

## ALGORITHM FOR TRAFFIC FLOW ASSIGNMENT AND DISTRIBUTION WITHIN A SIMULATED URBAN TRAFFIC NETWORK

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*Urban traffic control and management systems represent the answer to the users' need of increased road capacity and provide important benefits by reducing delays, stops and travel times. The present paper describes a proposed algorithm for traffic assignment and distribution within an urban network, which requires less inputs from the user when integrated within a simulation tool compared to the current applications that need traffic volumes specification for each link. The algorithm can also be used as a basis for modelling programs aiming to provide estimations of future traffic progress, to anticipate its evolution and prepare specific measures of improvement.*

**Keywords:** algorithm, directed graph, Poisson distribution, simulation, traffic assignment and distribution

### 1. Introduction

Urban traffic control and management systems are an important part of Intelligent Transportation Systems (ITS). They represent the answer to the users need of increased road network capacity and provide important benefits by reducing delays, stops and travel times. During the past decades, the development of advanced simulation programs to support ITS systems evolution has been an important focus point in the domain.

The simulation of transportation systems is a computer program that uses mathematical models to conduct experiments with traffic events on a transportation facility or system over extended periods of time [1].

The present paper aims to present a simple mathematical algorithm based on graph structures and discrete Poisson distribution for modelling the arrival rate of vehicles at the stop line of a junction at the border of an urban traffic system. Then, the traffic is assigned to the inner network based on specific characteristics and load of the links. It is basically a traffic distribution method that can be used

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within a traffic simulation application in order to minimize the data required from the user as inputs for modelling its desired network.

The optimal traffic assignment in dynamic networks with multiple origins and destinations has been an interest domain for many researchers in the last 2 decades. The first and main objective was to improve traffic flow and the user experience by minimizing travel times and delays. To achieve this purpose, authors have developed mathematical models with practical testing applications based on discrete-time dynamic systems and grouping vehicles into platoons irrespective of origin node and time of entry to network [2].

Similar approaches based on matrices and Poisson distribution to model the vehicles arrival rate have been employed to support traffic forecasting technologies and real-time control applications [3].

In the area of traffic simulations, different models have been developed for use in many ITS applications: traffic management systems, traffic control schemes and vehicle routing. For all these purposes, predictions play an important role, especially when it comes to enhancing current capabilities and increasing transportation systems efficiency [4].

Another area of research is focused on queuing models and the fact that the parameters of arrival and service rates in the system are not independent to each other except for the network's boundaries [5].

This idea is also followed in the present paper, as the proposed algorithm assumes that only the traffic injected in the network can be random and independent of other entries, but once inside, the original flows are influencing the distribution within the system.

Employed with real-time data as an input, a simulation model based on the proposed solution can be used for all the aforementioned purposes, with minimum manual adjustments or information required from the user. Current simulation applications that have proved their efficiency and accuracy through worldwide deployment (Synchro, Transyt) use specific flow values introduced by the user for every link of the traffic network.

A traffic simulation tool based on the proposed solution can also integrate a route selection option using Dijkstra algorithm for finding the minimum cost path using the attached cost matrix or the shortest path if a link length matrix is also defined.

Dijkstra's algorithm is a graph search algorithm that solves the single-source shortest path problem for a graph with nonnegative edge path costs, producing a shortest path tree [6].

The paper is structured as follows: section 2 describes the proposed traffic assignment and distribution algorithm and its operating mode, based on directed graphs structures (with cost matrices attached) and discrete Poisson distribution; next, in section 3, a concrete example of employing Poisson distribution at the

border of a traffic network is presented and concluded with the results and discussions.

## 2. Algorithm description

Some basic mathematical and traffic concepts were employed for building the proposed traffic assignment and distribution algorithm.

From a mathematical point of view, graphs and discrete Poisson distribution have been used. For what concerns the traffic theory, the network representation was described using the concepts of node and link.

A traffic node is a junction or a point along the way where a geometric property of the road is modified. A link can be interpreted either as the connection between 2 successive nodes or as every distinct situation for vehicles to form a traffic queue [7] [8].

Directed graph structures can be used for representing a traffic network from a mathematical point of view. In this way, it can easier be implemented in informatics applications and employed in traffic modelling and simulation tools [9].

This correspondence will be used next, with a few differences (which can be noticed in Figs 1 and 2):

- the links at the network's boundaries will not have a source node originating the traffic (Fig. 1);
- the conventional arcs will be replaced by 1 to 3 arcs between every 2 nodes, corresponding to the possible/permitted movements of vehicles in a specific junction (Fig. 2).

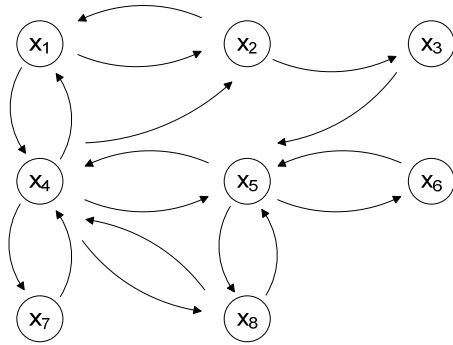


Fig. 1. Conventional graph.

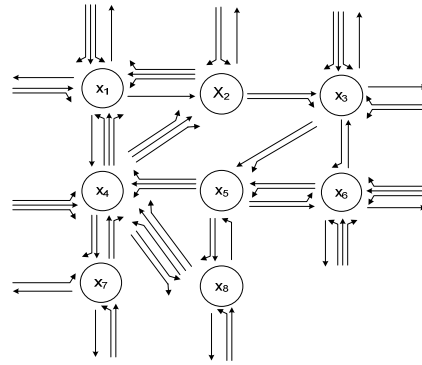


Fig. 2. Graph adjustment for using the representation in a traffic simulation

This will not affect in any way the graph's associated matrices, on the contrary, they facilitate the representation of vehicles leaving node  $i$  (entry node) and heading to node  $i$ 's neighbor, node  $k$ .

Matrix  $A$  is associated to the oriented graph and defined by the user of the simulation tool.  $A$  is a square matrix of  $n \times n$  elements with  $a_{ik}$  representing the element on line  $i$  and column  $k$ .

$$a_{ik} = \begin{cases} 1, & \text{there is a link between nodes } i \text{ and } k \\ 0, & \text{there is no link between nodes } i \text{ and } k \end{cases}$$

May  $C$  be the cost matrix asociated to each link of the network, also defined by the user when entering the input data for the simulated network.

$$c_{ik} = \begin{cases} 8, & \text{extremely high usage of the traffic link} \\ 6, & \text{very high usage of the traffic link} \\ 4, & \text{significant usage of the traffic link} \\ 3, & \text{average usage of the traffic link} \\ 2, & \text{low usage of the traffic link} \\ 1, & \text{very low usage of the traffic link} \end{cases}$$

The highest costs correspond to traffic links leading to major points of interest such as city center, turistic attactions, business parks, hypermarkets.

For the links entering the network, traffic flows will be calculated using Poisson discrete distribution.

In probability theory and statistics, the Poisson distribution is a discrete distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time elapsed since the last event [10].

The vehicles arriving at the stop line of the boundary links will be accepted between specific limits according to the time of the day (rush hours or not).

The following notations will be used in the next section:

$\tau$  = traffic signaling cycle length;

$x$  = number of vehicles arriving on one link during a traffic cycle interval (Poisson distribution parameter);

$D_{ik}$  = number of vehicles arriving at the stop line of a link which can leave the current node during the next green interval;

$V_{ik}$  = hourly traffic volume of the inbound link to node  $i$  which directs traffic towards node  $k$ ;

$v_{ik}$  = green light interval;

$\alpha$  = average rate of Poisson distribution;

$P_{ik}(x)$  = probability that a given number of  $x$  vehicles will reach the stop line of a link;

$F_{ik}(x)$  = calculated frequency for exactly  $x$  vehicles to reach the stop line of the inbound link to node  $i$  which directs traffic towards node  $k$ ;  
 $S_{ik}$  = saturation flow for the link representing the traffic flow source between nodes  $i$  and  $k$ ;  
 $l$  = average length of vehicles;  
 $d_{ik}$  = link  $ik$  length;  
 $q_{ik}$  = maximum number of vehicles that can be added to a queue before this is extended to the upstream junction.

The traffic network will be defined by the user, specifying for each node and link the following information:

- Cycle length and split times (green, yellow, all-red intervals for each approach);
- Average length of vehicles;
- Saturation flow as the maximum number of vehicles passing through the junction during the green interval;
- Link's length and speed or travel time (useful for more complex computation of delays, fuel consumption, routes, etc.);
- Link's cost (for all the links, excluding the boundary ones, represents the elements of matrix  $C$ );
- Hourly traffic volumes (only for boundary links).

It will be assumed next that the simulated network will use the same cycle length with only one green interval; the deployment of double cycles or various (equal or unequal) green intervals is usually an optimization option.

The link's capacity is the saturation flow multiplied by the proportion of the green interval during one traffic cycle [7]:

$$N_{ik} = S_{ik} \cdot \frac{v_{ik}}{\tau} \quad (1)$$

The vehicles arriving at the stop line of the boundary links will be accepted between specific limits according to the time of the day (rush hours or not):

$$x \in \begin{cases} \left[0, \frac{q_{ik}}{2}\right), & \text{if not a rush hour} \\ \left[\frac{q_{ik}}{2}, 0.9 \cdot q_{ik}\right], & \text{during rush hours} \end{cases}$$

With:

$$q_{ik} = \frac{d_{ik}}{l+1} \quad (2)$$

The probability for  $x$  vehicles to reach the stop line of one link during the cycle interval ( $\tau$ ) is:

$$P(N[t + \tau] - N[t]) = P(x) \quad (3)$$

Equal to the Poisson distribution:

$$P(x) = \frac{(\alpha\tau)^x}{x!} e^{-\alpha\tau} \quad (4)$$

The algorithm starts with evaluating the data for boundary nodes at time  $t=0$ .

Let  $i$  be a such a boundary node of the traffic network designed by the user.

Line  $i$  of matrix  $A$  is scanned in order to compute the exterior semigrade of node  $i$  ( $\delta_i^+$ ), that is the number of neighbors of node  $i$  representing destinations for traffic flow originating in node  $i$ .

In other words, if element  $a_{ik}$  of matrix  $A$  is equal to 1, it means there is one link inbound to node  $i$  directing traffic to node  $k$ .

For each neighbor  $k$  of node  $i$ , the average rate of Poisson distribution, as the total number of vehicles divided by the overall observation periods, is calculated [10]:

$$\alpha_{ik}\tau = \frac{V_{ik}\tau}{3600} \quad (5)$$

Using expression (3) for every possible value of  $x$ , the probability  $P_{ik}(x)$  of seeing exactly  $x$  vehicles arriving at the stop line is [11]:

$$P_{ik}(x) = \frac{(\alpha_{ik}\tau)^x}{x!} e^{-\alpha_{ik}\tau} = \frac{1}{x!} \cdot \left(\frac{V_{ik}\tau}{3600}\right)^x \cdot e^{-\frac{V_{ik}\tau}{3600}} \quad (6)$$

The calculated or theoretical frequency of seeing  $x$  vehicles arriving at the stop line is equal, for every value of  $x$ , to the corresponding Poisson distribution multiplied by the total number of observation periods [12]:

$$F_{ik}(x) = P_{ik}(x) \cdot \frac{3600}{\tau} \quad (7)$$

The algorithm then selects the values of  $x$  with a calculated frequency of appearance different from 0 and randomly assigns them to the boundary link on each cycle interval throughout an hour:

$$D_{ik}(t=0) = x \quad (8)$$

But, there is only a certain number of vehicles that can pass through the junction during the green interval, equal to the link capacity, therefore, if  $x > N_{ik}$  it means that  $x - N_{ik}$  vehicles will form a queue that won't be able to discharge during one green interval and will be added to the flow arriving at the stop line during the next cycle, which begins at moment  $t+z\tau$  (with  $z$  being an integer between 1 and the total number of cycles within one hour).

$$D_{ik}(t=t+z\tau) = x_{(t=t+z\tau)} + D_{ik}(t=t+(z-1)\tau) - N_{ik} \quad (9)$$

with  $z = 1, \frac{3600}{\tau}$ .

At step  $t = t + \tau$ , the breadth-first search of the network's associated graph continues.

For better understanding and visualizing, a random example shall be considered next, with the aid of the graph portion in Fig. 3.

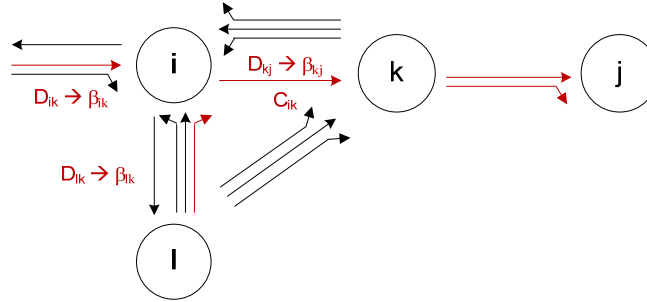


Fig. 3. Display of a small part of the simulated network.

Let  $k$  be a neighbor of node  $i$ .

Matrix  $A$  is now scanned on column  $k$  to compute the interior semigrade of node  $k$  (the number of inbound links,  $\delta_k^-$ ), but also matrix  $C$  on line  $k$ , to determine the associated cost to each link leading to node  $k$ 's neighbors.

Next, a proportionality factor is calculated for each traffic source of node  $k$ . Thus, for node  $i$  (considered the traffic source), the result should be:

$$\beta_{ik} = \begin{cases} \frac{D_{ik}}{\sum_{q=1}^n c_{kq}}, & \text{for } q \neq i \\ 0, & \text{for } q = i \end{cases} \quad (10)$$

Line  $k$  of matrix  $A$  is scanned in order to determine the number of outbound neighbors of node  $k$  (exterior semigrade,  $\delta_k^+$ ) and then the traffic flow sent from node  $k$  towards each neighbor  $j$  is calculated:

$$D_{kj} = \sum_{p=1}^n \beta_{pk} \cdot c_{kj} \cdot a_{kj} \quad (11)$$

Just like before, for the boundary links, a mandatory condition is not to exceed the pre-established saturation flow. Otherwise, the queue created at the stop line that was not able to discharge during the current cycle must be taken into account at the next step as initial flow.

$$D_{kj}(t=t+z\tau) = D_{kj}(t=t+z\tau) + D_{kj}(t=t+(z-1)\tau) - N_{kj} \quad (12)$$

Every time the algorithm computes the traffic distribution on a particular link, it has to check if the traffic queue on the link is extended to the upstream junction; if this situation occurs, the link is a critical one for the network. For this estimation, the average length of the vehicles and link length are employed, both introduced as manual inputs by the user.

If the report between the unserved vehicles multiplied by their average length (plus 1 meter of distance between every 2 consecutive cars) and the link length is bigger than 1, a correction should be applied to the program:

$$\frac{(D_{kj}(t=t+z\tau) - N_{kj})}{q_{kj}} \leq 1 \rightarrow D_{kj}(t=t+z\tau) = 0.9 \cdot q_{kj} + N_{kj} \quad (13)$$

The breadth-first search of the network's associated graph continues and the traffic distribution on links is determined and assigned until the simulation time interval ends.

### 3. Example, results and discussions

In this section, the Poisson distribution will be used for a link entering the traffic network to prove the fidelity of using this method for such traffic engineering applications [11]. The values used in the example are displayed in table 1. The cycle length to use will be  $\tau = 120$  seconds and the saturation flow on the chosen link will be equal to 50 vehicles.

The Poisson distribution uses variable  $x$  as taking consecutive integer values, but for simplification, a 5 vehicles increment will be considered, therefore the probability of 0, 5, 10... vehicles showing up at the stop line during one traffic cycle will be computed.

The observed frequency of event  $x$  or the arrival rate of  $x$  vehicles during one hour (on each cycle length) will also be assumed. The total number of vehicles reaching the stop line for each event is the product of  $x$  and the observed frequency  $F_o$ .



This theoretical example demonstrates how a Poisson distribution with the average rate  $\alpha$ , whose values have been determined using some observed data, describes pretty accurately the total volume of traffic.

The calculated values of  $x$  vehicles arriving at the stop line of a link vary with the observed frequency, but for the traffic distribution algorithm, this fluctuation is not so important as long as the hourly volume remains the same and this is the only parameter that the user needs to introduce and that is, only for the network's boundary links.

Table 1

Vehicles arriving rate at the stop line during a traffic signalling cycle

Number of vehicles arriving at the stop line ( $x$ )	Observed frequency ( $F_o$ )	Total observed number of vehicles ( $D_o$ )	Poisson distribution $P(x)$	Calculated (theoretical) frequency ( $F_c$ )	Total calculated number of vehicles ( $D_c$ )
0	0	0	0.0000	0.0	0.0
5	1	5	0.0000	0.0	0.0
10	1	10	0.0020	0.1	0.6
15	3	45	0.0529	1.6	23.8
20	6	120	0.2684	8.1	161.1
25	8	200	0.3974	11.9	298.0
30	6	180	0.2193	6.6	197.4
35	3	105	0.0531	1.6	55.8
40	2	80	0.0064	0.2	7.6
>40	0	0	0.0000	0.0	0.0
<b>Total</b>	<b>30</b>	<b>745</b>	<b>0.9996</b>	<b>30.0</b>	<b>744.3</b>

This is actually the main purpose of the proposed algorithm, to automatically distribute traffic within the network, with minimum effort required from the user.

The following characteristics can be observed in Fig. 4:

- The assumed (observed) values from table 1: the number of vehicles entering the link ( $x$ , with dark blue) and the observed frequency of each event (with red);
- The calculated values: the frequency of  $x$  vehicles arriving at the stop line during one traffic cycle (with light blue) and total number of vehicles (with green).

For better visualization, the chart is using two different scales: one for representing the values  $x$  and frequency of achieving each event  $x$  (on the left hand side, from 0 to 45) and the second for the total number of vehicles (with bigger values, from 0 to 250, on the right hand side of the chart).

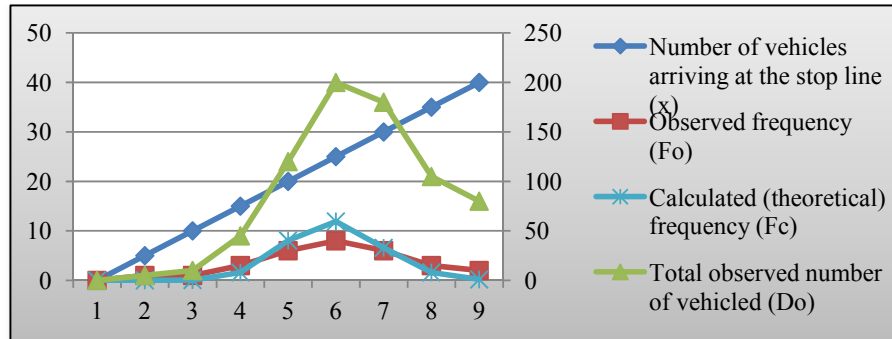


Fig. 4. Main elements for determining the total number of vehicles arriving at the stop line during 1 hour

By using a separate representation for the observed and calculated frequencies, it can be noticed in Fig. 5 how the Poisson distribution shapes the difference of level between the 2 data series, confirming the random character of the traffic distribution within the simulated network.

The last chart (Fig. 6) can be used for a better visualization the Poisson distribution of each event  $x$  or the probability of a certain number of vehicles ( $x$ ) to reach the stop line of a selected link during one traffic cycle interval. Similar to Fig. 5, 2 different scales have been used for representing the low values of the Poisson distribution ( $<1$ ) and the number of vehicles (from 0 to 45).

For the proposed algorithm, the starting point is the total observed number of vehicles introduced by the user for the links entering the network. Then, the probability of  $x$  vehicles arriving at the stop line of each boundary link is computed and the total (calculated) number of vehicles passing through the junction on each signalling cycle (the traffic flow distribution) is estimated.

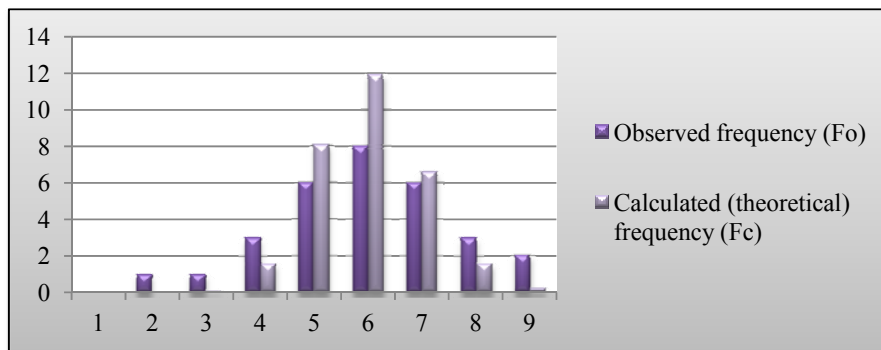


Fig. 5. The observed and calculated frequency of  $x$  vehicles arriving at the stop line during 1 hour

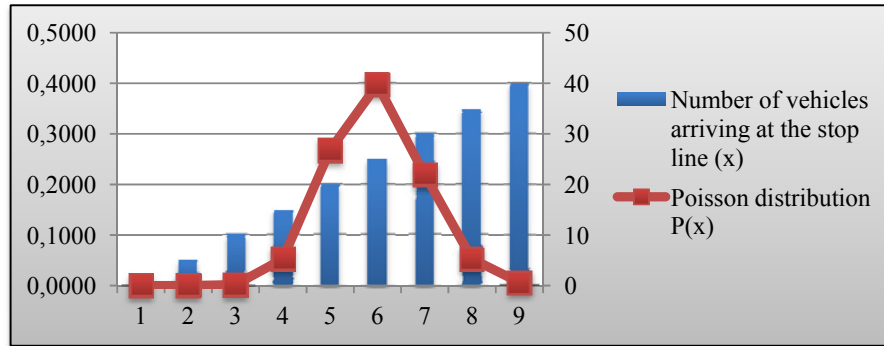


Fig. 6. The number of vehicles expected to arrive at the stop line and their Poisson distribution

#### 4. Conclusions

The present paper described a mathematical algorithm based on fundamental structures, like directed graphs with associated matrices and discrete Poisson distribution, used for vehicle assignment and distribution within a traffic network. The accuracy of the approximation of predetermined values has been demonstrated in section 3, so it can very well operate with real traffic data if connected to data acquisition equipment within the traffic management system through a compatible interface.

The present proposed algorithm represents a method of injecting and distributing traffic within an urban network, with less manual inputs from the user when integrated within a simulation tool compared to the applications available today. The algorithm can be used as a starting point for developing modelling programs offering advanced features such as queuing impact estimation and prevention, traffic forecast to anticipate its evolution and support for real time control strategies.

A traffic modelling tool based on the algorithm previously described could easily integrate a route optimization option using Dijkstra algorithm for finding the minimum cost path using matrix C or the shortest path if a link length matrix is also defined. The algorithm can also be further developed to cover the impact of real time speed and refine queuing interactions and impact at an area level.

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