SUBBAND STRUCTURE AND EXCITONIC STATES IN INVERSE PARABOLIC QUANTUM WELLS UNDER LASER RADIATION

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Dependenţa stărilor excitonice de câmpul laser în gropi cuantice invers parabolice din GaAs/Al_xGa_{1-x}As este studiată teoretic folosind aproximaţia masei efective şi ţinând cont de neparabolicitatea benzii de conducţie. Se demonstrează că i) potenţialul de confinare al electronului este puternic distorsionat prin creşterea intensităţii radiaţiei laser; ii) pentru structurile iradiate tăria oscilatorului în tranzitiile excitonice se modifică, mai ales pentru structurile cu barieră centrală înaltă. Prin urmare, stările excitonice în aceste gropi cuantice pot fi modificate substanţial prin alegerea corespunzătoare a parametrilor de structură (dimensiunile gropii şi forma potenţialului) precum şi prin variaţia câmpurilor externe aplicate.

Within the framework of the effective mass approximation and taking into account the conduction-band nonparabolicity the laser field dependence of the excitonic states in GaAs/Al_xGa_{1-x}As inverse parabolic quantum wells is theoretically investigated. We found that: i) the increase of the laser intensity dramatically modifies the electron confinement potential; ii) for irradiated structures a variation of the excitonic oscillator strength is expected, more pronounced for higher inverse parabolic barriers. Therefore, the exciton states in the inverse parabolic quantum wells can be tuned in a wide range by properly tailoring the structure parameters (well size and potential shape) as well as by varying the external applied fields.

Keywords: quantum well, laser dressed potential, excitonic states, oscillator strength

1. Introduction

The nature of exciton states has a significant influence on the optical properties of semiconducting nanostructures. Quantum confinement effects arise when the size of the nanostructure is of the order of the exciton radius in the bulk semiconductor, leading to an enhancement of the exciton binding energy and an increase of the oscillator strength. More important, changes of nanostructures' electronic and optical properties may be controlled by a proper selection of the sample geometry and material parameters, as well as by applying external

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perturbations, which allows new applications in optoelectronics [1-6]. In order to understand their dependence on material and geometrical parameters, exciton states and related binding energies have been calculated in a large number of different semiconducting nanostructures by using variational or numerical techniques [7-11] and also the fractional-dimensional space formalism [12,13].

Recently, the investigations have been extended to the quantitative depiction of the laser field effects on the excitonic states in low-dimensional semiconductor heterostructures. Brandi *et al.* [14,15] have proposed a model in which the effect of the laser-semiconductor interaction is taken into account through renormalization of the electron/hole effective masses and semiconductor band gap. Based upon a nonperturbative theory which has been originally developed to describe the atomic behavior in intense high-frequency laser fields [16], the exciton binding energy in quantum well wires of GaAs/$A1_xGa_{1-x}As$ [17-20] and in quantum wells with [21,22] and without [23,24] dielectric mismatch at interfaces has been calculated.

The previous studies have been mostly concentrated on laser dressed square quantum wells. With the development of semiconductor crystal growth techniques, quantum well structures can be successfully fabricated with various shapes, such as the triangular and parabolic quantum wells, as well as asymmetric graded-gap structures. Such systems offer new degrees of freedom in achieving tailorable optical response and more reliable performances. In particular, the inverse parabolic quantum well (IPQW) has received an increasing attention because it exhibits a significant Stark effect and a much larger amplitude reduction rate of the excitonic resonance [25,26] than other QWs. Several studies on the physical properties of the GaAs/AlGaAs IPQW for different Al concentrations at the well centre have been reported [25-31].

The present work is concerned with the theoretical study of the excitonic states in GaAs/AlGaAs IPQWs under the action of non-resonant intense laser fields. We use a nonperturbative theory developed by Gavrila *et al.* [16, 32-34] to include the laser-dressing effects on both the confinement potential of the structure and the electron-hole Coulomb interaction. The oscillator strength associated with the exciton ground state is found to be significantly changeable under intense laser fields. The radiation effect on the exciton states could be experimentally confirmed by measuring the radiativ lifetime variations in the dressed structure, which is related to the oscillator strength.

The paper is organized as follows. Section 2 describes the theoretical framework. The results regarding the binding energy dependence on the well size and applied laser field are presented in Section 3. The radiation effects on the oscillator strength associated with the ground exciton state are also discussed. Finally, our conclusions are summarized in Section 4.

2. Theory

2.1. Subband levels and exciton energy in IPQWs

In GaAs the inverse parabolic shape of the confinement potential may be obtained by properly controlling the Al concentration in the epitaxial process. This is variable inside the well and constant in the barrier layers. We consider an AlGaAs IPQW with the *z*-axis along the growth direction. The valence band mixing effect may be neglected in strong quantum confinement regime. Within the single band effective-mass theory, in order to describe the electron-hole states of the well, we solve the Schrödinger equation

$$H\,\Psi(\vec{r}_e,\vec{r}_h) = E\,\Psi(\vec{r}_e,\vec{r}_h) \qquad (1)$$

with the Hamiltonian

$$H = H_e + H_h + U_{eh} \qquad (2)$$

where $H_{e(h)}$ refers to the single electron (hole) Hamiltonian and U_{eh} is the potential energy associated with the Coulomb interaction between the electron and the hole

$$U_{eh}(\rho, z_e, z_h) = -\frac{e^2}{4\pi\varepsilon}\frac{1}{\sqrt{\rho^2 + (z_e - z_h)^2}}. \qquad (3)$$

Here e is the elementary charge, ε is the static absolute permittivity and ρ is the in-plane electron-hole distance.

Because of the strong quantum confinement in IPQW structures, the anisotropy and the corrections due to the conduction band nonparabolicity are important, so that in the absence of the laser field the electron's Hamiltonian reads

$$H_e(\vec{r}_e) = \frac{p_{xe}^2 + p_{ye}^2}{2m_{\|e}} + \frac{p_{ze}^2}{2m_{\perp e}} + V_e(z_e). \qquad (4)$$

$m_{\|e}$ and $m_{\perp e}$ are, respectively, the parallel and perpendicular effective masses of the electron, which we may obtain from the first order nonparabolicity approximation [35]

$$m_{\|e} = m^*\left[1 + (2\beta + \delta)E\right]; \qquad (5a)$$

$$m_{\perp e} = m^*(1 + \beta E). \qquad (5b)$$

m^* denotes the bulk electron masse, β and δ are the nonparabolicity parameters (see Table I) and E is the electron ground state energy obtained as a solution of the single particle Schrödinger equation by using the bulk electron masses. The parallel mass determines the electron energy in the *xy*-plane whereas the perpendicular mass determines the quantized energy of the electrons in the *z*-direction.

Table 1

Material parameters used in the calculations: the bulk electron mass m^*, the band discontinuity for electron and hole, Luttinger parameters γ_1 and γ_2, and nonparabolicity parameters β and δ

Parameter	GaAs	$Al_{0.3}Ga_{0.7}As$
m^*/m_0	0.0665	0.0960
$V_{0e}\,(meV)$	228	0
$V_{0h}\,(meV)$	152	0
γ_1	6.98	5.85
γ_2	2.06	1.63
$\beta\,(eV^{-1})$	0.64	-
$\delta\,(eV^{-1})$	0.70	-

For the heavy-hole the mass in each direction is the same and corresponds to the curvature of the valence band around the Γ-point, i.e.

$$\frac{m_0}{m_h} = \gamma_1 - 2\gamma_2, \tag{6}$$

where γ_1 and γ_2 are the Luttinger parameters (see Table I), and m_0 is the free electron mass. The single particle Hamiltonian for the hole is

$$H_h(\vec{r}_h) = \frac{p_{xh}^2 + p_{yh}^2 + p_{zh}^2}{2m_h} + V_h(z_h) \tag{7}$$

The functional form of the confinement potential for the electron (hole) is given as

$$V_j(z_j) = \begin{cases} \dfrac{V_{0j}}{\sigma}\left[1 - \left(\dfrac{2z_j}{L}\right)^2\right], & |z_j| \le L/2; \\ V_{0j}, & |z_j| > L/2, \end{cases} \qquad j = e, h. \tag{8}$$

Here $\sigma = x_{\max}/x_c$, $x_{\max}$ being the Al concentration in the barriers and x_c the Al concentration at the well center, $V_{0e(h)}$ is the conduction (valence) band discontinuity for $x_{\max} = 0.3$ (Table I) and L is the width of the well.

As the in-plane electron momentum is an exact integral of motion, the eigenfunction $\Phi_e(\vec{r}_e)$ of the electron Hamiltonian (Eq. 4) is given by

$$\Phi_e(\vec{r}_e) = \exp\left[\frac{i}{\hbar}\left(p_x x_e + p_y y_e\right)\right]\varphi_e(z_e). \tag{9}$$

For the ground state $\vec{p}_\rho = (p_x, p_y) = 0$, so that $\varphi_e(z_e)$ is the eigenfunction of the Hamiltonian

$$H_{ez} = \frac{p_{ze}^2}{2m_{\perp e}} + V_e(z_e). \tag{10}$$

By assuming a strong quantization in the QW potentials, the separable trial wave function for the 1*s*-exciton state is chosen as

$$\Psi(z_e, z_h, \rho) = N\varphi_e(z_e)\varphi_h(z_h)\chi(\rho, \lambda). \tag{11}$$

Here N is the normalization constant and $\varphi_j(z_j)$ is the ground state eigenfunction of Hamiltonian H_{jz}, $j = e, h$. The 1s variational wave function is taken by the form

$$\chi(\rho, \lambda) = \exp(-\rho/\lambda). \tag{12}$$

The variational parameter λ can be obtained by minimizing the exciton energy

$$E_{exc} = \min_{\lambda} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}. \tag{13}$$

The exciton binding energy E_b may be calculated as

$$E_b = E_e + E_h - E_{exc}, \tag{14}$$

where $E_{e(h)}$ is the single-particle energy of the electron (hole).

2. 2. The laser field effect on the exciton states

We consider an electron-hole pair in an IPQW subjected to a laser field of frequency ω, whose polarization direction is parallel to the *z*-axis. Following Refs. [34,36], in the high frequency limit the electron (hole) sees a laser dressed potential given by

$$\tilde{V}_j(z_j, \alpha_{0j}) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_j(z_j + \alpha_j(t))dt, \quad j = e, h, \tag{15}$$

which is obtained by averaging the confining potential $V_j(z_j + \alpha_j(t))$ over a period of the laser. Here

$$\alpha_j(t) = \alpha_{0j} \sin(\omega t) \tag{16}$$

describes the electron (hole) motion in the laser field. α_{0j} are the laser-dressing parameters associated with the carriers

$$\alpha_{0j} = \frac{e A_0}{m_j \omega}, \tag{17}$$

where A_0 is the vector potential amplitude of the laser field. With these expressions for the dressed potentials the ground subband states are obtained from the time-independent Schrödinger equations

$$\left[-\frac{\hbar^2}{2m_j}\frac{d^2}{dz_j^2}+\widetilde{V}_j(z_j,\alpha_{0j})\right]\widetilde{\varphi}_j(z_j)=E_{0j}\,\widetilde{\varphi}_j(z_j),\quad j=e,h\,. \tag{18}$$

Eq. (18) can be numerically solved by using a finite difference method providing excellent precision.

In the Ehlotzky approximation [37] the laser dressed Coulomb potential takes the form

$$U_{eh}(\rho,z_e,z_h)=\frac{-e^2}{8\pi\varepsilon}\left\{\left[\rho^2+(z_e-z_h+\alpha_0)^2\right]^{-\frac{1}{2}}+\left[\rho^2+(z_e-z_h-\alpha_0)^2\right]^{-\frac{1}{2}}\right\}, \tag{19}$$

with $\alpha_0=\dfrac{eA_0}{\mu\omega}$ denoting the laser parameter corresponding to the exciton and $\mu=\dfrac{m_{\perp e}m_h}{m_{\perp e}+m_h}$ being the reduced exciton mass.

The oscillator strength f associated with an optical transition involving the exciton state of energy E_{exc} may be calculated as [38]

$$f=\frac{E_p}{E_{exc}}\left|\int_0^{\infty}\Psi(\rho=0,z_e=z_h=z)dz\right|^2, \tag{20}$$

where the Kane energy of GaAs is $E_p=25.7\ meV$ [39].

3. Results and discussion

For numerical calculations we used the material parameters listed in Table I and the same value of the absolute permittivity ε in the whole structure: the static value $\varepsilon_s=12.58\varepsilon_0$ for $\alpha_0=0$ and $\varepsilon_\infty=10.89\varepsilon_0$ in the high-frequency laser field (ε_0 being the electrical constant).

Figs. 1(a,b) display the variation of the ground subband energy as a function of the well width for different laser field parameters and two different Al concentrations in the center of the IPQW, for electron and heavy-hole, respectively. When the well width increases, the geometric confinements of electron and hole become weaker and therefore the ground state energies decrease with a relatively constant rate for higher inverse parabolic barriers ($\sigma=1$) and more rapidly at thinner wells for smaller barriers ($\sigma=3$).

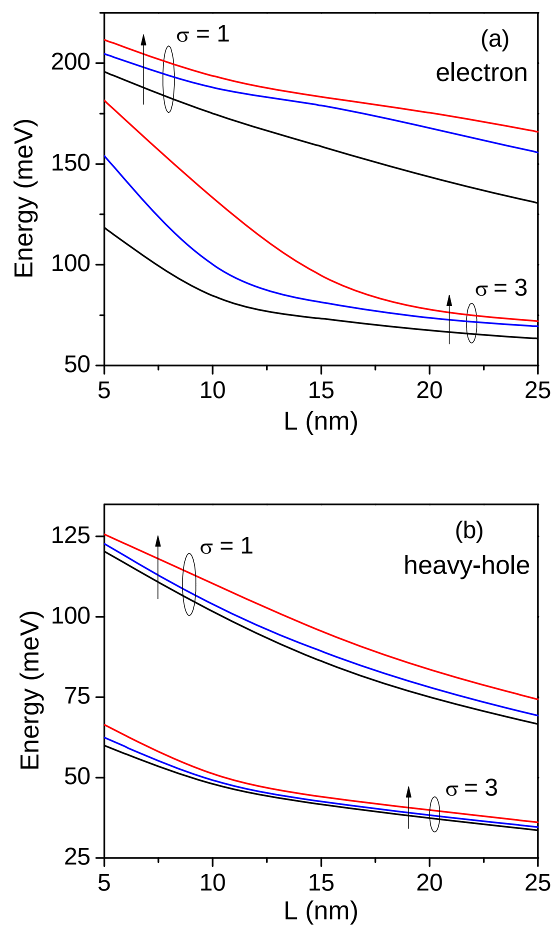

Fig. 1. The well size dependence of the ground subband energy in GaAs-$Ga_{0.67}Al_{0.33}As$ IPQWs. Two different Al concentrations at the well center and various laser intensities have been considered. The arrows indicate the laser parameter rising: $\alpha_{0e} = 0, 3, 6$ nm .

As seen in Fig. 2, for high central Al concentrations, IPQW structure behaves like double quantum well having a parabolic potential barrier. Thus, single particle energies are larger than for the other σ values since the electron and hole are confined in the two narrow well sides. Figure 1(a) shows that the electron energy presents a significant increase with the laser intensity which is more pronounced for thinner wells, when the contribution of the confinement potential to the energy becomes dominant. In contrast, the valence band states (Fig. 1(b)) are less affected by the radiation field. These results are in good agreement with previous theoretical studies on quantum wells under intense laser fields [40,41].

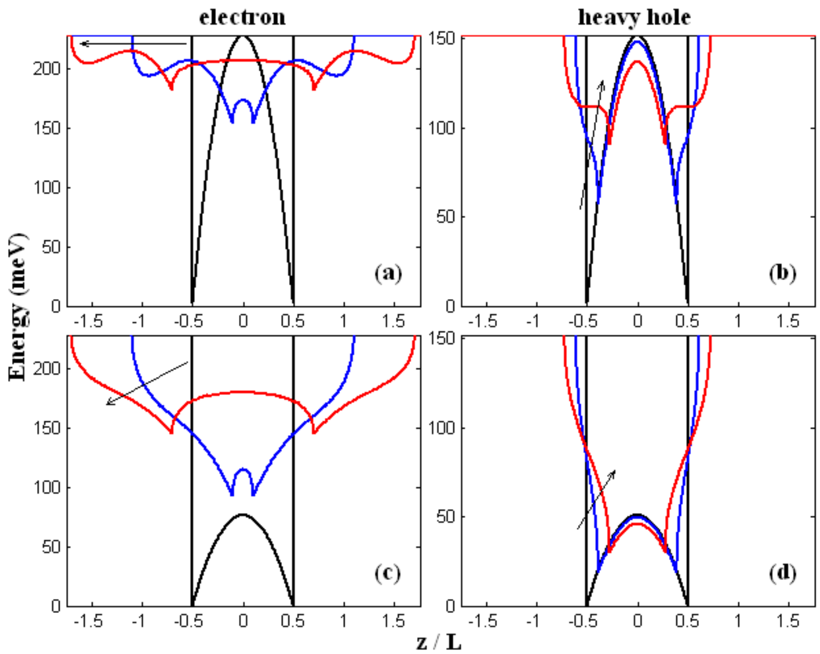

Fig. 2. The dressed confinement potential for the electron (a,c) and hole (b,d) in a GaAs/$Al_{0.33}Ga_{0.67}As$ IPQW with $L = 5$ nm, $\sigma = 1$ (a,b) and $\sigma = 3$ (c,d). The arrows indicate the laser parameter rising: $\alpha_{0e} = 0, 3, 6$ nm.

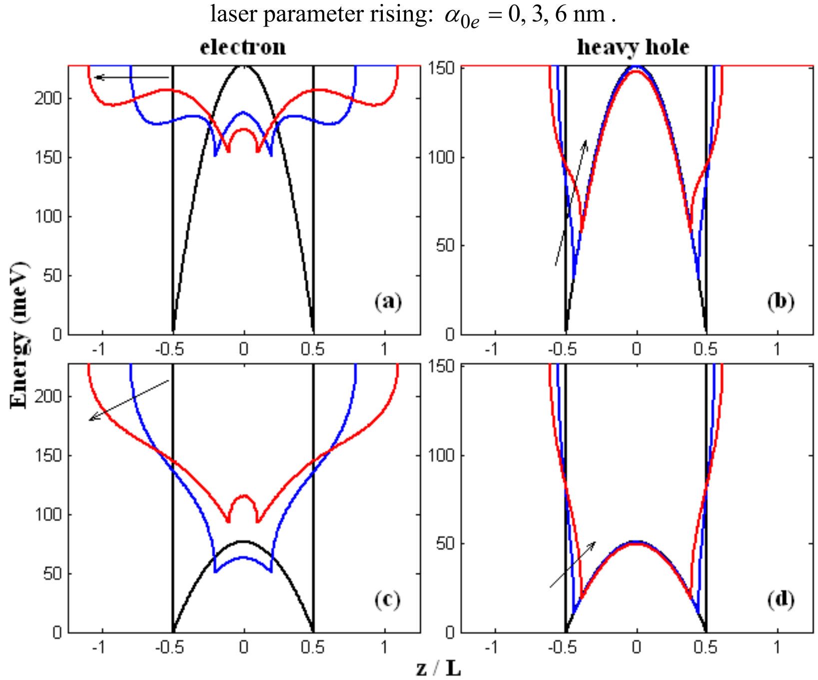

Fig. 3. The same as Fig. 2 but for $L = 10$ nm.

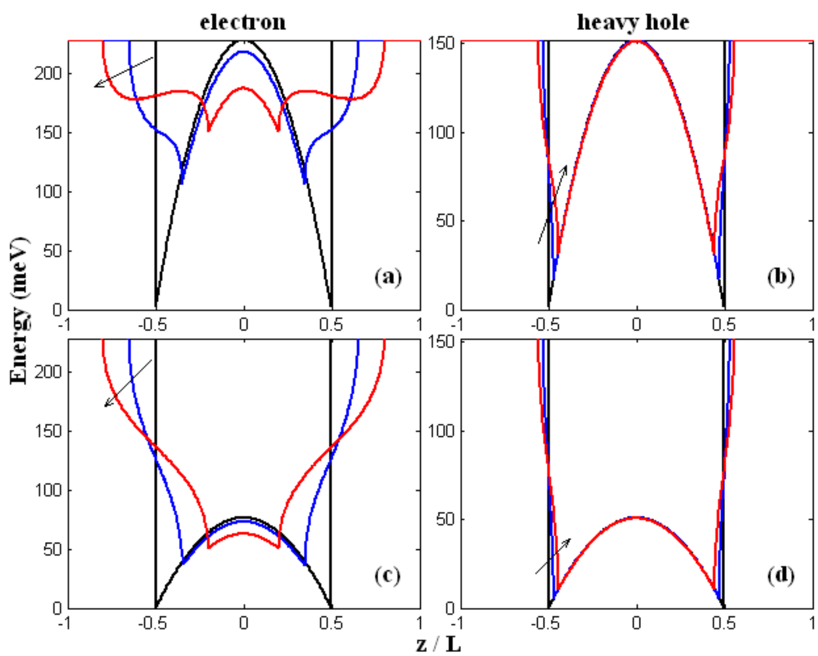

Fig. 4. The same as Fig. 2 but for $L = 20$ nm .

As expected, the energy levels' behavior is associated with a shape variation of the confinement potentials, which affects the carriers' distribution within the structures. The dressed potential profiles of the conduction and valence bands for different laser field parameters and two central Al concentrations are shown in Figs. 2-4 for different sizes of the IPQW.

It is observed that the electron confinement potential significantly changes under the laser field. The radiation reduces the effective barrier height in the central region of the well leading to a confinement potential flattening, more pronounced for very thin wells. In contrast, as $\alpha_{0j} \propto 1/m_j$, the laser intensity enhancing has a weaker effect on the hole potential. It is clear from Figs. 2-4 that a large field intensity is needed to induce an appreciable change of the valence potential profile so that, for the studied laser parameter range, only a slow increasing of the hole energy is obtained.

In order to further investigate the effect of the competition between the quantum confinement and the laser field effect, we plot the squared modulus of the ground state functions for three different laser parameters , in IPQWs of sizes $L = 5$ nm (Fig. 5), $L = 10$ nm (Fig. 6) and $L = 20$ nm (Fig. 7), for both values of σ . The probability of the electron and the heavy hole to be inside the quantum well is indicated in each figure. As expected, the well shape changes depicted in Figs. 2-4 are associated with an obvious variation in the electron cloud distribution.

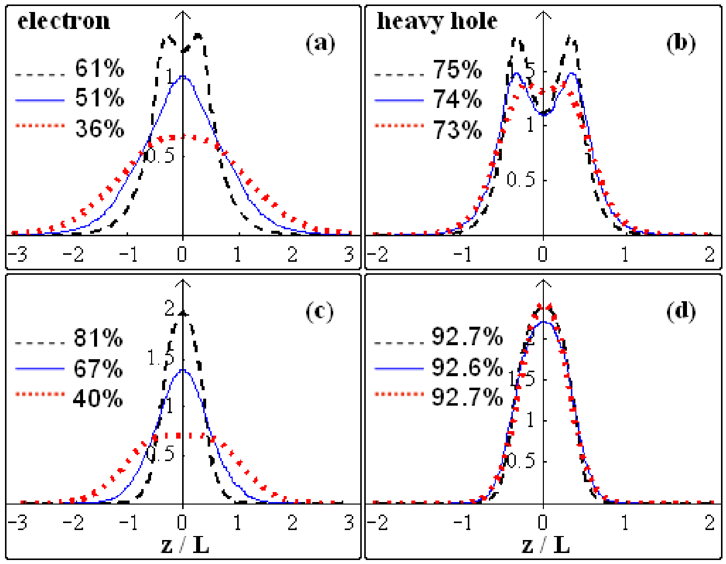

Fig. 5. The probability density for the electron (a,c) and the heavy hole (b,d) along the growth direction of an IPQW with $L = 5$ nm for $\sigma = 1$ (a,b) and $\sigma = 3$ (c,d) and three values of the laser parameter: $\alpha_{0e} = 0$ (dashed lines), $\alpha_{0e} = 3$ nm (solid lines), and $\alpha_{0e} = 6$ nm (dotted lines).

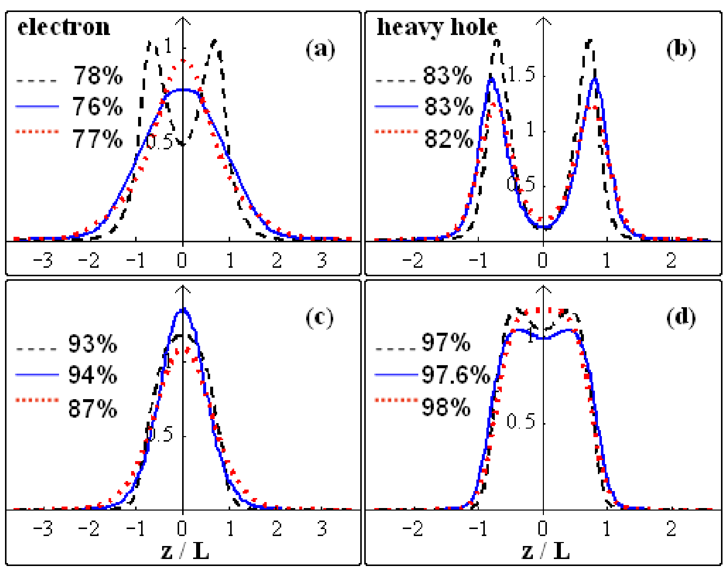

Fig. 6. The same as Fig. 5 but for $L = 10$ nm .

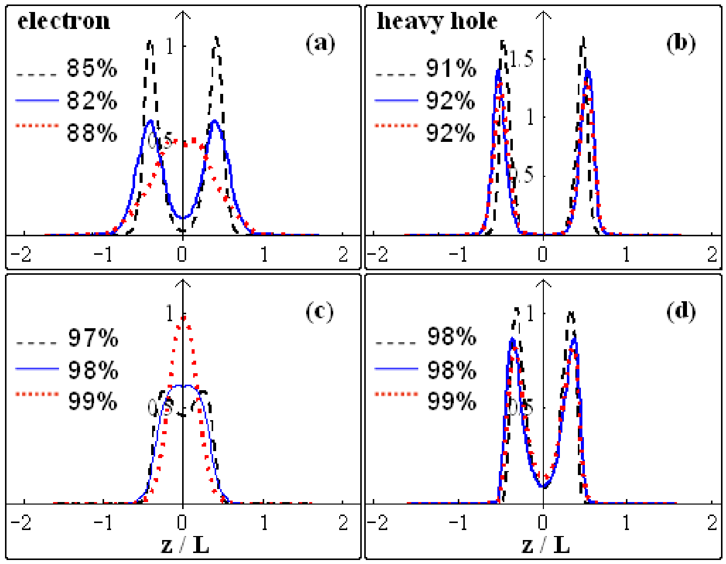

Fig. 7. The same as Fig. 5 but for $L = 20$ nm .

We observe that for $\sigma = 3$, when the deviation from the flat structure is less significant, the peak of the electron wave function is located at the well center. The effective well height decreases under the laser field, more pronouncedly for thinner QWs, with the consequent spreading of the electron wave function which easily penetrates into the potential barriers. But in the case of large deviation from the flat structure ($\sigma = 1$) we found that as α_0 increases the ground state wave function changes from a two-peaked function (with maxima around the middle of the two well sides, $z_{\pm} \cong \pm L/4$) to a single-peak function, which is practically confined in the center of the well. Thus the increase of the laser intensity is equivalent to a larger σ parameter value. This can be used to investigate these quantum systems in regimes of interest without growing many different samples. As expected, the hole wave function is less sensitive to the radiation field increase and a slightly reduced localization is observed in both σ cases.

Figure 8 shows the binding energy as a function of L for different laser intensities. The general behavior is the same for the two structures. However, for $\sigma = 1$ the exciton binding energy is smaller because the higher height of the parabolic potential barrier reduces the overlap integral magnitude. Increasing the well width under large laser intensities (see the insets) leads to lesser bound excitons due to the weaker Coulomb interaction. As expected, the energy difference between parabolic and nonparabolic cases is larger for smaller sizes.

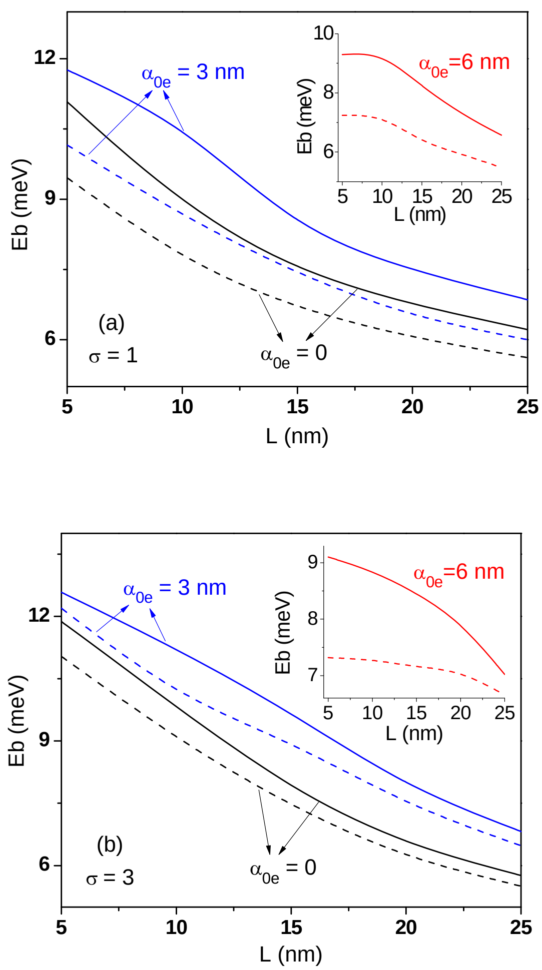

Fig. 8. The variation of the exciton binding energy as a function of the well width, for different laser field parameters. The results are with (solid lines) and without (dashed lines) the inclusion of the band nonparabolicity.

In Fig. 9 we show the oscillator strength given by Eq. (20) as a function of the well size and laser parameter. This quantity determined by the exciton energy E_{exc} and the overlap integral $\left|\langle\tilde{\varphi}_e(z_e)|\tilde{\varphi}_h(z_h)\rangle\right|^2$ decreases with the laser parameter increase due to the spreading of carriers wave functions into the barrier regions of the dressed IPQW. We can also observe that, if α_{0e} is kept unchanged, there is an optimal range of the well thickness for which the oscillator strength has a broad maximum. The peak moves to the larger well sizes as the laser field increases, more pronouncedly for $\sigma = 3$. In this case, the magnitude of the oscillator strength rapidly decreases with L, according to the wave functions behavior, but it becomes less sensitive to the laser field.

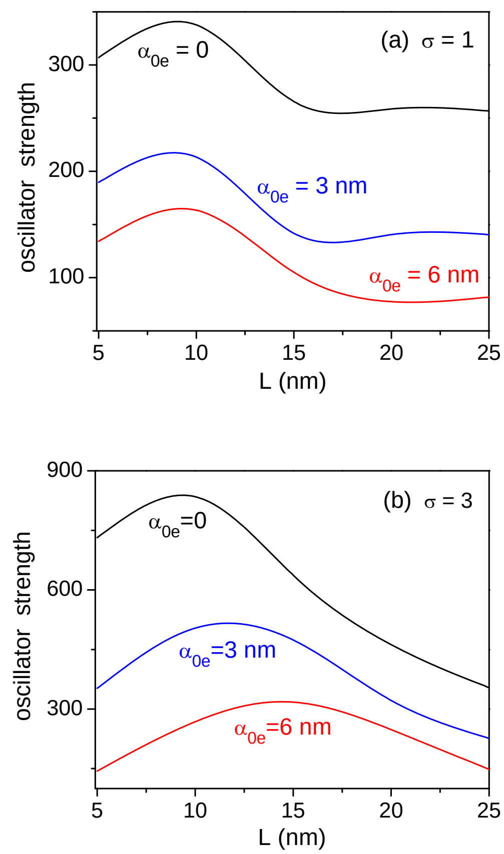

Fig. 9. The oscillator strength of the ground exciton state as a function of well size, for different laser intensities.

4. Conclusions

A variational calculation is presented for the ground-state properties of excitons confined in GaAs/AlGaAs inverse parabolic quantum wells with different Al concentrations at the well center under an intense, high frequency laser field. Single particle energy levels are obtained with a finite difference method and for various laser field parameters. The relationship between the exciton binding energy and the well size is investigated by making use of a theory which "dresses" both the confinement potentials and the Coulomb interaction between the electron and hole and taking into account the conduction-band nonparabolicity. Calculated results reveal that in these structures the exciton behavior is quite different from that in a square quantum well, and strongly dependent on the radiation intensity. For irradiated structures we predict important changes of the oscillator strength associated with the exciton ground level. These effects, as well as the large energy levels tunability in laser dressed IPQWs are expected to be useful for new electronic device applications.

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