

# INVESTIGATION OF THE EVEN-EVEN N=106 ISOTONIC CHAIN NUCLEI IN THE GEOMETRIC COLLECTIVE MODEL

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*Geometric-Collective-Model (GCM) is applied in this paper for the first time to the chain of even-even N=106 isotones:  $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$ ,  $^{184}\text{Pt}$ ,  $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ ,  $^{190}\text{Po}$ . Experimentally known low lying energy spectra, and electromagnetic properties are used in order to establish the parameters required by the model. The observables are analyzed as trend on the isotonic chain. In the GCM calculation, the absolute values of the ground state energies are taken to be equal with the experimental binding energies per nucleon. By using this novel procedure a self consistent set of GCM parameters has been obtained, allowing more realistic predictions for the absolute values of the nuclear bound state energies. Moreover, comparable sets of GCM parameters can be obtained for different isotopic and isotonic chains.*

**Keywords:** The Geometric Collective Model (GCM), spherical, triaxial, prolate, oblate deformation, nuclear structure

## 1. Introduction

The Geometric Collective Model (GCM) offers a very powerful instrument for the study of nuclear structure [1], and contains the main limiting cases of nuclear collective features- spherical, triaxial, prolate, oblate, and also nuclei with two minima in the potential energy. The GCM can be successfully used to calculate the low lying energy spectra,  $B(E2)$  values, and quadrupole moments of even-even nuclei from almost entire nuclear chart. There are reported several works on GCM investigations of the nuclear isotopic chains. For example collective states of  $^{124-132}\text{Ba}$  in the GCM framework are reported in [2]. Also the GCM analysis was applied to the isotopic chain of Pt, Os, and W in [3].  $^{108,110,112}\text{Ru}$  and  $^{152}\text{Sm}$  are studied in [4] and [5] respectively. To our knowledge there is no systematic GCM study on a nuclear isotonic chain reported in the literature up to now. In the present work it is presented such a study for the even-even N=106 isotonic chain nuclei. It is chosen this isotonic chain due to the richness of available experimental data and variety of nuclear structure phenomenology. It is the main idea in this paper that, in calculating the spectroscopic properties (level energies states, electromagnetic transition strengths, quadrupole deformation, etc) the first step in importance is to identify a

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correct absolute energy reference for entire nuclear chain, and even for all nuclei. In this work this absolute energy reference is considered to be build on the basis of the experimental Binding-Energy per nucleon (BE/A) value, which is associated to the ground state (g. s.) energy value [6]. In this way a coherent set of model parameters has been obtained for the entire isotonic chain (Table 1-a, b). Staying on this point of view, after a brief description of the GCM formalism and some details of the calculation procedure, there are presented the nuclear observables obtained with GCM and their comparison with the experimental data of the even-even N=106 isotonic chain nuclei:  $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$ ,  $^{184}\text{Pt}$ ,  $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ , and  $^{190}\text{Po}$ . Discussions in terms of nuclear symmetries and  $\gamma$ -band staggering are done. A brief summary and conclusions are given in the end.

## 2. GCM formalism

The collective model ([1], [8],[12]) was build in the collective space's deformation coordinates  $\alpha_{2m}$  defined via the expansion of the nuclear surface in terms of spherical harmonics  $Y_{2m}(\theta, \varphi)$ :

$$R(\theta, \varphi, t) = R_0(1 + \sum_m \alpha_{2m}(t) Y_{2m}(\theta, \varphi)) \quad (1)$$

$R(\theta, \varphi, t)$  is the radius of the nucleus along the direction  $(\theta, \varphi)$  in the laboratory-fixed principal-axes center-of-mass system and  $R_0 = 1.1A^{1/3}$  fm. The conjugate momentum operator  $\hat{\pi}_{2m}$  to the corresponding  $\hat{\alpha}_{2m}$  operator coordinates is defined by requiring canonical commutation relation:

$$[\hat{\pi}_{2n} \hat{\alpha}_{2m}] = -i\hbar \delta_{mn}, \quad (m, n = -2, -1, 0, +1, +2) \quad (2)$$

The kinetic energy is constructed to contain the two lowest-order terms proportional to the square of the momentum:

$$\hat{T} = \frac{1}{B_2} [\hat{\pi} \times \hat{\pi}]^{[0]} + \frac{P_3}{3} \{ [\hat{\pi} \times \hat{\alpha}]^{[2]} \times \hat{\pi}^{[0]} \}, \quad (3)$$

where  $\{.....\}$  is the sum over all permutation between the  $\hat{\pi}$  and  $\hat{\alpha}$ .  $B_2$  is the mass parameter and  $P_3$  is the anharmonic kinetic-energy term. The general form of Hamiltonian is:

$$\hat{H} = \hat{T} + \hat{V} \quad (4)$$

The multiplicative potential operator  $\hat{V}$ , constructed to be a polynomial expansion in the deformation coordinates, containing all possible independent terms up to sixth order (Ref.[1]):

$$\begin{aligned} \hat{V}(\hat{\alpha}) = & C_2 [\hat{\alpha} \times \hat{\alpha}]^{[0]} + C_3 [[\hat{\alpha} \times \hat{\alpha}]^{[2]} \times \hat{\alpha}]^{[0]} + C_4 ([\hat{\alpha} \times \hat{\alpha}]^{[0]})^2 + \\ & C_5 [[\hat{\alpha} \times \hat{\alpha}]^{[2]} \times \hat{\alpha}]^{[0]} [\hat{\alpha} \times \hat{\alpha}]^{[0]} + C_6 ([\hat{\alpha} \times \hat{\alpha}]^{[2]} \times \hat{\alpha})^{[0]} + D_6 ([\hat{\alpha} \times \hat{\alpha}]^{[0]})^3 \end{aligned} \quad (5)$$

The square of form  $[\hat{\pi} \times \hat{\alpha}]^{[2]}$  and  $[\hat{\pi} \times \hat{\alpha}]^{[0]}$  is the product of spherical tensor operators  $\hat{\pi}$  and  $\hat{\alpha}$  coupled to angular momentum two, and zero respectively.

It is more convenient to study the nucleus in the “intrinsic” system of the body-fixed principal-axis system, transforming the laboratory-fixed  $\alpha_{2m}$  coordinates axis in to the corresponding intrinsic  $\hat{a}_{2\mu}$  ( $\mu = -2, -1, 0, +1, +2$ ) ones [9]. Further on, we transform this orthogonal intrinsic coordinates in to spherical coordinates and, keeping in mind the axial symmetry relation:

$a_{2-2} = a_{2+2}$ , we can retain only two intrinsic polar coordinates:  $\beta$ -the quadrupolar deformation of nucleus, and the angle  $\gamma$  defined by the relation [6]):

$$\beta \equiv \sqrt{\left(\frac{1}{\sqrt{2}}\beta \sin \gamma\right)^2 + \left(\frac{1}{\sqrt{2}}\beta \sin \gamma\right)^2 + (\beta \cos \gamma)^2} = \sqrt{a_{2-2}^2 + a_{2+2}^2 + a_{20}^2} \quad (8) \quad \text{Results: } a_0 \stackrel{\text{def}}{=} \beta \cos \gamma = a_{20}$$

$$a_2 \stackrel{\text{def}}{=} \beta \sin \gamma \equiv \sqrt{\left(\frac{1}{\sqrt{2}}\beta \sin \gamma\right)^2 + \left(\frac{1}{\sqrt{2}}\beta \sin \gamma\right)^2} = \sqrt{a_{2-2}^2 + a_{2+2}^2} \quad (9)$$

$a_0, a_2$  are the cartesian intrinsic axial symmetry quadrupolar deformation coordinates used in this paper. Represented in this intrinsic referential, the multiplicative operator potential  $\hat{V}(\hat{\alpha})$  has the expression ([4]):

$$\hat{V}(\beta, \gamma) = C_2 \frac{1}{\sqrt{5}} \beta^2 - C_3 \sqrt{\frac{2}{35}} \beta^3 \cos 3\gamma + C_4 \frac{1}{5} \beta^4 - C_5 \sqrt{\frac{2}{175}} \beta^5 \cos 3\gamma + C_6 \frac{2}{35} \beta^6 (\cos 3\gamma)^2 + D_6 \frac{1}{5\sqrt{5}} \beta^6 \quad (10)$$

and may present or not the axial symmetry. This is the analytical expression of the Potential-Energy-Surface (PES) employed in the present study. The real six numbers  $C_2, C_3, C_4, C_5, C_6, D_6$ , are the PES parameters which, together with the kinetic-energy parameters  $B_2$  and  $P_3$ , are the GCM parameters used in our study.

Using the spherical harmonics in the intrinsic system in polar angle, for radius (1) of the nucleus results Refs.([9],[12]):

$R(\theta, \varphi) = R_0(1 + \delta R_k)$ , with  $\delta R_k$  being the increments of the nucleus' radius along the three semi-axes  $\hat{a}_{2\mu}$  in the intrinsic referential ([1]):

$$\delta R_\mu = R_0 \sqrt{\frac{5}{4\pi}} \beta \cos \left( \gamma - \frac{2\pi}{3} \mu \right), \quad \text{with } \mu = 1, 2, 3 \quad (9)$$

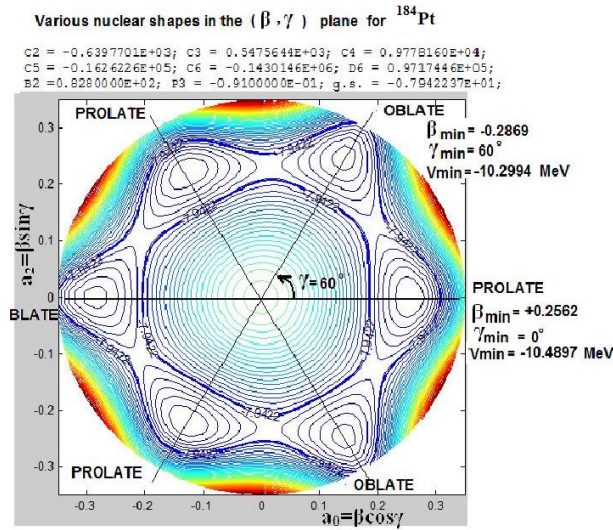


Fig.1 Various axial symmetries in the  $(\beta, \gamma)$  intrinsic deformation plane for Potential-Energy-Surface contour plot of  $^{184}\text{Pt}$  nucleus, calculated with the GCM parameters obtained in this work

From (9), results for a nucleus various nuclear shape in  $(\beta, \gamma)$  intrinsic deformation plane for axial symmetric deformation:

- 1)  $\gamma$  equal with  $0^\circ, 120^\circ, 240^\circ$  results axial prolate spheroids ( $\beta > 0$ ).
- 2)  $\gamma$  equal with  $180^\circ, 300^\circ, 60^\circ$  results axial oblate spheroids ( $\beta < 0$ ).

For  $\gamma$  not multiple of  $60^\circ$  results non axial, but triaxial symmetry shapes ([9]).

In Fig.1 it is shown an exemplification for axial symmetries in  $(\beta, \gamma)$  intrinsic deformation plane present in the potential of  $^{184}\text{Pt}$  nucleus established with the GCM parameters obtained in the present study.

### 3. GCM code description

The GCM code employed in the present work has been developed in Frankfurt by D. Troltenier, J.A. Maruhn, and P.O. Hess and published in [1]. In this code, the eigenstates of Hamiltonian (4) are calculated by diagonalization in the basic functions of the five dimensional harmonic oscillator as described in ([1], [12]).

For calculating matrix elements of the quadrupole moments and transition probabilities it is used the quadrupole operator, whose expression in the laboratory-fixed pricipale-axes is:

$$\hat{Q}_{2m} = \frac{3Z}{4\pi} R_0^2 (\hat{a}_{2m}^* - \frac{10}{\sqrt{70\pi}} [\hat{a} \times \hat{a}]_{2m}^*), \quad (11)$$

Transforming  $\hat{Q}_{2m}$  in the  $\hat{Q}_{2\mu}$  given in intrinsic  $a_{2\mu}$  referential, the quadrupole moment of excited states becomes:

$$Q(\delta, L) = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} L & 0 & L \\ -L & 0 & L \end{pmatrix} \langle \delta, L M = L | \hat{Q}_2 | \delta, L M = L \rangle,$$

where the angular momentum dependence is expressed through the Wigner 3J-symbol,  $|\delta, L M\rangle$  is a collective state with angular momentum L, projection M, and  $\delta$  summarizing all other quantum numbers,  $\hat{Q}_2$  -intrinsic quadrupol operator, ([1],[12]).The reduced transition probability is:

$$B(E2, \delta_i, L_i \rightarrow \delta_f, L_f) = \frac{1}{2L_i + 1} \sum_{M_i, M_f, \mu} |\langle \delta_f, L_f M_f | \hat{Q}_{2\mu} | \delta_i, L_i M_i \rangle|^2$$

$$= \frac{2L_f + 1}{2L_i + 1} |\langle \delta_f, L_f || \hat{Q}_2 || \delta_i, L_i \rangle|^2 = B(E2, L_i \rightarrow L_f),$$

where  $\langle \delta_f, L_f || \hat{Q}_2 || \delta_i, L_i \rangle$  is the reduced matrix element of the quadrupole operator between "i" and "f" states. The GCM code calculates only the upwards transition probabilities [1] (from smaller to larger angular momentum L transition); All experimental data has been taken from Ref.[10] where are given also the experimental adopted upwards transition probabilities and has been adopted in the present work. The usual downwards formula transformation for reduced transition probabilities is:

$$B(E2, L_f \rightarrow L_i) = \frac{2L_i + 1}{2L_f + 1} B(E2, L_i \rightarrow L_f)$$

#### 4. GCM calculations along the N=106 isotonic chain

For the N=106 isotonic chain considered in the present work, the GCM parameters are calculated separately for each nucleus. The GCM code performs an automatic fitting of a set of experimental data by adjusting the eight GCM parameters of Hamiltonian (4).

Table 1-a

**The Potential-Energy-Surface parameters for even-even N=106 isotonic chain nuclei obtained in the present study**

**PES parameters for even-even N=106 isotonic chain nuclei**

	C2 [MeV]	C3 [MeV]	C4 [MeV]	C5 [MeV]	C6 [MeV]	D6 [MeV]
176yb	-475.7760	562.2006	6040.9880	-12113.9600	-107940.4000	60390.4200
178hf	-478.4294	561.2617	6053.3420	-12122.9600	-107503.3000	60545.7500
180W	-493.1937	558.2894	6111.5780	-12174.5200	-104691.1000	61455.8200
182Os	-531.7013	617.8264	7360.2050	-15068.4000	-120823.7000	68460.8300
184pt	-639.7700	547.5644	9778.1600	-16262.2600	-143014.6000	97174.4600
186hg	-667.9884	539.8515	9640.4280	-16033.1900	-141000.1000	95805.7000
188pb	-9012.6180	24278.7600	1699287.0000	-10940150.0000	-347290900.0000	190025700.0000
190Po	-8896.6900	16619.1500	1633871.0000	-10689190.0000	-320687700.0000	185695955.0000

Table 1-b

The kinetic-energy parameters  $B_2$ ,  $P_3$ , and the absolute values of the ground state (g. s.) energy obtained in the present study, (the last value almost equal with the experimental BE/A for each nucleus, last column) for even-even  $N=106$  isotonic chain

**Kinetic-energy parameters for even-even  $N=106$  isotonic chain nuclei**

	$B_2 [10^{-42} \text{ MeV s}^2]$	$p_3 [10^{-42} \text{ MeV}^{-1} \text{ s}^{-2}]$	g.s. [MeV] (ABSOLUTE VALUE)	EXPERIMENTAL BE/A [MeV]
176yb	110.0423	-0.04976855	-8.064563	8.064075 (13)
178hf	99.1140	-0.05876855	-8.049599	8.049456 (11)
180W	89.0523	-0.04976855	-8.025908	8.025456 (11)
182Os	84.5000	-0.08409269	-7.989213	7.98973 (12)
184pt	82.8000	-0.0910000	-7.942237	7.94260 (8)
186hg	34.54529	-0.2354907	-7.888495	7.88826 (6)
188pb	167.5362	-0.1617555	-7.824681	7.82484 (6)
190Po	325.5313	-0.07878919	-7.749831	7.74946 (7)

The least-square fitting procedure has been used in the GCM code, with a optimal agreement of some experimental data (level energies,  $B(E2)$ , electric quadrupole moment). For the PES' parameters calculation, the fitting routine was completely computerized by GCM code [1]. The fit of the following available experimental data has been done: energy levels with  $J \leq 6^+$ , electric quadrupole moments of the  $2_1$  states and several  $B(E2)$  values if experimentally known. It is started from the  $^{186}\text{Os}$  parameters given in Ref.[1]. Then, have been determined GCM parameters (PES and Kinetic-energy parameters) of  $^{182}\text{Os}$ , and so on. For fitting procedure in GCM code, it has found the experimental data for the following nuclei in the  $N=106$  isotonic chain:  $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$ ,  $^{184}\text{Pt}$ ,  $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ , and  $^{190}\text{Po}$ . Then, the PES and the kinetic-energy parameters for these nuclei have been determined and are reported in Tables 1-a,1-b. With the eight determined GCM parameters, the GCM code calculates the theoretical model observables: the low-lying energy levels,  $B(E2)$  values, absolute value of ground state, and quadrupole moments for each nucleus.

In Fig. 2-a it is presented a comparison along  $N=106$  isotonic chain of the ground state band (gsb) experimental energy levels with GCM calculations. A spin-parity limit  $10^+$  has been considered. For nuclei where nuclear collectivity is well established ( $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ) the precision of the levels energy position obtained with GCM calculation is better than 6% in comparison with the experiment, for all states in the g. s. bands. For  $^{180}\text{W}$  nucleus this precision is even better than 1.8%. For the nuclei near closed proton shells the quality of the fit is less precise, for example in  $^{186}\text{Hg}$  where the precision in describing experimental results is about 10.5%. In these nuclei, single-particle mixing in the low-lying states is expected, and relative poor description in GCM theory is obtained, since these degrees of freedom are outside this collective model.

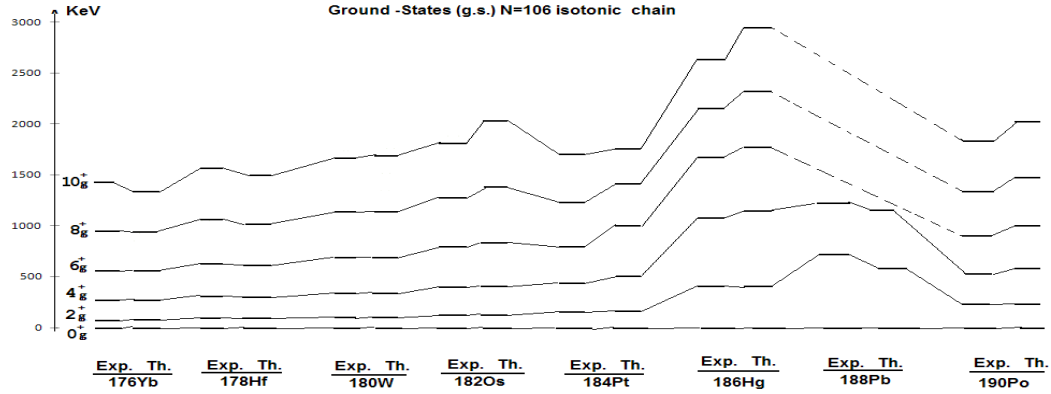


Fig. 2-a Ground state bands of even-even isotonic N=106 chain nuclei obtained in the present study, compared with experiment

In the present work, GCM predicts for all considered nuclei very good results for absolute values of ground state (g.s.) energy (Table 1-b), allowing more realistic prediction for the absolute values of the energies nuclear bound states than in other references. For example, in the case of  $^{180}\text{W}$  the g.s. absolute value obtained is equal with  $-8.0259\text{MeV}$ , much realistic value than the value  $-1.707\text{MeV}$  obtained in Ref.[2] for the same nucleus.

As a general evolution feature along the even-even N=106 isotonic chain, in Fig.2-b it is present the ratio  $E_{4_1^+}/E_{2_1^+}$  as a fingerprint of the collective structure in nuclei, even for the nucleus with closet proton shell ( $^{188}\text{Pb}$ ). The collectivity found in  $^{188}\text{Pb}$  by GCM calculation appears to be compatible with the valence p-n interaction scheme (Ref.[8]) which extends the nucleus' collectivity.

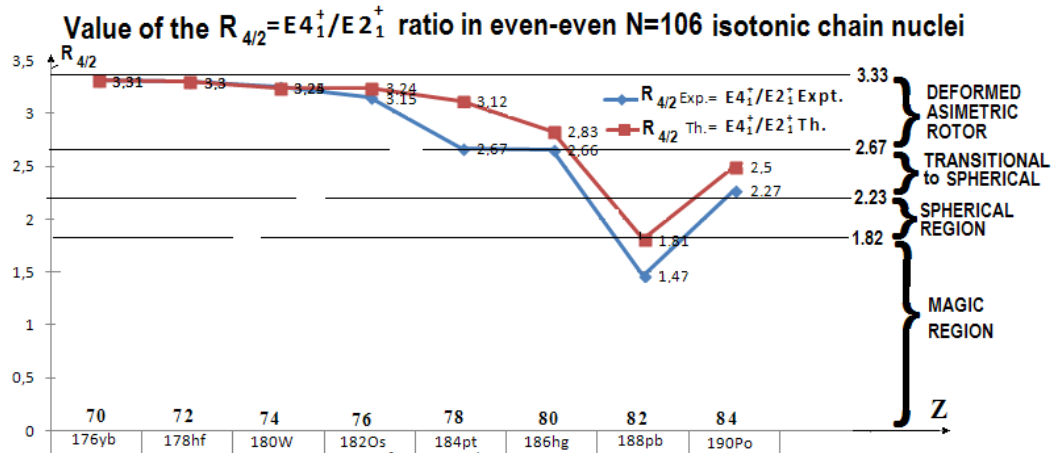


Fig.2-b Value of the  $E_{4_1^+}/E_{2_1^+}$  ratio in even-even N=106 isotonic chain nuclei obtained in the present study (Model of presentation inspired from Ref.[9])



The deformation shape changes in the even-even  $N=106$  isotonic chain nuclei, from deformed asymmetric rotor for ( $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$  with nearly ideal rotor ratio  $R_{4/2} = 3.33$ ,  $3.33 > R_{4/2} > 2.67$ ) to the transitional and spherical region with transitional point ( $R_{4/2} = 2.67$ ,  $2.67 > R_{4/2} > 2.23$  for  $^{184}\text{Pt}$ ), and ( $^{186}\text{Hg}$  to,  $^{190}\text{Po}$ ) respectively (with  $2.23 > R_{4/2} > 1.82$ , at the point  $R_{4/2} = 2.23$  the moment of inertia  $J_0$  vanishes), to magic region (for  $^{188}\text{Pb}$ ,  $R_{4/2} = 1.47$ ,  $1.82 > R_{4/2} > 1$ ) is shown.

Gamma band reproduction of the experimental energies levels it is shown in Fig. 2-c for the entire  $N=106$  isotonic chain up to the state  $5^+$ . Higher spin states are also compared with calculation in the nuclei where the experimental data are reported, as indicated in the figure. An overall agreement of the absolute values and  $\gamma$  staggering is obtained with the smooth variation of the model parameters obtained in this work. The  $\gamma$ -band heads of the lighter isotones from the chain ( $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ) has a poor description comparative with the heavier isotones from the chain ( $^{180}\text{W}$ ,  $^{182}\text{Os}$ ,  $^{184}\text{Pt}$ ,  $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ ), although the staggering in these  $\gamma$ -bands is well described.

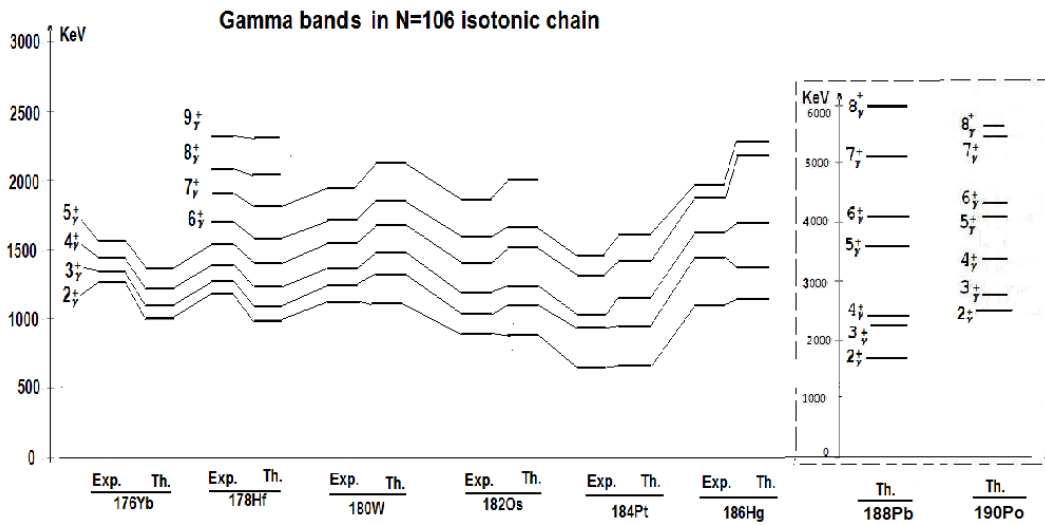


Fig. 2-c Gamma bands of even-even isotonic  $N=106$  chain nuclei obtained in the present study, compared with experiment (For  $^{188}\text{Pb}$  and  $^{190}\text{Po}$ , no experimental data has been found)

The staggering in  $\gamma$ -band as defined in Ref. [11], with  $E(I_\gamma^+)$  the excited energy of the  $\gamma$ -band level with angular momentum  $I_\gamma^+$ , is:

$$S(I_\gamma^+) = \frac{\{E[I_\gamma^+] - E[(I-1)_\gamma^+]\} - \{E[(I-1)_\gamma^+] - E[(I-2)_\gamma^+]\}}{E_{2_1^+}}$$



As shown in Ref.[11], the staggering  $S(I_\gamma^+)$  in  $\gamma$ -band is very sensitive to structural potential shape change in nuclei. For a nucleus with axial symmetry,  $S(I_\gamma^+)$  is constant with the spin  $I_\gamma^+$  variation in  $\gamma$ -band. For deformed  $\gamma$ -soft  $S(I_\gamma^+)$  oscillates between negative values for even-spin states ( $(S(4_\gamma^+), S(6_\gamma^+), S(8_\gamma^+))$ ), and positive values for odd-spin states ( $(S(5_\gamma^+), S(7_\gamma^+))$ ,  $(S(9_\gamma^+))$ ). For triaxial rigid potential,  $S(I_\gamma^+)$  oscillates in the (quasi)  $\gamma$ -band between positive values for even-spin states and negative values for odd-spin states. In the case of triaxiality, the magnitude of the  $S(I_\gamma^+)$  increases more rapidly with spin as compared with the gamma soft potential. Fig.2-e shows that no triaxial potential shapes are present in the N=106 isotonic chain, but only axial symmetry and gamma soft feature are present. Because the  $\gamma_{\text{rms}}$  is nearly the same as the  $\gamma$  value in  $\gamma$  rigid triaxial cases most predictions are similar for the two cases, but the staggering  $S(I_\gamma^+)$  can make the difference between them ([8], [11]).

#### EXPERIMENTAL AND THEORETICAL STAGGERING IN THE GAMMA-BANDS FOR EVEN-EVEN N=106 ISOTONIC CHAIN NUCLEI

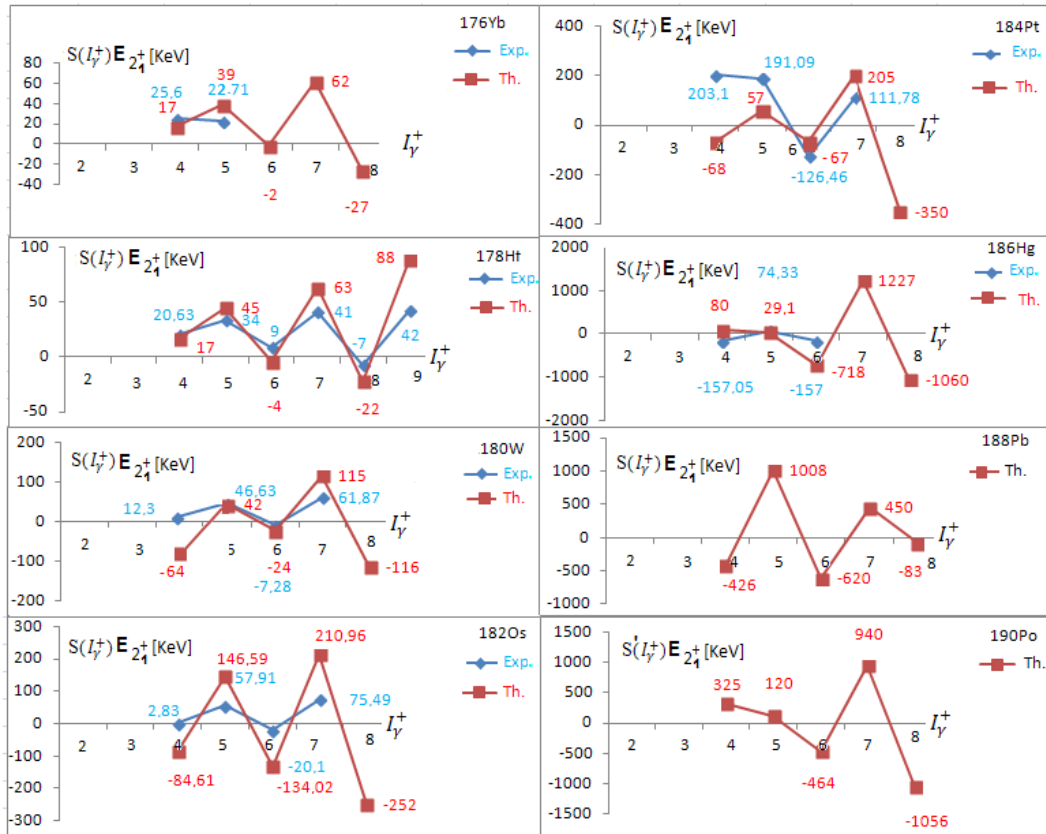


Fig.2-e Staggering  $S(I_\gamma^+)$  evolution of the even-even isotonic N=106 chain nuclei obtained in the present study, compared with experiment

The staggering  $S(I_\gamma^+)$  calculated with GCM parameters obtained (Fig.2-e) in this study, compared with experiment one for all considered nuclei changes the sign between negative values for even-spin states and positive values for odd-spin states. The situation is consistent with  $\gamma$ -soft potential, but not with the triaxial one. The experimental poor data, with  $S(3_\gamma^+) \approx S(4_\gamma^+) \approx 25$  in  $^{176}\text{Yb}$  confirms only axial symmetry in potential, but the theoretical data completes this potential picture with  $\gamma$ -soft features for larger excitation energy  $E(I_\gamma^+) > E(6_\gamma^+)$ . For the first 3 nuclei ( $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ),  $S(4_\gamma^+)$  and  $S(5_\gamma^+)$ , are positive in their values, resulting that the potential of this nuclei presents axial symmetry features, arising transition to  $\gamma$ -soft feature in potential only for larger excitation energies  $E(I_\gamma^+) \gtrsim E(5_\gamma^+)$ . For the  $^{182}\text{Os}$  nucleus, the experimental  $S(3_\gamma^+) > 0$  and  $S(4_\gamma^+) < 0$ , resulting a transition from axial symmetry potential region to a  $\gamma$ -soft potential region with a deformed  $\gamma$ -valley, at the excitation energies transition point  $E(I_\gamma^+) \approx E(4_\gamma^+)$ . The transition from axial symmetry potential region to the  $\gamma$ -soft potential region arises in all nuclei from this chain but only the excitation energy transition point differs, being lower for  $^{188}\text{Pb}$  and gradually increases with proton valence number until  $^{176}\text{Yb}$ , as can be seen in Fig.2-e. This fact is consistent with the shown potential barrier in the middle of  $\gamma$ -valley region, lower for  $^{188}\text{Pb}$ , but increasingly in height with proton valence number.

In Fig. 2-d, the experimental levels energy position of the excited  $K^\pi = 0^+$  bands are compared with GCM theory prediction. The same GCM “global” parameters from Table 1-a,b are employed in the calculation. As can be observed, the overall agreement is poorer than in g.s.b. and gamma band. But ordering of the experiment band energy-levels position is correctly predicted. Also, in most cases, in-band staggering of the energy levels is qualitatively well reproduced. It is well known that a correct understanding of the excited  $K^\pi = 0^+$  bands is difficult to be obtained in a pure collective model ([7]).

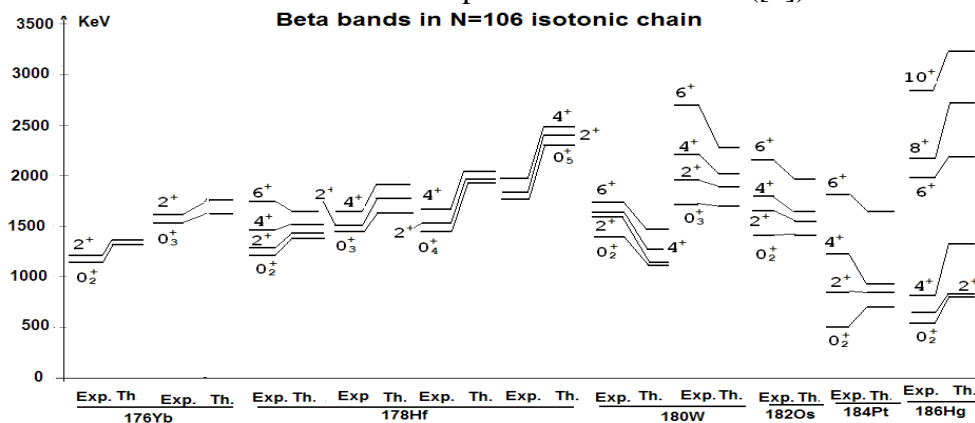


Fig. 2-d Beta bands of even-even isotonic N=106 chain nuclei obtained in the present study, compared with experiment

Mixing of the single-particle and collective degrees of freedom is a strong feature in these bands, and part of the energy levels is not included in the GCM model space.

In Fig.2-e, the  $BE2(0_1^+ \rightarrow 2_1^+)$  values evolution of the even-even isotonic N=106 chain nuclei obtained in the present study, compared with experiment is given. As expected, the smallest value of this observable is associated with the shell closure and a smooth variation is obtained from a nucleus to other. For  $^{190}\text{Po}$ , is found no experimental  $BE2(0_1^+ \rightarrow 2_1^+)$  values, but the GCM predicts for this nucleus a value of  $1.41 e^2 b^2$ .

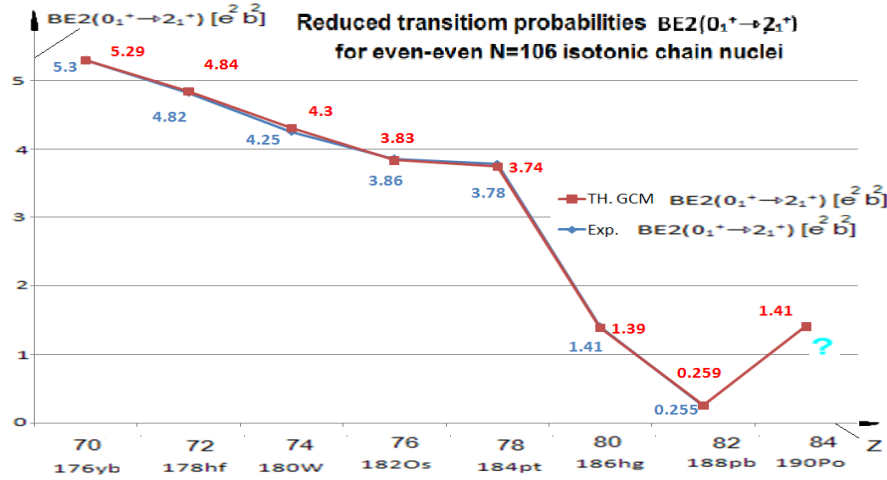


Fig.2-e The  $BE2(0_1^+ \rightarrow 2_1^+)$  evolution of the even-even isotonic N=106 chain nuclei obtained with the used fitting procedure in the present study, compared with experimental data [10].

As expected, from the reduced transition probabilities  $BE2(0_1^+ \rightarrow 2_1^+)$  in Fig.2-e and excitation energies of  $2_1^+$  states in Fig. 2-a as functions of proton number, it appears less collectivity in the closed shell proton and vicinity ( $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ ,  $^{190}\text{Po}$ ), but enhanced collectivity with the increasing of the proton valence number in the case of  $^{184}\text{Pt}$ ,  $^{182}\text{Os}$ ,  $^{180}\text{W}$ ,  $^{178}\text{Hf}$ , and  $^{176}\text{Yb}$  nuclei ([8],[9],[12]).

### 3. Conclusions

In this paper, it is applied the GCM to the even-even isotonic N=106 chain nuclei ( $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$ ,  $^{184}\text{Pt}$ ,  $^{186}\text{Hg}$ ,  $^{188}\text{Pb}$ , and  $^{190}\text{Po}$ ). The model parameters are obtained imposing for the g.s. absolute values, the experimental  $BE/A$ . The “in-band” energy levels ordering and the band-heads values are well predicted. A good agreement is obtained for the g. s. bands and  $\gamma$ -bands. Less accurate description of the bandheads position is obtained for the  $K^\pi = 0^+$  bands, but the “in-band” structures are reasonable described. For some cases where experimental information is scarce, the GCM calculations provide predictions that can guide experimental investigation.

It is shown that the geometrical shape of the potential determines very well the low-lying energy spectra, and the absolute  $B(E2)$  values are in general,

well reproduced. From the staggering  $S(I_\gamma^+)$  in  $\gamma$ -band, results for all nuclei a transition from a potential region with axial symmetry, to a potential region with  $\gamma$ -soft valley, at an excitation energy transition point smaller for  $^{188}\text{Pb}$  but increasing with the proton valence number, in agreement with the potential barrier from the middle of the  $\gamma$ -valley.

Evolution of the nuclear shape in the even-even  $N=106$  chain nuclei ( Fig.2-b) shows a transition from deformed asymmetric rotor (for  $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ ,  $^{180}\text{W}$ ,  $^{182}\text{Os}$ ) to transitional region (for  $^{184}\text{P}$ ) and then to spherical region (for  $^{186}\text{Hg}$  and  $^{190}\text{Po}$ ) and magical region (for  $^{188}\text{Pb}$ ) respectively.

By using the GCM parameters from the present work, a detailed presentation of the PES and two proton separation energies will be given in Ref.[7], for the even-even  $N=106$  isotonic chain nuclei. A detailed comparison between GCM and experimental data applied to each nucleus from the even-even  $N=106$  isotonic chain will be presented in a future work.

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