

## FINITE ELEMENT ANALYSIS, EXPERIMENTAL VALIDATION AND OPTIMIZATION OF A ROTATION SPEED TRANSDUCER

Felicia GHEORGHE<sup>1</sup>

*Scopul acestei lucrări este evaluarea caracteristicilor de funcționare ale unui traductor pentru măsurarea turației bazată pe modele numerice în element finit. Două geometrii și diferite materiale au fost luate în considerare pentru a determina influența numărului de dinți și a sursei câmpului magnetic asupra mărimii de ieșire a traductorului. Elementele de noutate constau în prezentarea unor rezultate experimentale pentru validarea modelelor numerice și determinarea configurație geometrice a traductorului prin asocierea modelelor în element finit și a unor algoritmi de optimizare.*

*The goal of this paper is the evaluation of operation characteristics of a rotation speed transducer, based on numerical models in finite element. In order to determine the influence of the number of teeth and the influence of the magnetic field source on the transducer's output quantity two geometries and different materials were considered. The innovations consist in the presentation of experimental results to validate numerical models and the determination of the geometric configuration of the transducer by associating finite element models and optimization algorithms.*

**Keywords:** rotation speed transducer, finite element model, optimal design

### 1. Introduction

Magnetic field transducers are widely used in many applications to control rotational speed, linear or angular position. The rotation speed transducers are used to measure the speed of car wheels, respectively rotation angles or intervals of time [1] [2]. The determination with high enough accuracy of their operation characteristics is very important for the work of different car systems receiving communicable signals from [3] [4] [5] the rotation sensors.

In combination with the information from the position sensors of angular position, the car navigation system calculates the direction and the beam of the curve to follow [6].

The numerical models of the transducer studied in this paper are used to evaluate the voltage induced in the coil for the rotation speed measuring. The

---

<sup>1</sup> PhD student, Faculty of Electrical Engineering, University POLITEHNICA of Bucharest, Romania, e-mail: felicia.gheorghe@amotion.pub.ro

optimization algorithms are based on finite element models, in order to determinate the optimal configuration of the transducer.

## 2. Electromagnetic configuration of the rotation speed transducer

The operation model of a rotation speed transducer is based on Faraday's law:

$$u = -N \frac{d\Phi(\theta)}{dt} \quad (1)$$

where  $u$  is the induced voltage in the sensor coil,  $\Phi$  is the magnetic flux in the coil depending on  $\theta$ , the angular position,  $t$  is the time and  $N$  is the number of turns of the sensor coil.

The rotation speed transducer studied in this paper consists in a mobile ferromagnetic-toothed wheel and a coil around a permanent magnet [7]. The rotation of the wheel in the proximity of the sensor coil is responsible for the variation in time of the magnetic flux generated by the permanent magnet placed inside the coil, Fig.1. The variation of the magnetic flux in time is influenced by  $n_R$ , the number of teeth of the mobile wheel:

$$\Phi(\theta) = \Phi_m \sin n_R \theta \quad (2)$$

The angular position is depending in time on the angular speed of the mobile wheel:

$$\theta = \Omega t \quad (3)$$

Introducing relations (2) and (3) in relation (1), results:

$$u = -\Omega N n_R \Phi_m \cos(n_R \Omega t) \quad (4)$$

The rms value and the frequency of the sensor output voltage, recuperated at the coil terminals, depends on the rotation speed of the mobile wheel, relation (4).

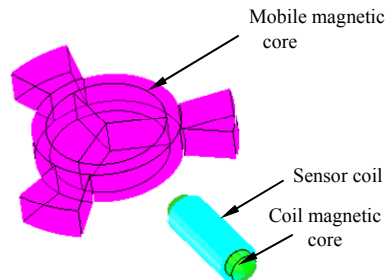


Fig.1. Components of the rotation speed transducer

In order to measure the sensor output voltage, the circuit model in Fig. 2 contains the resistance  $R = 1 \text{ M}\Omega$  of the voltmeter connected to the coil terminals.

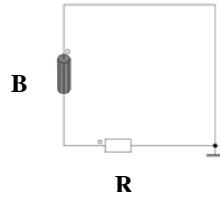


Fig. 2. The circuit model of the rotation speed transducer

### 3. Finite element 3D models and evaluation of the static characteristics

The main parameters of the magnetic coupling [8] between the fixed coil and the proximal mobile magnetic core are:

- the geometric configuration and the number of coil turns;
- the geometric configuration of the proximal magnetic core;
- the magnetic properties of the permanent magnet inside the coil and of the proximal mobile magnetic core.

The differential model of the quasi-static magnetic field expressed by magnetic vector potential  $\mathbf{A}(\mathbf{r}, t)$ , space ( $\mathbf{r}$ ) and time ( $t$ ) dependent, is:

$$\text{curl}[(1/\mu)\text{curl}\mathbf{A}(\mathbf{r}, t)] + (\partial\mathbf{A}/\partial t)/\rho = \text{curl}[(1/\mu)\mathbf{B}_r] \quad (5)$$

where  $\mu$  is the magnetic permeability,  $\rho$  is the electrical resistivity and  $B_r$  is the remanent magnetic flux density.

#### A. Mobile magnetic core with tree teeth

The nonconductive 3D computation domain of the magnetic field, Fig. 3, includes:

- the mobile wheel with three teeth. This wheel is made of nonlinear magnetic steel, with saturation flux density 1.9 T and initial relative permeability  $\mu_{ri} = 1500$ ;
- the sensor coil, placed around a permanent magnet. This magnet is SMCO made, with remanent magnetic flux density 0.4 T and relative permeability  $\mu_{ri} = 1.075$ ;
- the surrounding air, respectively the Air and Infinity regions. The last represents the transformation of the infinite region outside of the Air region, so that the computation domain is infinitely extended.

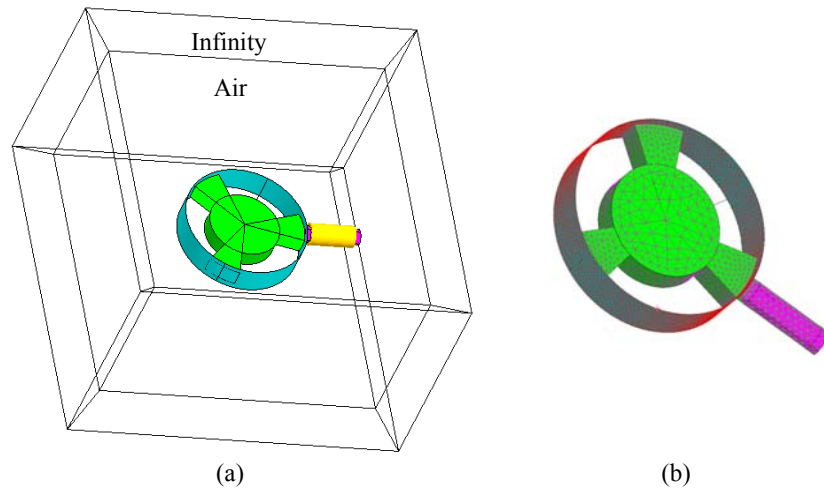


Fig.3. Regions of the computation domain (a) and a mesh zoom (b) – wheel with tree teeth

The time variation of sensor output voltage represented in Fig. 8 corresponds to the rotation speed 1,000 rpm.

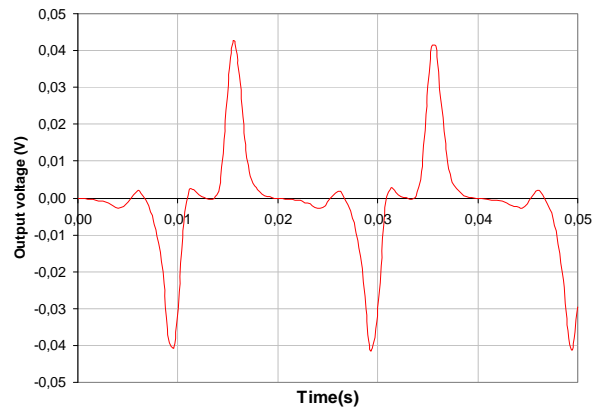


Fig. 4. Time variation of output voltage – wheel with tree ferromagnetic teeth

The dependence of the rms value of the sensor output voltage with respect to the rotation speed in Fig. 5 represents the static characteristic of the studied sensor. This voltage has practically a linear variation between 7.6 mV for 500 rpm and 45.7 mV for 3,000 rpm.

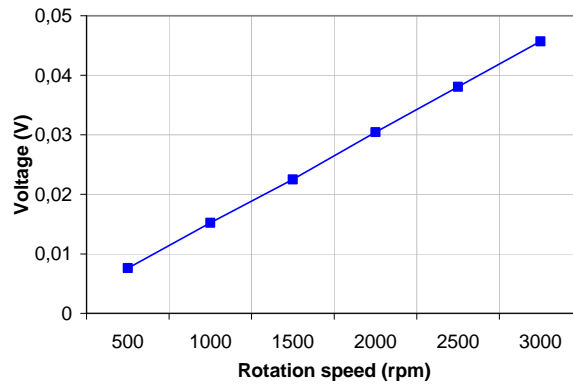


Fig.5. Static characteristic for the rotation speed transducer – wheel with three teeth

### ***B. Mobile magnetic core with four teeth***

The modification of the mobile magnetic wheel geometry influences the sensor output voltage [9]. In this section a wheel with four teeth, Fig. 6, is considered. The physical structure is the same as in the first case presented.

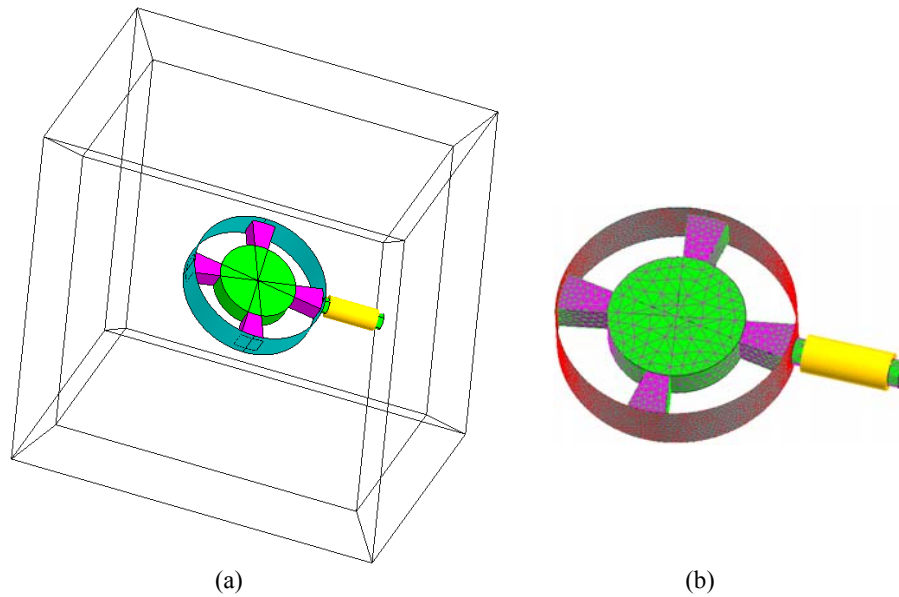


Fig.6. Regions of the computation domain (a) and a mesh zoom (b) – wheel with four teeth

The time variation of the output voltage of the four teeth speed sensor, Fig. 7, corresponding to the rotation speed of 1,000 rpm, shows practically the same peak value as for three teeth, around 40 mV. The frequency of the output voltage is 4/3 times greater.

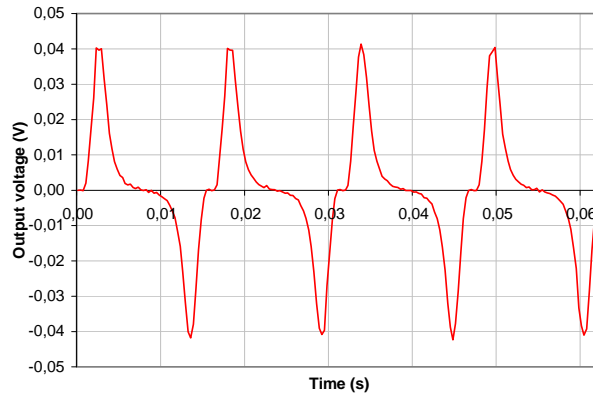


Fig. 7. Time variation of output voltage – wheel with tree ferromagnetic teeth

The static characteristic of the rotation speed transducer with four teeth is practically that presented in Fig. 5.

Another physical structure of the four teeth mobile wheel transducer was also studied. The geometry is the same, but the nonconductive 3D computation domain of the magnetic field includes a mobile wheel with four teeth. This wheel has permanent magnets teeth, SMCO made, with remanent magnetic flux density 0.4 T and relative permeability  $\mu_{ri} = 1.075$ . The hub of the transducer mobile wheel is nonlinear magnetic steel made, with saturation flux density 1.9 T and initial relative permeability  $\mu_{ri} = 1500$ . The sensor coil is placed around a magnetic core, the same nonlinear magnetic steel made as the hub.

The time variation of the output voltage, Fig. 12, of this last transducer corresponds to the rotation speed 1,000 rpm. The increase of the rms value in comparison with the first physical structure, from 15.2 mV to 317.3 mV, is a very important advantage.

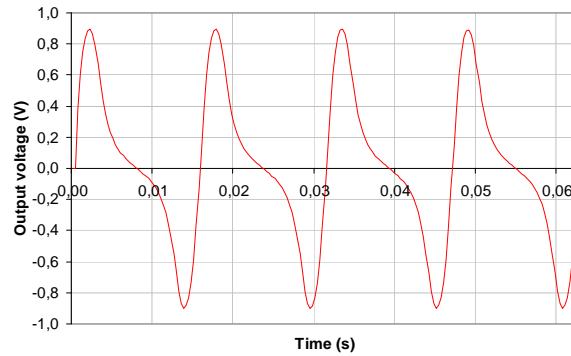


Fig. 8. Time variation of output voltage – wheel with three permanent magnet teeth

As waited, the static characteristic, Fig. 9, reflects the increase of the output voltage when the rotation speed increases. From 500 rpm to 3,000 rpm the output voltage linearly increases from 157.6 mV to 913.7 mV.

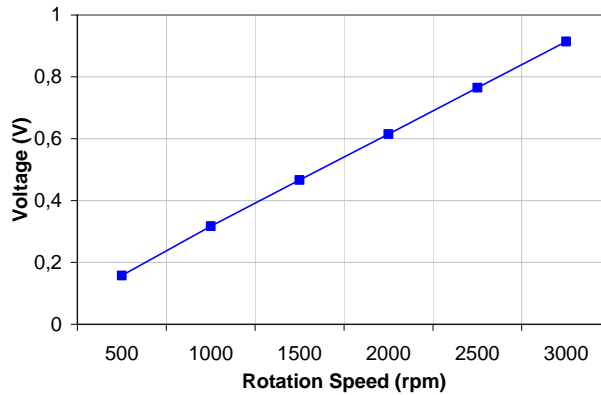


Fig.9. Static characteristic for the rotation speed transducer – wheel with four teeth

The Joule losses in the magnetic mobile wheel have low values, being around 1.5 mW.

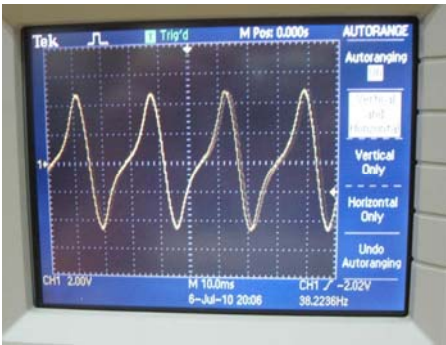
#### 4. The comparison between the finite element results and experimental results

Fig. 9 shows the sensor coil and the mobile wheel on the shaft of the motor with variable rotation speed. The configuration of the mobile wheel is with four teeth. This electrical ensemble is used for the experimental validation of the finite element results.

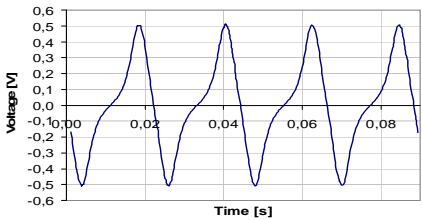


Fig.9. Electrical montage for experimental validation

The experimental and finite element results, Fig. 10, correspond to the rotation speed of 682 rpm. The output voltage in the experimental testing is 0.293 V, and the output voltage resulted from the finite element analysis is 0.286 V. The difference between the two results increases with the high values of the rotation speed, Fig. 11.



(a)



(b)

Fig.10. Experimental (a) and finite element (b) time variation of the output voltage

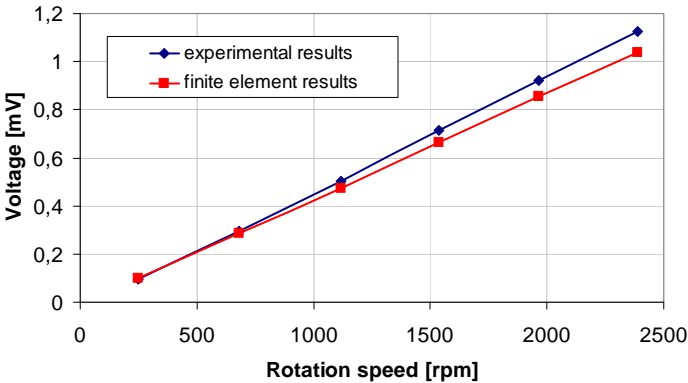


Fig.11. Experimental and finite element results

### 5. Optimization of the wheel geometry based of finite element model

The interest for the transducer optimization consists in the maximization of the output voltage. The optimization study in this section concerns in two processes: a process with one optimization parameter, respectively a process with two optimization parameters. For the one-dimensional optimization are presented two methods: an approximation algorithm and a search algorithm [10].

The starting geometry of the optimization process is that presented in Fig. 12 with the following optimization parameters: the angle from the centre of the tooth wheel  $BETA = 20^\circ$  and the hub radius  $R = 12.5$  mm. The initial output voltage is  $U = 0.0691$  V. The optimization constraints reflects the range  $[0^\circ, 60^\circ]$  for the angle  $BETA$  and the range  $[0$  mm, 21 mm] for the radius  $R$ .

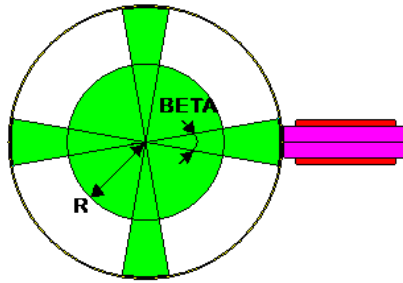


Fig.12. Parameters of geometry mobile wheel optimization

The one-dimensional process with the optimization parameter the angle, has as solution  $BETA_{OPT} = 3^\circ$ . The corresponding maximal value of the output voltage is  $U = 0.0908$  V. The same results are obtained with the both used, one-dimensional methods.

The used bidimensional process is Simplex algorithm [11], which considered the both optimization parameters. For the optimal solution, the values of the parameters are  $BETA_{OPT} = 3.21^\circ$  and  $R_{OPT} = 7.55$  mm. The corresponding maximal value of the output voltage is  $U = 0.1036$  V. The optimal geometry is presented in Fig.13.

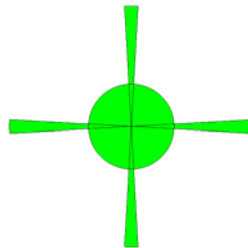


Fig.13. Optimized geometry of the transducer wheel

## 6. Conclusions

The finite element analyses of different geometric and electromagnetic configurations of the rotation speed transducer offer important information for optimal design. The equation obtained was used to analyze the output voltage. Using parameterized analysis the determination of new geometric configuration becomes very efficient. As example, the number of teeth of the transducer wheel influences the frequency of the output voltage. The making of the wheel teeth by permanent magnet determines the increasing of the output voltage by 20 times. Thus, the influence of the source of the magnet field is significantly.

The results of the experimental tests are in relatively good agreement with the finite elements results. The difference between the numerical and experimental values of the voltage slope (sensitivity) deviates as the rotation speed increases.

The optimization of the mobile magnetic wheel geometry significantly increases the transducer output voltage.

## REFERENCES

- [1] *J. E. Lenz*, A review of magnetic sensors, Proceedings of the IEEE, **vol. 6**, no. 6, pp. 973-989, June 1990
- [2] *S. Zurek, T. Meydan, A.J. Moses*, Analysis of twisting of search coil leads as a method reducing the influence of stray fields on accuracy of magnetic measurements, Sensors and Actuators, A: Physical, **vol. 142**, pp. 569 - 573, June 2007
- [3] *M. Díaz-Michelena*, Small magnetic sensors for space applications, <http://www.mdpi.com/journal/sensors>, February, 2009
- [4] *M. Vopálenský, P. Ripka, A. Platil*, Precise magnetic sensors, Sensors and Actuators, **vol. A**, pp. 38-42, 2008
- [5] *J. Brauer*, Magnetic actuators and sensors, John Wiley & Sons, New Jersey, 2007
- [6] *I.O. Bukar*, Position error compensation via a variable reluctance sensor applied to a hybrid vehicle electric machine, <http://www.mdpi.com/journal/sensors>, March, 2010
- [7] *J.G. Webster*, The Measurement Instrumentation and Sensors Handbook, New York, 1999, pp. 109-112
- [8] *J.E. Akin*, Finite Elements for Analysis and Design, Academic Press, 1994, United Kingdom
- [9] *V. Fireșteanu, F. Gheorghe*, Finite element models of angular position and rotation speed sensors, Advanced Topics in Electrical Engineering, București, România, 2008
- [10] *G. Ciuprina, D. Ioan, I. Munteanu, M. Rebican, R. Popa*, Optimizarea numerică a dispozitivelor electromagnetice (The numerical optimization of the electromagnetic devices), Editura Printech, București, 2002
- [11] *A. Antoniou, Wu-Sheng Lu*, Practical Optimization – Algorithms and Engineering Applications, Springer Science - Business Media, 2007.