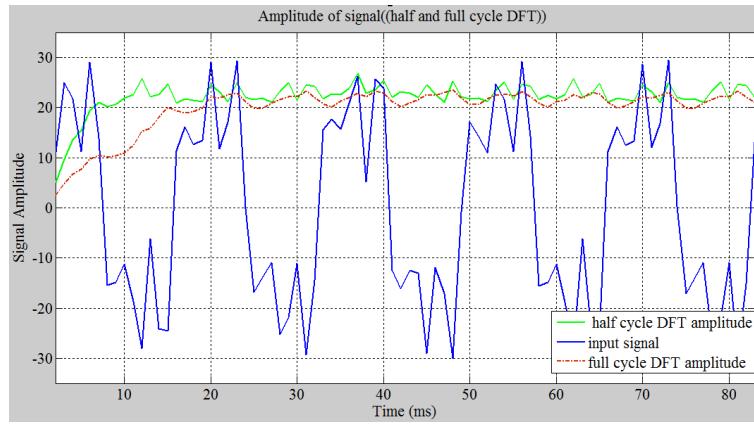


Fig. 5. Input signal and its resultant amplitude by means of Fourier filter

The performance of DFT is unstable in these cases, so that the usage of anti-aliasing filter is important to attenuate the effect of harmonics as much as possible.

### 2.1.1.1. A Comparison between accuracy and speed of the two approached Fourier filters

The overall decision-making time of the relay is based on the length of the moving scan window. The frequency response of full-cycle Fourier filter is shown in Fig. 6.

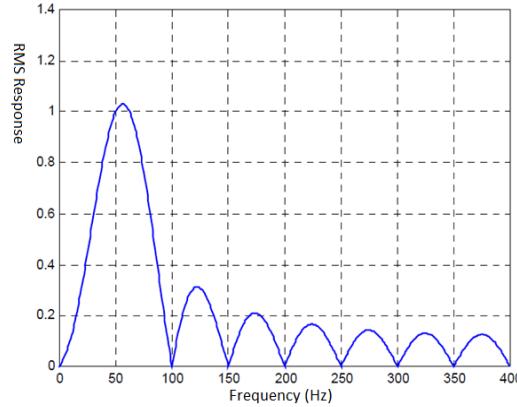


Fig. 6. Frequency response of full-cycle Fourier filter [13]

From Fig. 6, it is obvious that full-cycle DFT has a great capability for eliminating DC Component and all harmonics of the integer orders. The frequency response of half-cycle Fourier filter is shown in Fig. 7.

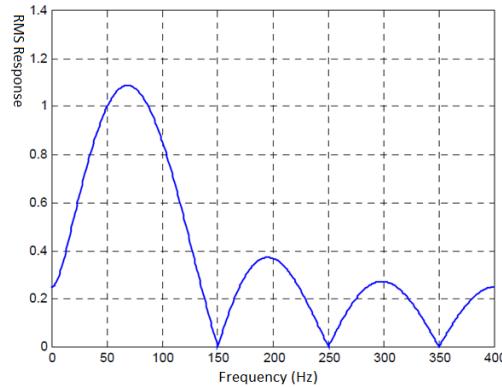


Fig. 7. Frequency response of half-cycle Fourier filter [1]

From Fig. 7 it is clear that half-cycle DFT is unable to reject the DC component and all the even harmonics orders. Consequently, the stability of the half-cycle DFT is very low if the input signals contain such components. From the previous section, for the half-cycle DFT the decision-making time is at least 10 ms and 20 ms for the full-cycle DFT. Finally, half-cycle filters improve the speed of the relay at the expense of accuracy, but half-cycle filters improve the accuracy of the relay at the expense of speed.

### 3.4.1.3 Walsh Filters

Walsh filters are a set of orthogonal signals which only take the values 0, 1. The fundamental components values depending on the Walsh filter can be calculated by the following equations [1], [2], [3], [6], [7], [8], [9], [12]:

$$\begin{aligned} \overset{\Lambda}{Y} cal &= \frac{1}{k} \sum_{k=1}^k y_k cal_1(k\theta) \\ \overset{\Lambda}{Y} sal &= \frac{1}{k} \sum_{k=1}^k y_k sal_1(k\theta) \end{aligned} \quad (7)$$

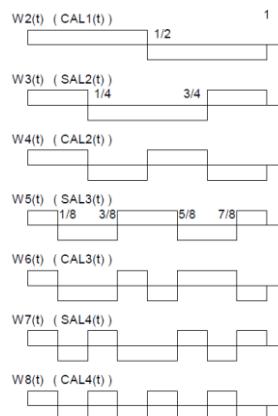


Fig. 8. The first 8 Walsh functions

The magnitude of Fourier filter is calculated by the equation:

$$|Y| = \sqrt{\left(\frac{Y_{cal}}{Y_{sal}}\right)^2 + \left(\frac{Y_{sal}}{Y_{cal}}\right)^2} \quad (8)$$

The Walsh filter needs to calculate a sufficient number of Walsh coefficients terms to obtain the same accuracy compared with the full-cycle Fourier filter [3].

i.e. if the input signal  $y_i$  is given by the following equation:

$$Y_i = (8 * \cos(\omega t)) + (6 * \cos(2\omega t)) + (8 * \exp(-t/0.08)) + (0.5 * \cos(3\omega t)) + (0.4 * \cos(4\omega t)) + (13 * \cos(5\omega t)) \quad (9)$$

the output of Walsh filter will be as shown in Fig. 8.

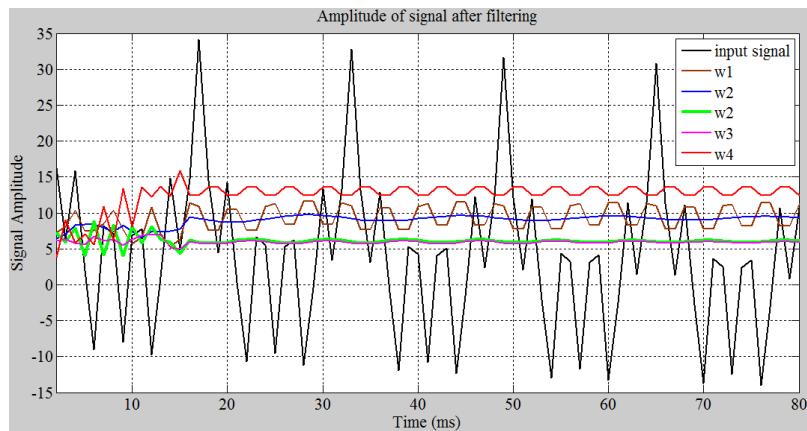


Fig. 8. Input signal and its resultant amplitude by means of Walsh filter

Fig. 9 shows the output of full-cycle DFT for the same previous wave.

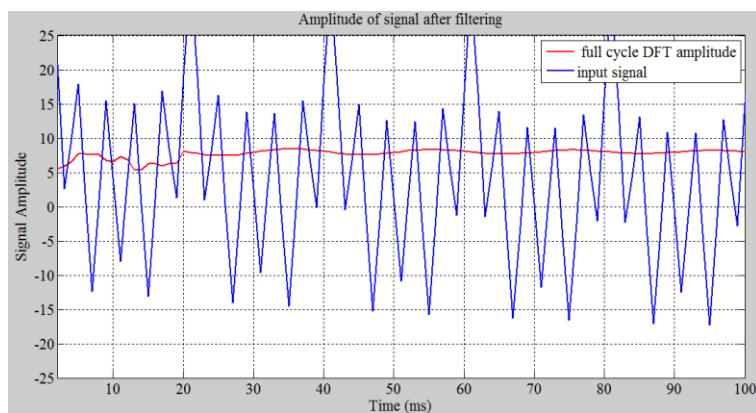


Fig. 9. Input signal and its resultant amplitude by means of full-cycle DFT

It is obvious that Fourier filters have higher speed than Walsh filter for the same accuracy, so that Fourier filter is widely used than other filters in computer relaying applications.

### 3.4.2 Digital Filters for Dual-Element Relay

A dual-element relay needs two input signals to perform its calculations, such as distance relays and power factor relays.

Two main algorithms which are used for that type of relays, are Fourier algorithm and Differential Equation algorithm.

#### 3.4.2.1 Fourier algorithm

There are no differences in the principles between the Fourier algorithm and the Fourier filter. Input signals of voltage and current, are divided into two orthogonal components (real part and imaginary part) by using the Fourier filters. According to those values, the apparent impedance is calculated as follows:

$$Z = \frac{U}{I} = \frac{U_C + jU_S}{I_C + jI_S} \quad (10)$$

$$R = \frac{U_C \cdot I_C + U_S \cdot I_S}{I_C^2 + I_S^2} \quad (11)$$

$$X = \frac{U_S \cdot I_C - U_C \cdot I_S}{I_C^2 + I_S^2} \quad (12)$$

$$Z = \sqrt{R^2 + X^2} \quad (13)$$

Where  $I_s$ ,  $I_c$ ,  $U_s$  and  $U_c$  are the orthogonal components of voltage and current Respectively [1], [2], [3], [7], [13].

## 3.5 Discussion and Conclusion

In some cases, half-cycle Fourier Filters are unable to remove undesired components like DC component and even harmonics orders from the input signal, and consequently the performance of this filter is low, so that the usage of anti-aliasing filter is necessary to attenuate the effect of these components as much as possible. Full-cycle Fourier Filters have a good ability to remove DC Component and all harmonics of integer orders.

Half-cycle DFT improves the speed of the relay at the expense of accuracy, while half-cycle DFT improves the accuracy of the relay at the expense of speed. It is clear that from the previous comparison between Fourier filters and Walsh filters for the same accuracy, Fourier filters have higher speed than Walsh filters, thus Fourier filters are widely used than other filters in computer relaying applications and by manufacturers of protection units. The choice option of any

filters depends on the protection system structures and the economic considerations. Each component of numerical relay unit needs a lot of study and analysis, because the total performance of the numerical relay is integrated. Ongoing monitoring efforts and case studies will provide the information to characterize system performance and to understand the problems well and to find the suitable solutions.

## R E F E R E N C E S

- [1] F. Wang, "On Power Quality and Protection by," no. 372, 2001.
- [2] Waldemar Rebizant, Janusz Szafran Andrzej Wiszniewski, Digital Signal Processing in Power System Protection and Control, Wroclaw University of Technology, Poland, 2011.
- [3] Arun G. Phadke, James S. Thorp, Computer Relaying for Power Systems, second edition, USA, 2009.
- [4] Zitouni Abdelkader, Abderrahmane Ouadi and Hamid Bentarzi, Measurement Quality Enhancement using Digital Filter in Power Grid integrating TCSC, IEEE, 2015.
- [5] Edmund O. Schweitzer, and Daqing Hou, Filtering for Protective Relays, Schweitzer Engineering Laboratories, USA, 1995.
- [6] J. M. Kennedy, G. E. Alexander, J. S. Thorp, Variable Digital Filter Response Time in a Digital Distance Relay, General Electric Company, USA, 2013.
- [7] E. Orduña, D.G. Colomé, G. D. G. Venerdini, G. Rattá, O. M. Torres and J. S. Ulloa, Behavior of the Digital Fourier Transform as Numerical filter in Distance Protection of Series Compensated Transmission Lines. Simulations with a Real Transmission System, Ieee Latin America Transactions, Vol. 10, No. 5, September 2012.
- [8] Chi-Shan Yu, Yi-Sheng Huang and Joe-Air Jiang, A Full- and Half-Cycle DFT-based Technique for Fault Current Filtering, IEEE, 2010.
- [9] Vidyarani K.R, R. Nagaraja, G.K. Purushothama and Somnath Guha, Implementation of Advanced DSP Techniques in Distance Protection Scheme, IEEE, 2012.
- [10] H. O. Pascual and J. A. Rapallini, Behaviour of Fourier, Cosine and Sine Filtering Algorithms for Distance Protection, under Severe Saturation of the Current Magnetic Transformer, IEEE, 2001.
- [11] Network Protection & Automation Guide, Edition MAY 2011, Published by Alstom Grid ISBN: 978-0-9568678-0-3.
- [12] Hector J. Pdtuve F., Ismael Dim V., Ernesto Vhzquez M, Fourier and Walsh digital filtering algorithms for distance protection, Mexico, 1995.
- [13] Q.H. Wu, Z. Lu and T.Y. Ji, Protective Relaying of Power Systems Using Mathematical Morphology, University of Liverpool, England, 2009.