

SIMILARITY ANALYSIS OF NUMERICAL SIMULATIONS OF TURBULENT FLOW IN A HYDRO- TURBINE

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In order to the explore similarity of turbulent flow in a hydro-turbine, according to the similarity principle, the model was obtained by the prototype, of which the geometric size was reduced to 10%. The number and topological structure of grids between the model and the prototype were the same. In the case, numerical simulation of 3D turbulent flow in the mode under the rating condition was carried out by means of the LES. The quantitative values of flow parameters and distribution characteristics, such as velocity, pressure, vorticity and turbulent kinetic energy were obtained. By comparing the calculation results of the prototype and the model, it is shown that the velocity, pressure, eddy viscous fore and inertial force, vortex structure evolution, vorticity, eddy viscosity and turbulent kinetic energy of the model and the prototype have similar laws in numerical values.

Keywords: hydro-turbine; large eddy simulation; Similarity; Channel vortex.

1. Introduction

Fluid flows in a hydro-turbine, which is complex geometrically, are most complicated three-dimensional movement of revolving turbulence. It exits poor hydraulic stability in a hydro-turbine, though hydro-turbine is used widely in the hydropower plant. It is difficult to effectively guaranteed operation stability of the unit. Failure or safety accidents induced by instability occur from time to time. For example, the Pakistan Tarbela hydropower station was forced to stop operation due to the excessive vibration caused by the blade vortex in 1992 [1]. And in its enforced frequency control mode on 17 August 2009, Unit 2 at the Sayano Shushenskaya plant in Russia entered the critical zone many times, resulting in considerable vibrations in the unit [2]. Consequently the head cover of unit 2 broke away from its restrains.

To ensure that the hydraulic turbine runs normally and stably with high efficiency on its working condition, the most frequently used method is the model hydro-turbine test [3,4] concerning the energy performance, cavitation performance as well as runaway speed of the hydraulic turbine in different working conditions. Model hydro-turbine test has many advantages, such as low

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cost and convenience. Theoretically the so-called "absolute mechanical similarity" should be satisfied with the prototype in the model test. However, it is impossible. If the viscous force of the model is proportionate to that of the prototype, the viscous force of a chosen liquid medium in the model should be much less than water in the prototype. Even using the liquid medium, it brings new problems. For instance, the ratio of gravity to inertial force will be different for the model and the prototype.

Therefore, due to the limitation of similarity conditions, there is no reliable conversion method for flow field characteristics between the prototype hydro-turbine and the correlative model. The similarity rules in some aspects of flow characteristics are still unclear. At present, it never exists an perfect prediction method, nor unified standards or norms to provide useful information [5,6]. So, it is of interest to explore the similar relationship of turbulent flow in the prototype and the model.

Under such circumstances, some extensive and intensive studies [7,8] have been done by many scholars. Wu et al. [9] and Liu et al. [10] applied the RNG $k-\varepsilon$ model to analyze the similarity of pressure fluctuation between the model and the prototype hydro-turbines. Under similar conditions, the pressure fluctuation of model and prototype has the same frequency multiplication and propagation law, but there is no similar relationship in amplitude between the model and the prototype. The hydraulic pressure pulsation value of hydro-turbine can't be converted from the correlative model test. Xu et al. [11] discussed the definition and similarity of the cavitation coefficient of the original model turbine. They pointed out the existence of the similarity relationship in the incipient cavitation coefficients of the model and the prototype turbines. Most work usually focus on the similarity of unit performance such as pressure fluctuation [12-14] and cavitation [15-17]. In addition, some progress has been achieved in similarity solutions [18,19]. Dagan et al. [20] found new similarity solutions for a polydisperse spray in vortex flows. Bosschers et al. [21] got cavitating similarity solutions for the viscous cavitating vortex. However, little research concentrates on similarity analysis of vortex characteristics, without vorticity and so on. There are still some fundamental questions that remain unanswered for the case of similar relationship of turbulent flow, such as, eddy viscous fore and inertial force, and vortex structure evolution.

The study of similar relationship contributes to further understand characteristics of turbulent flow in a hydro turbine, such that more accurate predictions in performance also may be made for the prototype. Ji et al. [22] found that the distribution of welding residual stress of the runner's simulative component was the same as that of the practical runner. Those results adequately proved that the runner's simulative component established on the basis of similarity theory can substitute the practical runner to carry out simulation, and

solved the difficulties in runner simulation. Li [23] found out a high level of similarity in flow patterns in the RSI (rotor-stator interaction) region. Tan et al. [24] contrasted the force produced on the tidal turbines' rotors with different diameters and analyzed the force-related characteristics of tidal turbines based on similarity theory of hydrodynamics. The correctness of the similarity theory in tidal turbines was validated.

In this paper, a pump-turbine was taken as the prototype. Some research on the prototype has been presented. Numerical simulations of 3D turbulent flow in the prototype under different conditions were carried out with LES by using software FLUENT. In one study [25], compared with the characteristics of the vortex structure in hydraulic turbine condition, the characteristics of the vortex structure in the runner region and guide vane apparatus region with pump operating condition were basically opposite, but some common characteristics existed. The currents flowed from large space to small space, the flow was blocked, the vortex structure increased significantly. On the contrary, the currents flowed more smoothly, the change of the vortex structure was not obvious. Other research [26] focused on vortex cascade features for the prototype in hydraulic turbine condition. The present research was one of the proceedings of the previous research, but the focus was placed on the probably similar relationship of turbulent flow in the prototype and the model. In order to explore the similarity of these features of turbulent flow in a hydro-turbine, a comparison between the model and the prototype was presented.

In this paper, the geometric scale was 10. The influential elements on the calculated results include computational grid, numerical method. In order to avoid these influences, the number and topological structure of grids, numerical method for the model were the same as these of the prototype in the previous research. 3D unsteady simulation of the model with rated case was conducted with LES. The quantitative values of flow parameters and distribution characteristics were obtained. By comparison, probably similar relationship of turbulent flow in the prototype and the model was explored.

2. Computation Method

2.1 Computation Domain and Mesh

In this study we considered a Francis pump-turbine prototype which has seven blades [25, 26]. The numbers of stay vanes and guide vanes were 19 and 20 respectively. The rated rotation speed of the runner is $n = 300$ rpm, the opening of guide vanes is $\alpha = 23^\circ$ in rated power. Geometric similarity is the premise and basis of motion similarity and dynamic similarity. In this case, in order to facilitate data conversion, we chose 10 as geometric scale. According to the similarity principle,

the model was obtained by which the geometric size of the prototype was reduced to 0.1 times. Their main geometry parameters are shown in Table 1.

Table 1

The main geometry parameters of the model and the prototype(m)

Sequence number	Name	The prototype	The model
1	runner diameter	4.6875	0.46875
2	inlet diameter	3.131	0.31310
3	height of gate operating mechanism	0.6263	0.06263
4	distribution circle diameter of guide vane	5.625	0.56250
5	outlet diameter of the runner	3	0.3
6	outlet diameter	6.2194	0.62194

In order to reduce given error, the full flow passage in hydro-turbine, from the inlet to the outlet, was chosen as computational domain which is shown in Fig. 1. It includes four components, which are the spiral case, the guide vanes, the runner and the draft tube. 3D computational domain was established by Pro/E.

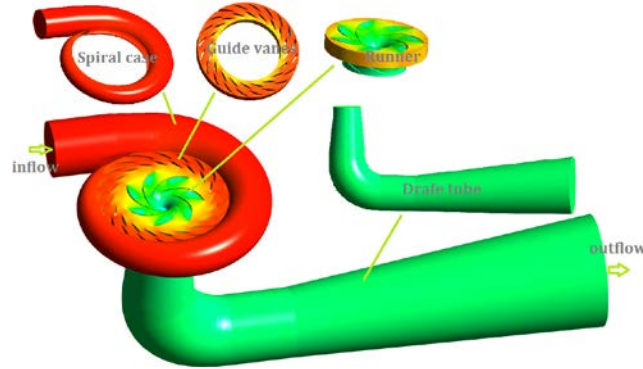


Fig. 1. Computational domain

In order to reduce the grid influence on calculations, the model and the prototype had the same grid generation method. Configuration of the computational domain is very complicate. In order to ensure the simulation accuracy and reducing computational cost, the hybrid grid was designed and generated by using HyperMesh according to the flow physics of the different regions and the characteristics of large-scale parallel computing. Structural meshes have high quality, while unstructured meshes are more fitful for the complicated geometry. The structured meshes were taken as precedence for consideration. The structured grids are applied in the relatively regular guide vanes mechanism and the spiral case, the draft tube where the flow is not changing rapidly. The complex shape of the large tongue plate and the runner is dealt with adaptable unstructured mesh. The number of computations meshes is about 20 million. The topological structure of the model and the prototype were the same. The total mesh number of the model and the prototype were all about 20 million. The local grids near to the middle surface of the guide vanes for the model are displayed in Fig. 2.

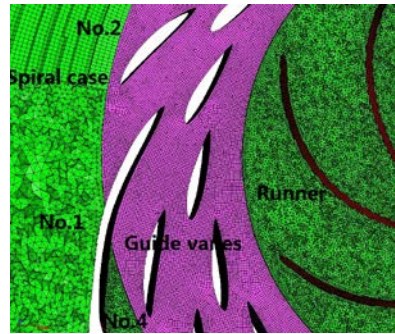


Fig. 2. Local grids near to the middle surface of the guide vanes for the model

2.2. Calculation Method and Initial Condition

The influential elements on the calculated results also include algorithm. In order to avoid the influence of algorithm, the calculation method of the model was the same as those of the prototype. The Navier-Stokes equations were discretized by the finite volume method in space and matched implicitly in time using the second order implicit scheme. SIMPLE algorithm is adopted to deal with the pressure-velocity coupling. The chosen scheme includes the second order scheme for the pressure term, the second order upwind scheme for the momentum term. One-coefficient dynamic SGS model [26] was used.

The similarity of boundary conditions and initial conditions is a sufficient condition to ensure the similarity of flow. So, in the case, the boundary conditions and the initial conditions chosen for the model and the prototype are similar. Turbulent flow is developed sufficiently in a long inlet channel. The effects of a homogenous velocity boundary can be neglected. Velocity inlet, outflow boundary conditions were applied. Steady calculation results were taken as initial conditions for unsteady calculation.

Whether in the model test or in the numerical simulation test, theoretically the so-called "absolute mechanical similarity" should be satisfied with the prototype. However, it is impossible. If the viscous force of the model is proportionate to that of the prototype, the viscous force of a chosen liquid medium in the model should be much less than water in the prototype. Even using the liquid medium, it brings new problems. For instance, the ratio of gravity to inertial force is different for the model and the prototype. So, water at 20 °C was chosen as the medium for the model and the prototype. Considering self-modelling region of the Reynolds number in the runner region, Euler criterion was selected as the model law. Three independent scales, namely, geometric scale, velocity scale and density scale, are used [27]. The scales were set 10, 3.16, 1, respectively. From the above three independent scales, it can be deduced that:

time scale

$$k_t = \frac{t_T}{t_M} = \frac{L_T / V_T}{L_M / V_M} = k_L k_V^{-1} = 3.16 \quad (1)$$

where, the subscript "T" represents the prototype turbine, the subscript "M" represents the model turbine.

angular velocity scale

$$k_w = \frac{w_T}{w_M} = \frac{V_T / L_T}{V_M / L_M} = k_V k_L^{-1} = 0.316 \quad (2)$$

Rated operation was selected for simulations. The calculated parameters of the model were obtained, as shown in Table 2.

Table 2

The Calculated parameters of the model and the prototype

Name	Inlet velocity(m/s)	The water head(m)	The rated rotation speed(rpm)
The prototype	19.28	200	300
The model	6.10	20	949

Considering the size of the calculation and the high rotate speed, the time-step size can be set to the time consumption that the runner rotates 1.5° . Namely, every rotationally periodic is divided into 240 time -steps. The convergent limits of the residuals were set to 10^{-3} .

The compute scale is large in the extreme. All simulations of the model and the prototype were carried out by means of parallel computations in the high performance computing platform, which can perform 46 billion floating-point operations per second. CPU cores and nodes chosen to parallel computing for the model were the same to that of the prototype [26]. The computation took about 200 hours.

3. Results and Similarity Analysis

In this section it was done a comparative analysis of calculation results of the model and the prototype.

3.1 Kinematic Similarity

The middle line was constructed in intermediate surface of blade channel. Divide the line into five equal parts. Along the flow direction, equal points were denoted by P1–P4 (see Figures 3). The flow structure in the channel is complex. The instantaneous velocity direction and the values proportion at the corresponding points are different. Therefore, the average velocities at the corresponding points were chosen as comparator in this section. The average velocity ratios of the prototype and the model at the corresponding points P1 to P4 are 2.87, 3.67, 3.22 and 3.17 respectively, which are very close to the speed scale 3.16. The average velocity satisfies kinematic similarity. In addition, the

maximum velocity of the prototype is 76.1m/s, which is about three times of the model machine 25.9m/s. It further verifies the kinematic similarity.

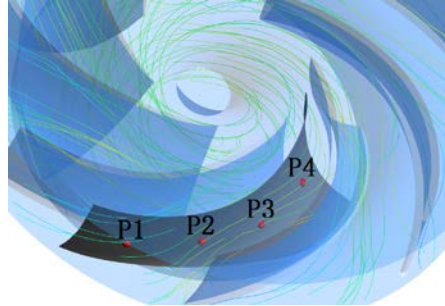


Fig. 3. Monitored points P1-P4

3.2 Dynamic Similarity

According to the literature [27], the mechanical similarity can't be fully satisfied for hydraulic turbine, sometimes even it is contradictory. Usually only the similarity of dominant forces can be satisfied, and it mainly follows the similarity of pressure and inertia forces.

a) Pressure similarity, that is, the Euler numbers should be equal for the prototype and the model. Accordingly, the pressure scale can be obtained.

$$Eu = P / \rho V^2 \quad (3)$$

$$k_p = P_T / P_M = k_v^2 = 10 \quad (4)$$

Similarly, the pressure ratios at points P1 to P4 were 9.4, 9.6, 8.9 and 9.7 respectively, which is close to the pressure scale 10. Moreover, the minimum and the maximum pressures of the prototype are -1107500 Pa and 4903850 Pa, respectively, while the minimum and the maximum pressures of the model are 476380 Pa and -96197.2 Pa, respectively. The minimum and the maximum pressure ratios are approximate to the pressure scale 10. It is thus clear that Pressure satisfies similarity.

b) The viscous force similarity, that is, the Reynolds number should be equal for the prototype and the model. In this paper, the inlet area of blade passage was chosen. Its square root was taken as the characteristic length. The Reynolds numbers of prototype at P1-P4 are 3.63×10^4 , 5.89×10^4 , 6.17×10^4 and 6.21×10^4 respectively, which are about 28.7, 36.7, 32.2 and 31.7 times of the Reynolds numbers of the model at corresponding points. Obviously, the Reynolds numbers are not equal. So, the viscous force doesn't satisfy the similarity. However, it should be noted that the Reynolds number of the internal flow in hydro-turbine is generally quite high. And the turbulent eddy viscosity is much larger than the molecular viscosity. The eddy viscosity is the controlling factor of

viscous force. So, the molecular viscous mass can be replaced by the eddy viscosity. The eddy Reynolds number based on the eddy viscosity is defined as:

$$\text{Re}_{\text{eddy}} = VL / n_{\text{eddy}} \quad (5)$$

According to this definition, the eddy Reynolds number at P1-P4 can be calculated (see Table 3). It is apparent from inspection of the table that the eddy viscosity ratios of the prototype to the model at P1-P4 are about 30, and the eddy Reynolds number are approximately equal for the prototype and the model. So, it satisfies the turbulent eddy viscosity force similarity.

Table 3

Eddy viscosity and Eddy Reynolds number for the model and the prototype

Points	P1	P2	P3
Eddy viscosity for the prototype	0.1187	0.1434	0.2613
Eddy viscosity for the model	0.0048	0.0043	0.0080

c) Inertial force similarity, that is, the Strohal number should be equal for the prototype and the model.

$$St = L / (VT) \quad (6)$$

$$St_T / St_M = k_L k_V^{-1} k_T^{-1} \quad (7)$$

Based on the above similarity rule, the Strohal number at corresponding points of the prototype and the model is basically equal. And the inertial force is similar.

To summarize, for high Reynolds number turbulence, it shows that some similar laws exist in numerical values for the prototype and the model with rated case.

3.3 Efficiency Comparison

Calculation results and some parameters for the model and the prototype are shown in table 4. The efficiency ratio of the model to the prototype is:

$$\eta_T / \eta_M = \frac{M_M \cdot n_M}{m_M \cdot H_M} / \frac{M_T \cdot n_T}{m_T \cdot H_T} = \frac{261.13352 \times 949 \times 139833.128 \times 200}{2660972.1 \times 300 \times 442.192 \times 20} = 98.17\% \quad (8)$$

where, h is efficiency, M represents torque, n is rotation speed, and m represents fluid mass.

Table 4

Calculation results and some parameters for the model and the prototype

Points	Torque of single blade (N · m)	Rotation speed (rpm)	Inlet mass (kg/s)	Water head (m)
The model	261.13352	949	442.192	20
The prototype	2660972.1	300	139833.128	200

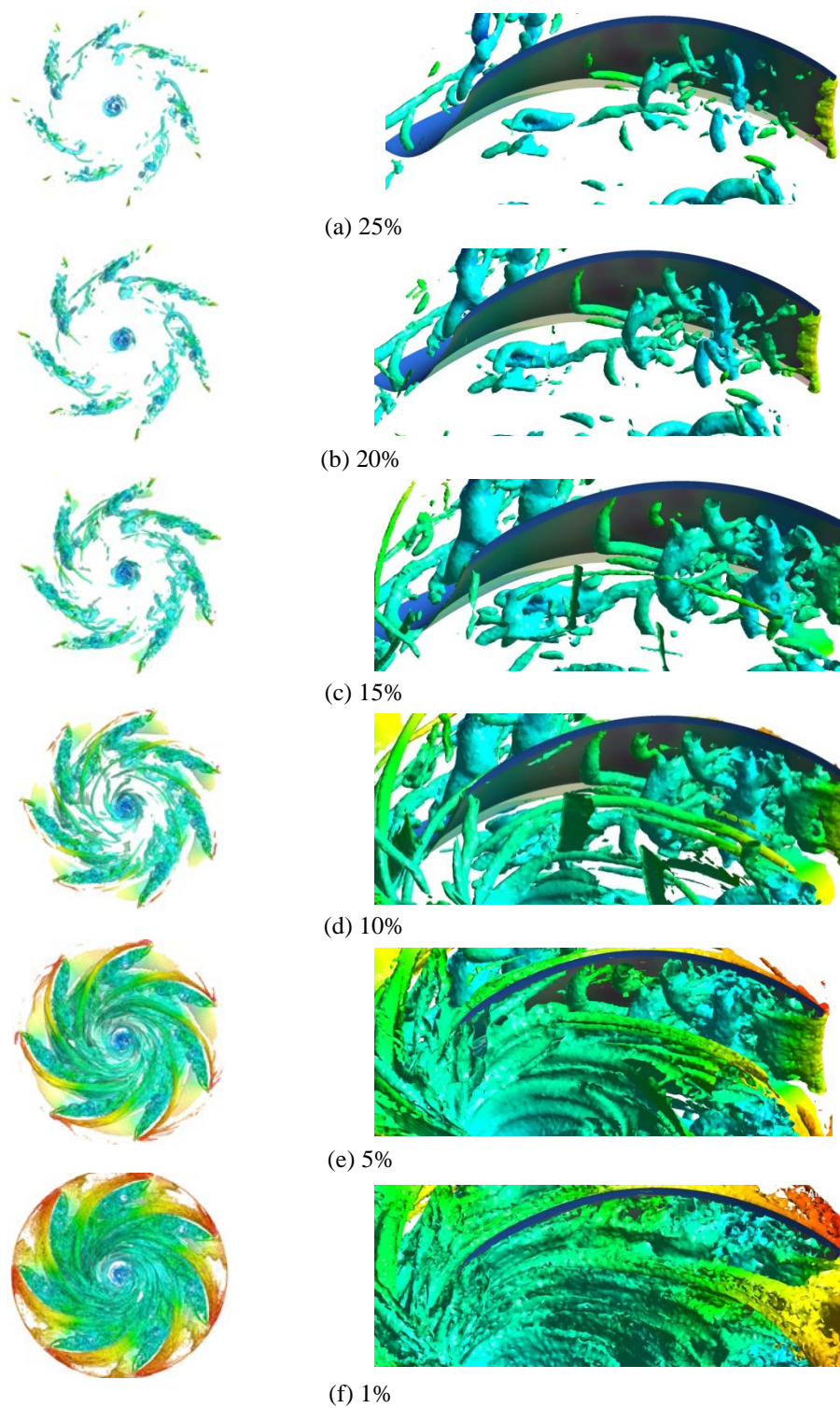


Fig. 4. Vortex structures of the model with rated case

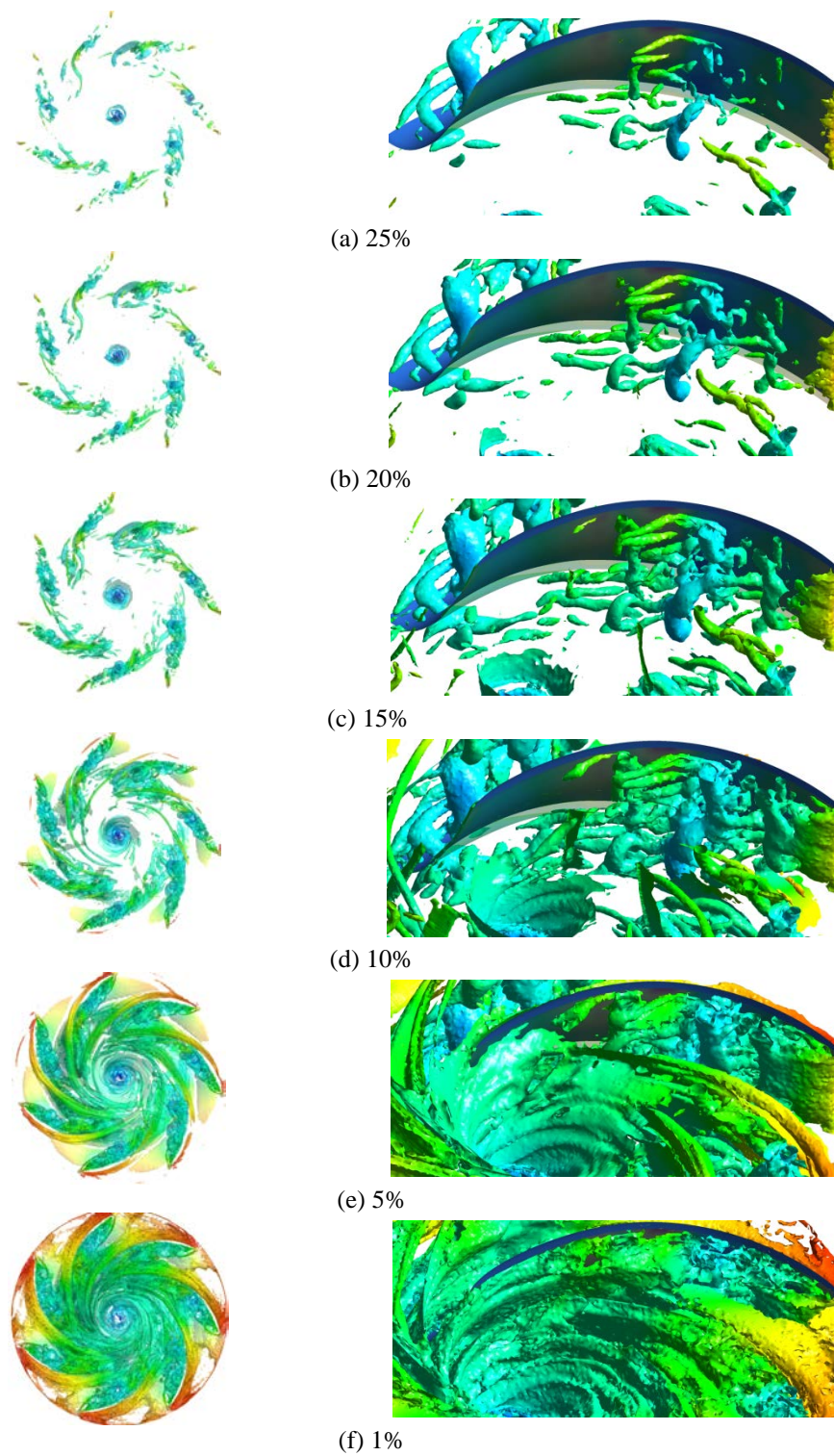


Fig. 5. Vortex structures of the prototype with rated case

The efficiency of the model is lower than that of the prototype. The engineering practice also shows that the energy characteristics of the model are always weaker than that of the prototype. It may be explained by the eddy viscous force similarity, that is, the energy dissipation of the model is always greater than that of the prototype for the same eddy Reynolds number.

4) Vortex Structure and Similarity Analysis of Vortex Characteristics

By referring to the definition of the eddy intersection ratio [28], a definition of survival volume ratio for the continuous scales eddies was given as following:

$$M(A, B) = V(B) / V(A) \quad (9)$$

where, V is the survival volume of eddies, A , B stand for continuous scales eddies, and the scale of B was much larger than that of A .

Eddies of the model and the prototype with rated case was decomposed six levels eddies. The scale levels were based on the characteristic length. The scale levels were 25%, 20%, 15%, 10%, 5%, 1%, respectively. The survival volume of the different scale levels eddies of the model and the prototype with rated case were shown in Figures 4 and 5.

From the Figures 4 and 5, the trend in vortex survival volume with scale levels eddies is the same for the model and the prototype. The survival volume of eddies increased as the scale of eddies decreased. The survival volume of the eddies of "small scale" 1% is much larger than that of the eddies of "large scale" 25%. The survival volume of the eddies of "large scale" 25% is herein relatively small and concentrated, while the survival rate of the eddies of "small scale" 1% is very large and widely spread. The eddies of "small scale" 1% fill the whole blade passage. The figures also show the spatial distribution of the eddies of different scale levels for the model is very similar to that of the prototype. But the corresponding scale vortex in the blade passage of the model is relatively thick and concentrated. In addition, it can be seen that vortex structure ranks near suction surface are more abundant.

This is mostly because of vortex breakdown. Vortex is extremely unstable in turbulent flow. Big whorls breakdown into little whorls, and little whorls break down into smaller whorls, and so it went on. The eddies are become smaller and smaller in scale level. And they are increasing in number.

The survival volume ratio for the continuous scales eddies with rated case for the model and the prototype is shown in Figure 6. As can be seen clearly from Figure 5, the survival volume increase multiples of continuous scale vortex in the model and the prototype are basically same, and the variation trends are same. The survival volume increase multiples of "large scale" to "small scale" vortices increased at first, and then decreased. The survival volume ratios of the continuous scales eddies are about between 1.2 and 1.6. The large survival volume ratios of the continuous scales eddies appear at the continuous scales

eddies of “intermediate-sized” 15% to 10%, which is closes to 1.55. Furthermore, the farther away from the peak, the smaller these ratios were. So, from a standpoint of the same survival volume ratios for the continuous scales eddies, the process of vortex evolution of the model and the prototype is similar.

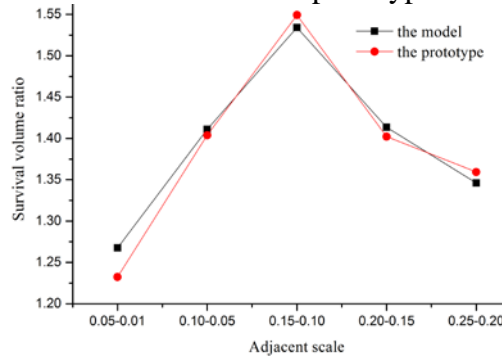


Fig. 6. Survival volume ratio for the continuous scales eddies with rated case

To achieve a clear and vivid vortex distribution, one of blade passages was divided into four equal parts along crosswise orientation, as was shown in Figure 7. The surfaces of equal parts were denoted by S1, S2, S3, S4, S5 in turn, which were along the distance from pressure surface to suction surface. According to the directions of flow, the vortex are usually broken down into streamwise vortex and crosswise vortex. It has complex direction of flow in a hydro-turbine. It is also a very hard question to decomposition of vortex. In order to facilitate the study, value of average vorticity are applied, as shown in Figure 8. From the figure, we can see that the vorticity trend of the monitored surfaces for the model is the same as that of the prototype. It is obvious to the change of the average vorticity at the monitored surfaces. The vorticity descended initially and then rose from pressure surface to suction surface for the model and the prototype. From large to small of the average vorticity, monitored surfaces in turn were S5, S1, S4, S3 and S2, which is completely consistent with the distribution of vortex structure. The average vorticity near the blade surface is higher than that of near the away blade surface. And the average vorticity near suction surface is higher than that of near pressure surface.

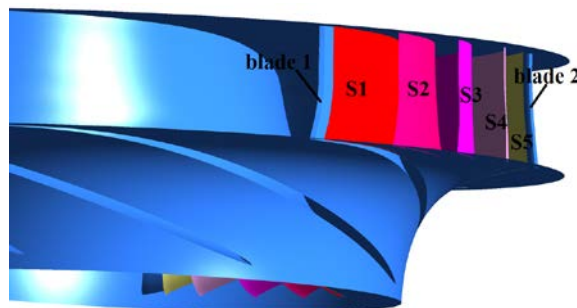


Fig. 7. Monitored surfaces

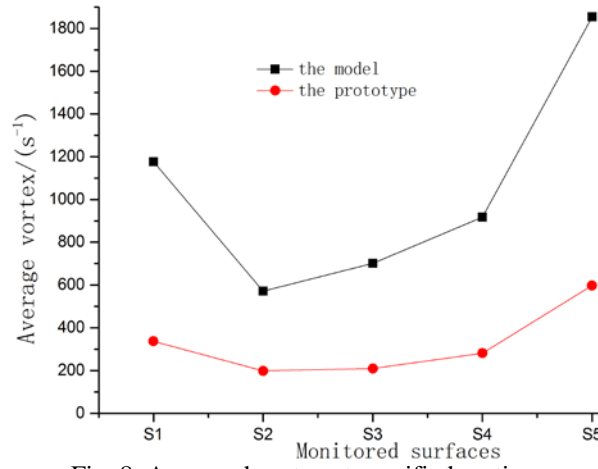


Fig. 8. Averaged vortex at specified sections

In addition to the same trend of the vorticity, the average vorticity of the prototype to the model at the corresponding monitoring surface S1-S5 has a certain regularity in the amplitude, that is, the average vorticity ratios of the prototype and the model are 0.29, 0.35, 0.30, 0.31 and 0.32 respectively, very close to the vorticity scale 0.316, which means that the prototype and the model have similarities in vorticity.

The turbulent kinetic energy:

$$k = \frac{3}{2}(VI)^2 \quad (10)$$

where, I is turbulent intensity.

$$I = 0.16 \text{Re}^{-1/8} \quad (11)$$

So, the turbulent kinetic energy scale is:

$$k_k = \frac{\frac{3}{2}(V_T I_T)^2}{\frac{3}{2}(V_M I_M)^2} = k_L^{-\frac{1}{4}} k_V^{-\frac{7}{4}} \quad (12)$$

Table 5 shows the turbulent kinetic energy of P1-P4 at each equal point of the model and the midline of the prototype blade passage. From Tables 2 and 5, it can be seen that the viscous and turbulent kinetic energy of the model and prototype turbines increase along the blade passage. Especially in second half of the blade passage, the viscous and turbulent kinetic energy of vortex become very large, which indicates that turbulent energy dissipation increases with the large increase of the small-scale vortices along the blade passage, and the obvious dissipation channel is formed. The eddy viscous ratio of the prototype to the model at the corresponding point is about 30.85, which is close to the viscous scale 31.6. The turbulent kinetic energy ratio ranges from 3.5 to 5.5, averaging

4.37, which is close to the turbulent kinetic energy scale 4.21. It shows that eddy viscous and turbulence kinetic energy satisfy similarity.

Table 5

Turbulent kinetic energy for the model and the prototype

Points	P1	P2	P3	P4
Turbulent kinetic energy for the prototype($m^2 \cdot s^{-2}$)	2.61	6.08	6.61	6.69
Turbulent kinetic energy for the model($m^2 \cdot s^{-2}$)	0.74	1.12	1.53	1.59
Turbulent kinetic energy ratio	3.53	5.44	4.32	4.21

4. Conclusions

The purpose of this article is to explore the similarity of turbulent flow in a hydro turbine. According to the similarity principle, the model was obtained by the prototype, of which the geometric size was reduced to 10%. The number and topological structure of grids for the model and the prototype were the same. Numerical simulation of 3D turbulent flow in the mode under the rating condition was carried out by means of LES. A comparison analysis of the model and the prototype was completed. The conclusions are as follows:

- 1) The velocity, pressure, eddy viscous force, inertia force, vorticity and turbulent kinetic energy are similar in the model and the prototype.
- 2) The spatial distribution of vortex at different scales in the blade passage is similar in the model and prototype. In addition, the survival volume ratios for the continuous scales eddies are largely the same, and cascade evolution characteristics of the vortices are similar in the model and prototype.

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